28.2 Rotational symmetry

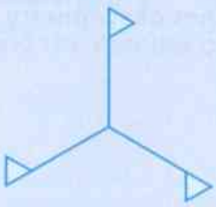
A two-dimensional shape has **rotational symmetry** if it can be rotated about a point to look exactly the same in a new position.

The **order of rotational symmetry** is the number of different positions in which the shape looks the same when it is rotated 360° about the point (that is, one complete turn).

The easiest way to find the order of rotational symmetry for any shape is to trace it and count the number of times that the shape looks the same as you turn the tracing paper through one complete turn.

Example 2

Find the order of rotational symmetry for this shape.



First, hold the tracing paper on top of the shape and trace the shape. Then rotate the tracing paper and count how many times the tracing matches the original shape in one complete turn.

You will find three different positions.

So, the order of rotational symmetry for the shape is 3.

EXERCISE 28B

CORE

1 Copy these shapes and write below each one the order of rotational symmetry. If it will help you, use tracing paper.

a

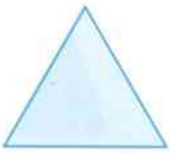
Square

b

Rectangle

c

Parallelogram

d

Equilateral triangle

e

Regular hexagon

Advice and Tips

Remember a shape with rotational symmetry of order 1 has no rotational symmetry.

2 Find the order of rotational symmetry for each of these shapes.

a


b


c

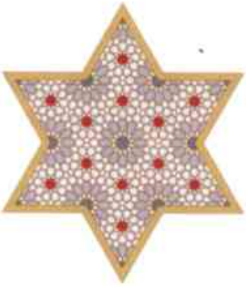

d


e


3 These are Greek capital letters. Write down the order of rotational symmetry for each one.

a Φ b Η c Ζ d Θ e Ξ

4 Here is a star pattern.

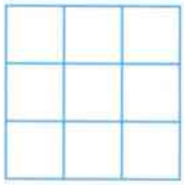


Inside the star there are two patterns that have rotational symmetry.

a What is the order of rotational symmetry of the whole star?

b What is the order of rotational symmetry of the two patterns inside the star?

5 Copy the grid on the right. On your copy, shade in four squares so that the shape has rotational symmetry of order 2.

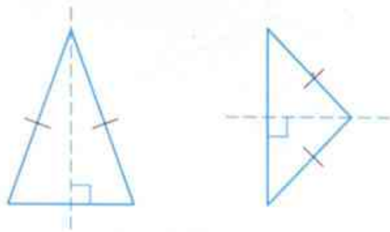


28.3 Symmetry of special two-dimensional shapes

Some three and four-sided shapes have special names such as **isosceles triangle** or **parallelogram**. You need to know the symmetry properties of these shapes.

For example:

- An isosceles triangle has one line of symmetry and no rotational symmetry.



- A parallelogram has no lines of symmetry and rotational symmetry of order 2.

EXERCISE 28C

CORE

1. Draw diagrams to show all the lines of symmetry on:
   a a **rectangle**
   b a **kite**
   c a **square**
   d an **equilateral triangle**
   e a **rhombus**.

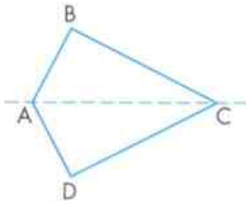
2. a Which shape in question **1** has no rotational symmetry?
   b Find the order of rotational symmetry for each of the others.

3. a What do you call a triangle with one line of symmetry?
   b Can you draw a triangle with exactly two lines of symmetry?

4. What is the name for:
   a a quadrilateral with no lines of symmetry and rotational symmetry of order 2
   b a quadrilateral with rotational symmetry of order 4?

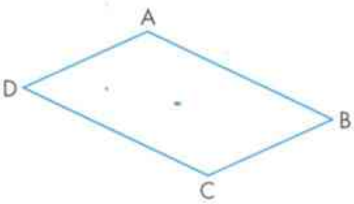
5. a Name two different quadrilaterals that have two lines of symmetry and rotational symmetry of order 2.
   b Can you draw a quadrilateral that has two lines of symmetry but no rotational symmetry?

6. The dotted line is a line of symmetry.
   a Which angles *must* be equal?
   b Which sides *must* be equal?
   c What is the name of this shape?



7. a What is the special name for a line of symmetry of a circle?
   b How many lines of symmetry does a circle have?
   c What is the order of rotational symmetry of a circle?

8. This shape has rotational symmetry of order 2.



   a Which angles *must* be equal?
   b Which sides *must* be equal?
   c What is the name of this shape?

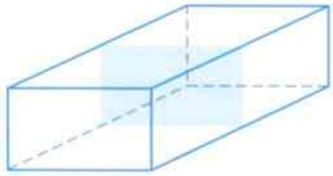
9. If a **trapezium** has a line of symmetry, what can you say about its angles?

28.4 Symmetry of three-dimensional shapes

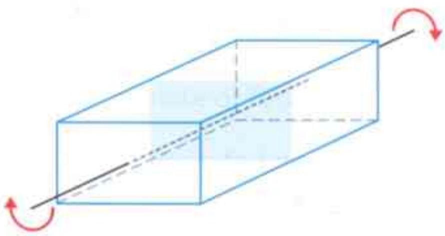
E

Three-dimensional shapes can be symmetrical in two ways.

A **cuboid** has a **plane of symmetry** that divides it into two halves. One is the **reflection** of the other half in a mirror on the plane of symmetry.



A cuboid has an **axis of symmetry**. It has rotational symmetry of order 2 about this axis.



EXERCISE 28D

EXTENDED

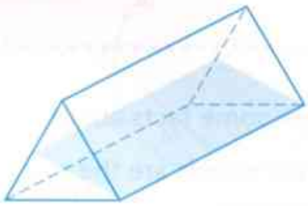
1 A cuboid has three planes of symmetry. Draw diagrams to show them.

2 A cuboid has three axes of symmetry.

- **a** Show them on a diagram.
- **b** What is the order of rotational symmetry about each axis?

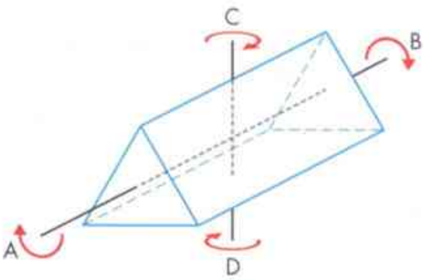
3 This is an equilateral triangular **prism**.
It has *four* planes of symmetry.
One is shown in the diagram.

Draw diagrams to show the other three.



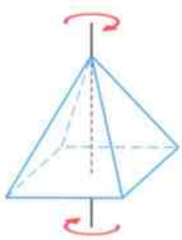
4 The diagram shows an equilateral prism.
AB and CD are axes of symmetry.

- **a** What is the order of rotational symmetry about each one?
- **b** The shape has two more axes of symmetry. Show them on a diagram.



5 The diagram shows an axis of symmetry for a square-based **pyramid**.

- **a** What is the order of rotational symmetry about the axis?
- **b** Does the pyramid have any other axes of symmetry?



EXTENDED

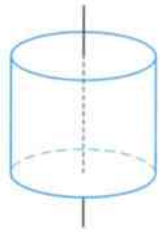
6 Draw diagrams to show the planes of symmetry of a square-based pyramid.

How many are there?

7 a Draw a diagram to show a plane of symmetry for a **cylinder**.

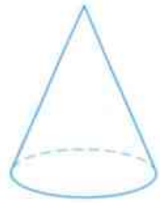
b One axis of symmetry for a cylinder is shown in this diagram.

Draw a diagram to show a different one.



8 a How many axes of rotational symmetry does a **cone** have?

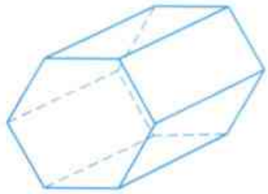
b How many planes of symmetry does a cone have?



9 The end of a prism is a regular hexagon.

a Show that the prism has seven planes of symmetry.

b How many axes of symmetry does it have?



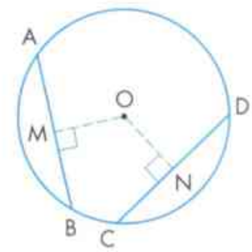
28.5 Symmetry in circles

E

Here are some facts about circles it is useful to know:

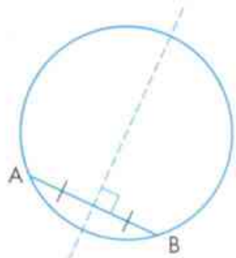
1 If two **chords** are the same length, they are the same distance from the centre.

If O is the centre of the circle and AB and CD are equal in length, then OM = ON.



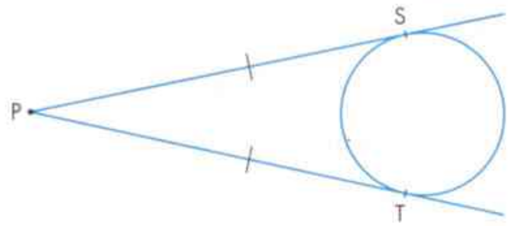
2 The **perpendicular bisector** of a chord passes through the centre of the circle.

In this diagram the **centre** must be on the broken line.



3 The two **tangents** from the point to a circle are equal in length.

PS and PT are the same length.



EXERCISE 28E

1 a Draw a circle. Use a pair of compasses.

b Draw two chords, AB and CD.

c Use compasses and a ruler to construct the perpendicular bisector of AB.

d Construct the perpendicular bisector of CD.

e You should find that the perpendicular bisectors cross at the centre of the circle. Explain why this is the case.

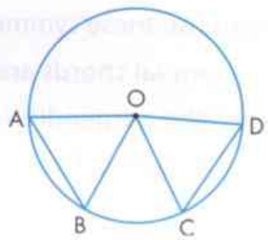
f You could draw a circle by drawing round a circular object such as a plate or food can. How could you use chords to find the centre of the circle?

2 AB and CD are two chords of a circle that are the same length.
O is the centre of the circle.

a What sort of triangle is AOB? Give a reason for your answer.

b Explain why triangles AOB and COD are congruent.

c If angle OAB is 65°, find the angle COD.

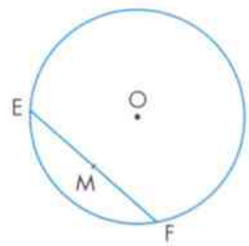


3 O is the centre of a circle. EF is a chord.
M is the mid-point of EF.

a Show that triangles EOM and FOM are congruent.

b Explain why angle EMO is a right angle.

c If angle MOF is 72°, find angle MEO.

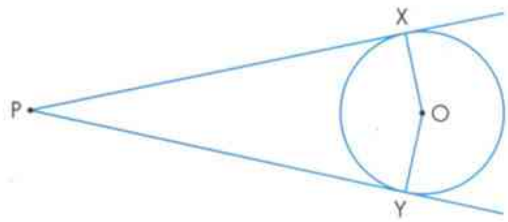


4 O is the centre of a circle.
PX and PY are tangents.

a Explain why angle PXO must be 90°.

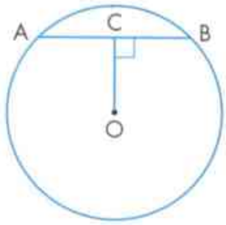
b Show that triangles XPO and YPO are congruent.

c Angle XPO = 17°. Calculate angle XOY.



5 O is the centre of the circle.
AB is a chord 6 cm long.
Angle ACO is 90°.
OC = 4 cm.

Calculate the radius of the circle.

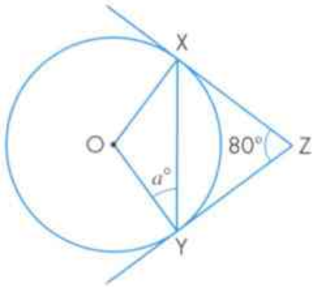


EXTENDED

6 O is the centre of the circle.

XZ and YZ are tangents.

Calculate the value of a.



Check your progress

Core

- I can recognise lines of symmetry
- I can recognise rotational symmetry and order of rotational symmetry in two dimensions

Extended

- I can recognise symmetry properties of prisms, cylinders, pyramids and cones
- I can use these symmetry properties of a circle:
  - equal chords are the same distance from the centre
  - the perpendicular bisector of a chord passes through the centre
  - tangents from an external point are the same length

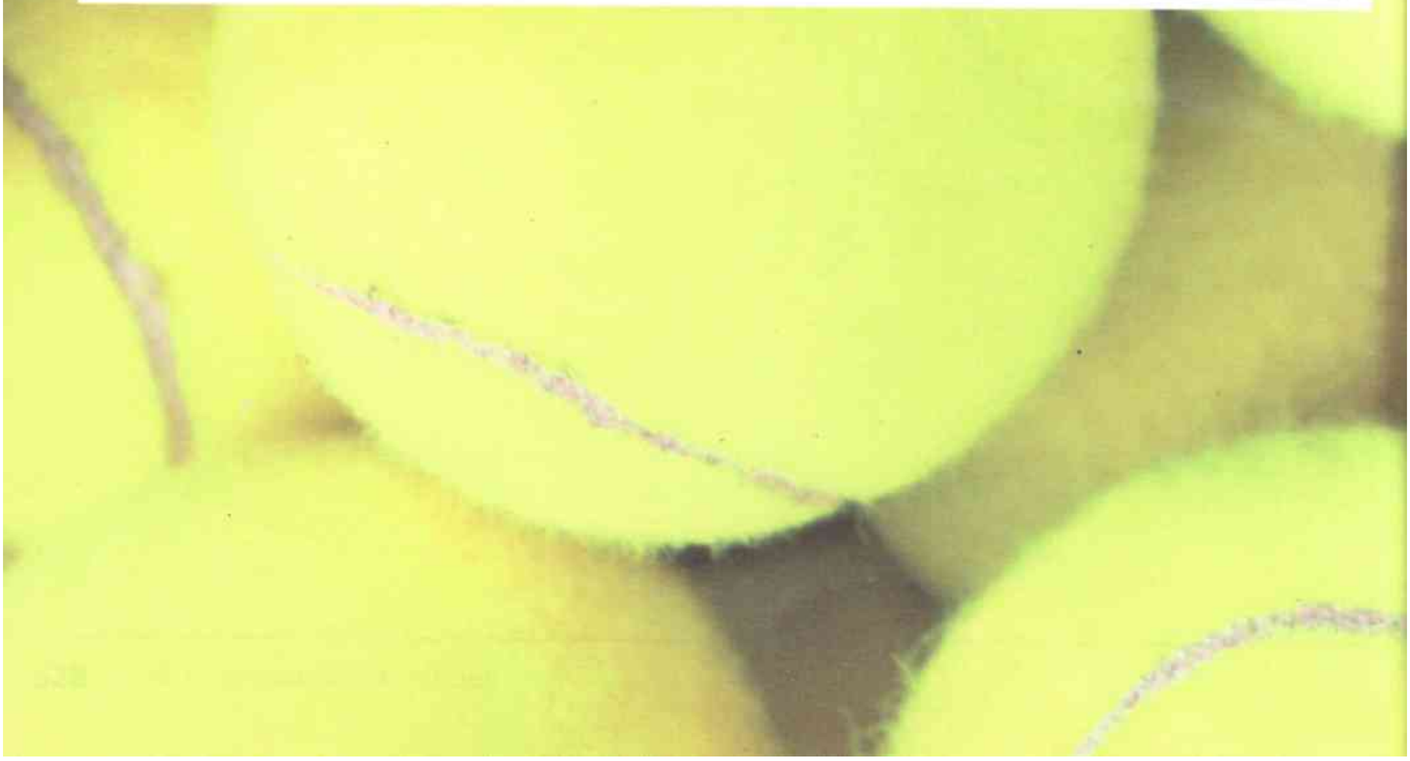
Chapter 29

Vectors

Topics	Level	Key words
1 Introduction to vectors	CORE	magnitude, direction, vector, column vector, scalar, coordinate grid
2 Using vectors	EXTENDED	position vector
3 The magnitude of a vector	EXTENDED	magnitude, Pythagoras' theorem

In this chapter you will learn how to:

CORE	EXTENDED
• Describe a translation by using a vector represented by eg: $\begin{pmatrix} x \\ y \end{pmatrix}$, $\overrightarrow{AB}$ or **a**. (C7.1 and E7.1) • Add and subtract vectors. (C7.1 and E7.1) • Multiply a vector by a scalar. (C7.1 and E7.1)	• Calculate the magnitude of a vector $\begin{pmatrix} x \\ y \end{pmatrix}$ as $\sqrt{x^2 + y^2}$. (E7.3) • Represent vectors by directed line segments. (E7.3) • Use the sum and difference of two vectors to express given vectors in terms of two coplanar vectors. (E7.3) • Use position vectors. (E7.3)



Why this chapter matters

Vectors are used to represent any quantity that has both magnitude and direction. The velocity of a speeding car is its direction and its speed. Velocity is a vector.

To understand how a force acts on an object, you need to know the magnitude of the force and the direction in which it moves – that is, its vector.

In science, vectors are used to describe displacement, acceleration and momentum.

But are vectors used in real life? Yes! Here are some examples.

In the 1950s, a group of talented Brazilian footballers invented the swerving free kick. By kicking the ball in just the right place, they managed to make it curl around the wall of defending players and go into the goal. When a ball is in flight, it is acted upon by various forces which can be described by vectors.

Formula One teams always employ physicists and mathematicians to help them build the perfect racing car. Since vectors describe movements and forces, they are used as the basis of a car's design.

Pilots have to consider wind speed and direction when they plan to land an aircraft at an airport. Vectors are an important part of the computerised landing system.

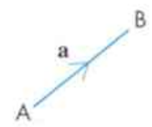
Vectors play a key role in the design of aircraft wings, where an upward force or lift is needed to enable the aircraft to fly.

Vectors are used extensively in computer graphics. Software for animations uses the mathematics of vectors.

Chapter 29 . Topic 1

29.1 Introduction to vectors

A **vector** is something that has both **magnitude** and **direction** and can be represented by an arrow.



Examples are velocity, acceleration, force and momentum.

Vectors can be written down in several ways:

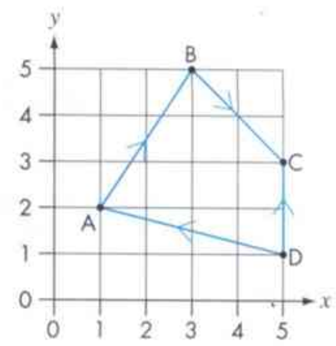
$\overrightarrow{AB}$ – giving the start and end points with an arrow over the top.

$\mathbf{a}$ – as a lower-case letter printed in **bold**. When you are writing vectors $\mathbf{a}$, $\mathbf{b}$ … by hand you cannot show them in bold type. Instead write them with a line underneath as $\underline{a}$, $\underline{b}$ … to show that they stand for a vector and not a number.

When vectors are drawn on a **coordinate grid** they can be represented by two numbers in brackets in a **column**.

The top number shows how far the line moves from one side to the other between its start and end points, and the bottom number shows how far it moves up or down.

For example, on this grid:



$\overrightarrow{AB} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ means move 2 right and 3 upwards to get to point B from point A.

If a line moves either downwards or left the coordinate is negative. So:

$\overrightarrow{BC} = \begin{pmatrix} 2 \\ -2 \end{pmatrix}$ means move 2 right and 2 downwards to get to point C from point B.

$\overrightarrow{DA} = \begin{pmatrix} -4 \\ 1 \end{pmatrix}$ means move 4 left and 1 upwards to get to point A from point D.

$\overrightarrow{DC} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$ means move neither left or right, just upwards to get to point C from point D.

Notice that the line joining A and B can be written as AB and BA and these both have the same magnitude (in this case length).

But the vectors $\overrightarrow{AB}$ and $\overrightarrow{BA}$ are not the same because their directions are different:

$\overrightarrow{AB} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ and $\overrightarrow{BA} = \begin{pmatrix} -2 \\ -3 \end{pmatrix}$

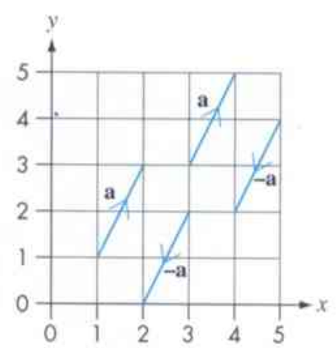
Therefore $\overrightarrow{BA} = -\overrightarrow{AB}$

Advice and Tips

Do not forget the arrow to indicate a vector.

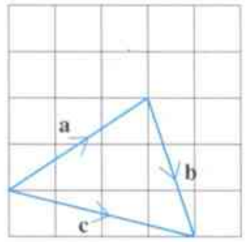
The vectors on this grid show that if $\mathbf{a} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ then $-\mathbf{a} = \begin{pmatrix} -1 \\ -2 \end{pmatrix}$.

$-\mathbf{a}$ is a vector with the same length (magnitude) but acting in the opposite direction as the vector $\mathbf{a}$.



Adding, subtracting and multiplying vectors

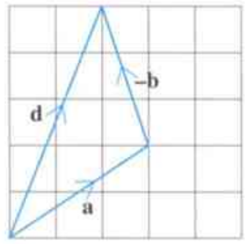
Vectors are added together by placing them end to end.



On this grid $\mathbf{a} + \mathbf{b} = \mathbf{c}$

or $\begin{pmatrix} 3 \\ 2 \end{pmatrix} + \begin{pmatrix} 1 \\ -3 \end{pmatrix} = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$

Notice that you add the top figures together (3 + 1 = 4) and the bottom figures together (2 + –3 = –1).



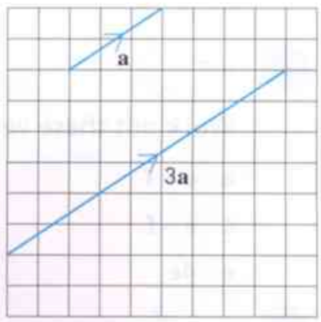
Vectors can be subtracted too.

$\mathbf{a} - \mathbf{b} = \mathbf{d}$

$\mathbf{a} - \mathbf{b}$ is the same as $\mathbf{a} + (-\mathbf{b})$

$\begin{pmatrix} 3 \\ 2 \end{pmatrix} - \begin{pmatrix} 1 \\ -3 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \end{pmatrix} + \begin{pmatrix} -1 \\ 3 \end{pmatrix} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$

Notice that 3 – 1 = 2 and 2 – –3 = 5.



Vectors can be multiplied by a number.

If $\mathbf{a} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

$3\mathbf{a} = 3 \times \begin{pmatrix} 3 \\ 2 \end{pmatrix} = \begin{pmatrix} 9 \\ 6 \end{pmatrix}$

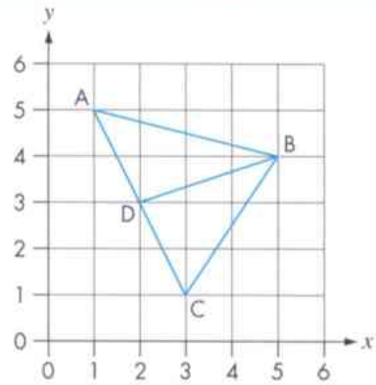
Notice that 3 × 3 = 9 and 3 × 2 = 6.

Here 3 is called a **scalar**, to distinguish it from a vector.

If k is a scalar, then $k\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} kx \\ ky \end{pmatrix}$.

EXERCISE 29A

1



a Write these as column vectors.

i $\overrightarrow{AB}$ **ii** $\overrightarrow{DB}$ **iii** $\overrightarrow{CB}$

iv $\overrightarrow{CA}$ **v** $\overrightarrow{AC}$ **vi** $\overrightarrow{DA}$

b Show that $\overrightarrow{AD} = \overrightarrow{DC}$. What does this tell you about the position of D on line AC?

CORE

CORE

2 F has the coordinates (4, 2), G has the coordinates (2, 6), M is the midpoint of FG and O is the origin.

a Mark F, G and M on a coordinate grid.

b Write these as column vectors.

i $\overrightarrow{FG}$ ii $\overrightarrow{GF}$ iii $\overrightarrow{OM}$ iv $\overrightarrow{MG}$ v $\overrightarrow{GO}$

3 A has the coordinates (3, 4).

$\overrightarrow{AP} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$, $\overrightarrow{AQ} = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$, $\overrightarrow{AR} = \begin{pmatrix} -2 \\ 0 \end{pmatrix}$

Mark A, P, Q and R on a coordinate grid.

4 $\mathbf{a} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 1 \\ -4 \end{pmatrix}$

Draw diagrams to show these vectors.

a $\mathbf{a} + \mathbf{b}$ b $-\mathbf{b}$

c $\mathbf{a} - \mathbf{b}$ d $\mathbf{b} + \mathbf{a}$

e $\mathbf{b} - \mathbf{a}$ f $2\mathbf{b}$

5 $\mathbf{e} = \begin{pmatrix} -3 \\ -2 \end{pmatrix}$ and $\mathbf{f} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$

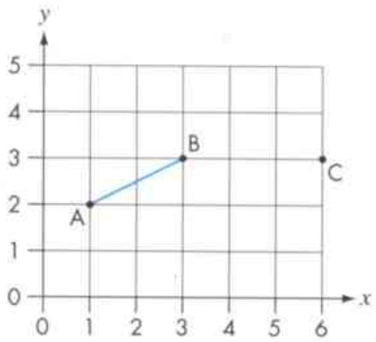
Work out these vectors.

a $\mathbf{e} + \mathbf{f}$ b $3\mathbf{f}$

c $\mathbf{e} - \mathbf{f}$ d $\mathbf{f} - \mathbf{e}$

e $4\mathbf{e}$ f $2\mathbf{f} + \mathbf{e}$

6



Copy this diagram.

a $\overrightarrow{AE} = 2\overrightarrow{AB}$. Mark E on the grid.

b $\overrightarrow{CD} = -2\overrightarrow{AB}$. Mark D on the grid.

c $\overrightarrow{AB} = 2\overrightarrow{AM}$. Mark M on the grid.

d $\overrightarrow{CN} = 2\overrightarrow{CB}$. Mark N on the grid.

e If $\overrightarrow{DC} = k\overrightarrow{AM}$, what is the value of k?

29.2 Using vectors

E

This grid is made of identical parallelograms.

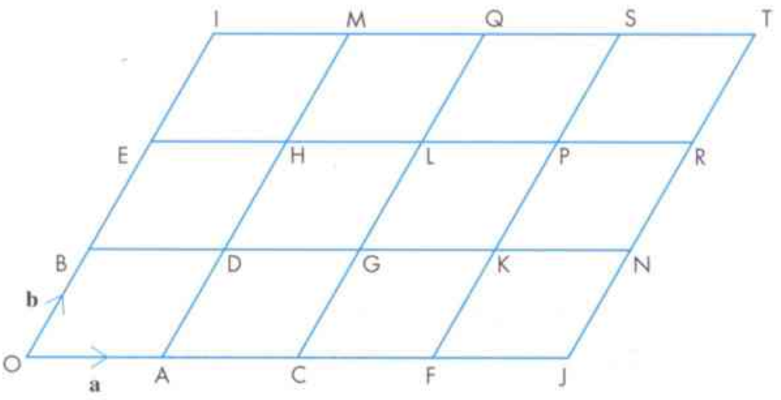
O is the origin.

The **position vector** of A is

$\overrightarrow{OA} = \mathbf{a}$

The **position vector** of B is

$\overrightarrow{OB} = \mathbf{b}$



You can give the position vectors of the other points in terms of **a** and **b**. For example:

The position vector of G = $\overrightarrow{OG} = 2\mathbf{a} + \mathbf{b}$

The position vector of S = $\overrightarrow{OS} = 3\mathbf{a} + 3\mathbf{b}$

You can write other vectors in terms of **a** and **b**. For example:

$\overrightarrow{CL} = 2\mathbf{b}$

$\overrightarrow{CP} = \overrightarrow{CL} + \overrightarrow{LP} = 2\mathbf{b} + \mathbf{a}$ or $\mathbf{a} + 2\mathbf{b}$

$\overrightarrow{CH} = \overrightarrow{CL} + \overrightarrow{LH} = 2\mathbf{b} + -\mathbf{a} = 2\mathbf{b} - \mathbf{a}$

Example 1

a Using the grid above, write down these vectors in terms of **a** and **b**.

i $\overrightarrow{BH}$ ii $\overrightarrow{HP}$ iii $\overrightarrow{GT}$

iv $\overrightarrow{TI}$ v $\overrightarrow{FH}$ vi $\overrightarrow{BQ}$

b What is the relationship between the following vectors?

i $\overrightarrow{BH}$ and $\overrightarrow{GT}$ ii $\overrightarrow{BQ}$ and $\overrightarrow{GT}$ iii $\overrightarrow{HP}$ and $\overrightarrow{TI}$

c Show that B, H and Q lie on the same straight line.

a i $\mathbf{a} + \mathbf{b}$ ii $2\mathbf{a}$ iii $2\mathbf{a} + 2\mathbf{b}$ iv $-4\mathbf{a}$ v $-2\mathbf{a} + 2\mathbf{b}$ vi $2\mathbf{a} + 2\mathbf{b}$

b i $\overrightarrow{BH}$ and $\overrightarrow{GT}$ are parallel and $\overrightarrow{GT}$ is twice the length of $\overrightarrow{BH}$.

ii $\overrightarrow{BQ}$ and $\overrightarrow{GT}$ are equal.

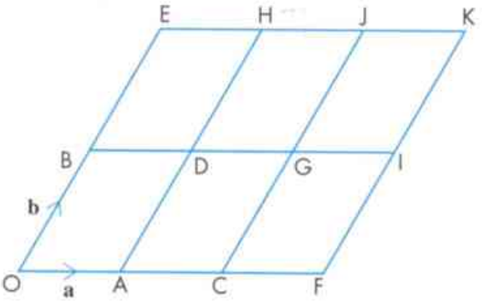
iii $\overrightarrow{HP}$ and $\overrightarrow{TI}$ are in opposite directions and $\overrightarrow{TI}$ is twice the length of $\overrightarrow{HP}$.

c $\overrightarrow{BH}$ and $\overrightarrow{BQ}$ are parallel and start at the same point B. Therefore, B, H and Q must lie on the same straight line.

EXERCISE 29B

EXTENDED

1 On this grid O is the origin, $\overrightarrow{OA}$ is **a** and $\overrightarrow{OB}$ is **b**.



a Name three other vectors equivalent to **a**.

b Name three other vectors equivalent to **b**.

c Name three vectors equivalent to −**a**.

d Name three vectors equivalent to −**b**.

2 Using the same grid as in question **1**, write these vectors in terms of **a** and **b**.

a the position vector of C

b the position vector of E

c the position vector of K

d $\overrightarrow{OH}$

e $\overrightarrow{AG}$

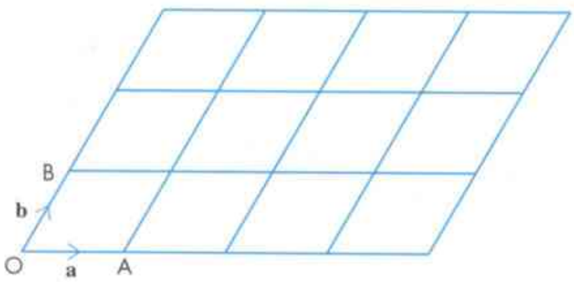
f $\overrightarrow{AK}$

g $\overrightarrow{BK}$

3 On the grid in question **1**, there are three vectors equivalent to $\overrightarrow{OG}$. Name all three.

4 On the grid in question **1**, there are three vectors that are three times the magnitude of $\overrightarrow{OA}$ and act in the same direction. Name all three.

5 Copy this grid.



On your copy, mark the points C to G to show these vectors.

a $\overrightarrow{OC} = 2\mathbf{a} + 3\mathbf{b}$

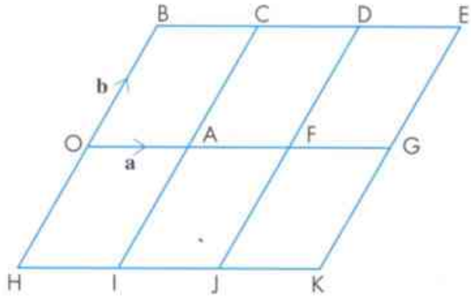
b $\overrightarrow{OD} = 2\mathbf{a} + \mathbf{b}$

c $\overrightarrow{OE} = 4\mathbf{a}$

d $\overrightarrow{OF} = 4\mathbf{a} + 2\mathbf{b}$

e $\overrightarrow{OG} = \frac{1}{2}\mathbf{a} + 2\mathbf{b}$

6 On this grid, $\overrightarrow{OA}$ is $\mathbf{a}$ and $\overrightarrow{OB}$ is $\mathbf{b}$.



Write down vectors in terms of $\mathbf{a}$ and $\mathbf{b}$.

a $\overrightarrow{OH}$ **b** $\overrightarrow{OK}$

c $\overrightarrow{OJ}$ **d** $\overrightarrow{OI}$

e $\overrightarrow{OC}$ **f** $\overrightarrow{CO}$

g $\overrightarrow{AK}$ **h** $\overrightarrow{DI}$

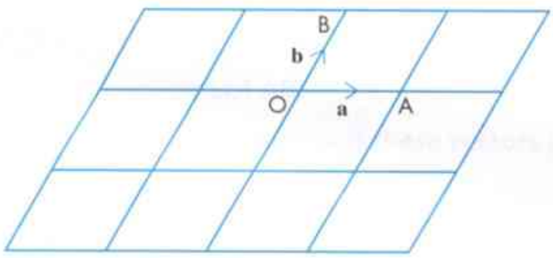
i $\overrightarrow{JE}$ **j** $\overrightarrow{AB}$

k $\overrightarrow{CK}$ **l** $\overrightarrow{DK}$

7 **a** On the grid in question **6**, there are two vectors that are twice the size of $\overrightarrow{AB}$ and act in the opposite direction. Name both of them.

b On the grid in question **6**, there are three vectors that are three times the size of $\overrightarrow{OA}$ and act in the opposite direction. Name all three.

8 Copy this grid.



On your copy, mark the points C to P to show these vectors.

a $\overrightarrow{OC} = 2\mathbf{a} - \mathbf{b}$ **b** $\overrightarrow{OD} = 2\mathbf{a} + \mathbf{b}$

c $\overrightarrow{OE} = \mathbf{a} - 2\mathbf{b}$ **d** $\overrightarrow{OF} = \mathbf{b} - 2\mathbf{a}$

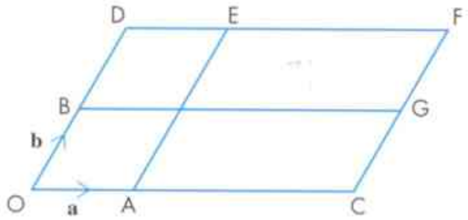
e $\overrightarrow{OG} = -\mathbf{a}$ **f** $\overrightarrow{OH} = -\mathbf{a} - 2\mathbf{b}$

g $\overrightarrow{OI} = 2\mathbf{a} - 2\mathbf{b}$ **h** $\overrightarrow{OJ} = -\mathbf{a} + \mathbf{b}$

i $\overrightarrow{OK} = -\mathbf{a} - \mathbf{b}$ **j** $\overrightarrow{OM} = -\mathbf{a} - \frac{3}{2}\mathbf{b}$

k $\overrightarrow{ON} = -\frac{1}{2}\mathbf{a} - 2\mathbf{b}$ **l** $\overrightarrow{OP} = \frac{3}{2}\mathbf{a} - \frac{3}{2}\mathbf{b}$

9 The diagram shows two sets of parallel lines. O is the origin.



$\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$

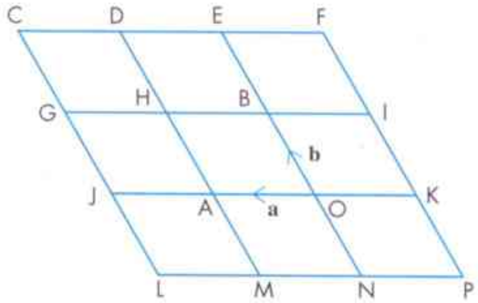
$\overrightarrow{OC} = 3\overrightarrow{OA}$ and $\overrightarrow{OD} = 2\overrightarrow{OB}$

a Write down these vectors in terms of **a** and **b**.

i $\overrightarrow{OF}$ ii $\overrightarrow{OG}$ iii $\overrightarrow{EG}$ iv $\overrightarrow{CE}$

b Write down two vectors that can be written as $3\mathbf{a} - \mathbf{b}$.

10 This grid shows the vectors $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$. O is the origin.



Write down the position vectors of these points.

a G

b F

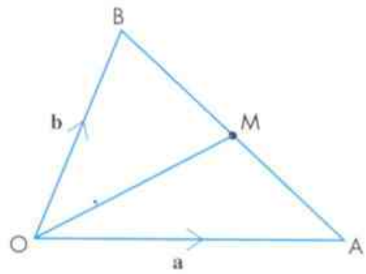
c The midpoint of DH

d The centre of OAHB

e The centre of DCGH

f The centre of AMLJ

11 The diagram shows the vectors $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$. M is the midpoint of AB.



a i Work out the vector $\overrightarrow{AB}$ in terms of **a** and **b**.

ii Work out the vector $\overrightarrow{AM}$.

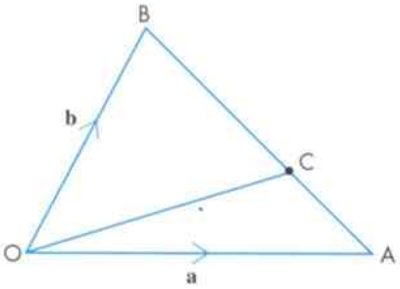
iii Explain why $\overrightarrow{OM} = \overrightarrow{OA} + \overrightarrow{AM}$.

iv Using your answers to parts **ii** and **iii**, work out $\overrightarrow{OM}$ in terms of **a** and **b**.

b Copy the diagram and show on it the vector $\overrightarrow{OC}$ which is equal to $\mathbf{a} + \mathbf{b}$.

c Describe in geometrical terms the position of M in relation to O, A, B and C.

12 The diagram shows the vectors $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.
The point C divides the line AB in the ratio 1:2.



Advice and Tips

AC is $\frac{1}{3}$ the distance from A to B.

a **i** Work out the vector $\overrightarrow{AB}$.

ii Work out the vector $\overrightarrow{AC}$.

iii Work out the vector $\overrightarrow{OC}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

b If C now divides the line AB in the ratio 1:3, write down the vector that represents $\overrightarrow{OC}$.

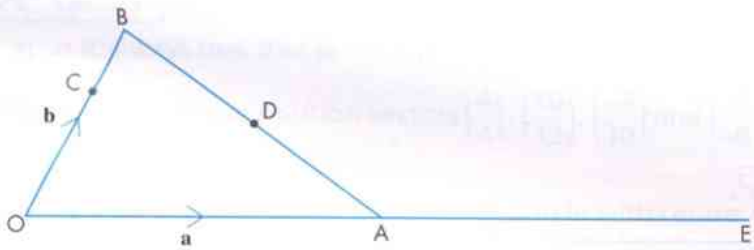
Advice and Tips

AC is now $\frac{1}{4}$ the distance from A to B.

13 The diagram shows the vectors $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.

Advice and Tips

OC is $\frac{2}{3}$ the distance from O to B.



The point C divides OB in the ratio 2:1.

The point E is such that $\overrightarrow{OE} = 2\overrightarrow{OA}$.

D is the midpoint of AB.

a Write down (or work out) these vectors in terms of $\mathbf{a}$ and $\mathbf{b}$.

i $\overrightarrow{OC}$

ii $\overrightarrow{OD}$

iii $\overrightarrow{CO}$

b The vector $\overrightarrow{CD}$ can be written as $\overrightarrow{CD} = \overrightarrow{CO} + \overrightarrow{OD}$. Use this fact to work out $\overrightarrow{CD}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

c Write down a similar rule to that in part **b** for the vector $\overrightarrow{DE}$. Use this rule to work out $\overrightarrow{DE}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

d Explain why C, D and E lie on the same straight line.

29.3 The magnitude of a vector

E

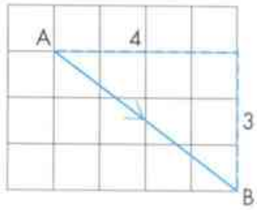
The size or **magnitude** of a vector is represented by two vertical lines which stand for 'magnitude of' or 'length of', eg $|\overrightarrow{AB}|$ or $|\mathbf{a}|$.

If a vector is drawn on a rectangular coordinate grid you can use **Pythagoras' theorem** to calculate the magnitude.

For example:

if $\overrightarrow{AB} = \begin{pmatrix} 4 \\ -3 \end{pmatrix}$

then it can form the hypotenuse of a triangle with sides of lengths 3 and 4 as shown on the grid.



The square of the hypotenuse is equal to the sum of the squares of the other two sides so:

$$|\overrightarrow{AB}| = \sqrt{4^2 + 3^2}$$
$$= \sqrt{25}$$
$$= 5$$

In general, if $\mathbf{a} = \begin{pmatrix} x \\ y \end{pmatrix}$ then $|\mathbf{a}| = \sqrt{x^2 + y^2}$.

Example 2

If A has coordinates (3, –2) and B has coordinates (–3, 5), calculate $|\overrightarrow{AB}|$.

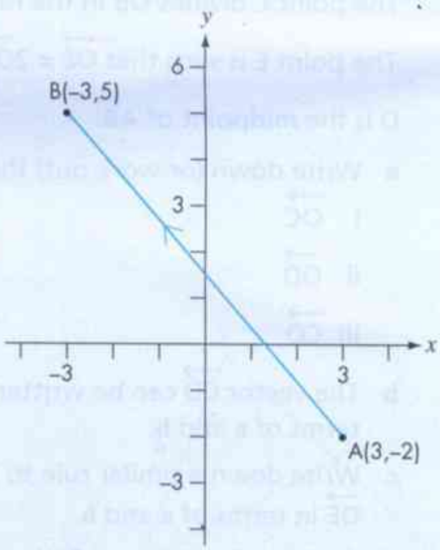
$\overrightarrow{AB} = \begin{pmatrix} -6 \\ 7 \end{pmatrix}$

$|\overrightarrow{AB}| = \sqrt{(-6)^2 + 7^2}$

$= \sqrt{36 + 49}$

$= \sqrt{85}$

= 9.22 to 2 decimal places



Advice and Tips

Remember that $(-6)^2 = 6^2$

EXERCISE 29C

1 O is the origin.

P, Q and R have position vectors $\begin{pmatrix}3\\5\end{pmatrix}$, $\begin{pmatrix}6\\-2\end{pmatrix}$, $\begin{pmatrix}0\\-4\end{pmatrix}$.

- **a** Show O, P, Q and R on a diagram.
- **b** Calculate $|\overrightarrow{OP}|$, $|\overrightarrow{OQ}|$ and $|\overrightarrow{OR}|$. You can leave square root signs in your answers.
- **c** Calculate $|\overrightarrow{PQ}|$.
- **d** Calculate $|\overrightarrow{QR}|$.

2 $\mathbf{a} = \begin{pmatrix}6\\8\end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix}5\\-12\end{pmatrix}$

- **a** Calculate $|\mathbf{a}|$ and $|\mathbf{b}|$.
- **b** Calculate $\mathbf{a} + \mathbf{b}$.
- **c** Calculate $|\mathbf{a} + \mathbf{b}|$.
- **d** Is it true that $|\mathbf{a} + \mathbf{b}| = |\mathbf{a}| + |\mathbf{b}|$? Give a reason for your answer.
- **e** Calculate $|\mathbf{a} - \mathbf{b}|$.
- **f** Calculate $|\mathbf{b} - \mathbf{a}|$.
- **g** Is it always true that $|\mathbf{a} - \mathbf{b}| = |\mathbf{b} - \mathbf{a}|$? Give a reason for your answer.

3 A, B, C and D have position vectors $\begin{pmatrix}4\\4\end{pmatrix}$, $\begin{pmatrix}10\\12\end{pmatrix}$, $\begin{pmatrix}-4\\10\end{pmatrix}$ and $\begin{pmatrix}4\\-6\end{pmatrix}$.

- **a** Calculate $|\overrightarrow{AB}|$, $|\overrightarrow{AC}|$ and $|\overrightarrow{AD}|$.
- **b** Explain why B, C and D must lie on a circle with centre A and state the radius of the circle.

4 $\mathbf{c} = \begin{pmatrix}1\\4\end{pmatrix}$ and $\mathbf{d} = \begin{pmatrix}-2\\5\end{pmatrix}$

Calculate:

- **a** $|\mathbf{c}|$
- **b** $|3\mathbf{d}|$
- **c** $|2\mathbf{c} + \mathbf{d}|$
- **d** $|4\mathbf{c} - 2\mathbf{d}|$.

Check your progress

Core

- I can describe a translation with a vector of the form $\begin{pmatrix}x\\y\end{pmatrix}$
- I can add and subtract vectors
- I can multiply a vector by a scalar

Extended

- I can calculate the magnitude of a vector
- I can represent vectors by line segments
- I can use sums and difference to express one vector in terms of two others
- I can use position vectors

Chapter 30

Transformations

Topics	Level	Key words
1 Translations	CORE	transformation, translation, vector
2 Reflections: 1	CORE	reflection, object, image, mirror line
3 Reflections: 2	EXTENDED	
4 Rotations: 1	CORE	rotation, centre of rotation, angle of rotation, clockwise, anticlockwise
5 Rotations: 2	EXTENDED	
6 Enlargements: 1	CORE	scale factor, enlargement, centre of enlargement, ray methods, coordinate method
7 Enlargements: 2	EXTENDED	negative enlargement
8 Combined transformations	EXTENDED	

In this chapter you will learn how to:

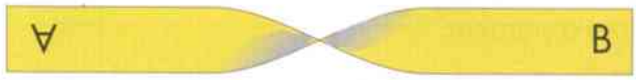
CORE	EXTENDED
• Reflect simple plane figures in horizontal or vertical lines. (C7.2 and E7.2) • Construct given translations and enlargements (with positive and fractional scale factors) of simple plane figures. (C7.2) • Recognise and describe reflections, rotations, translations and enlargements (with positive and fractional scale factors). (C7.2) • Rotate simple plane figures about the origin, vertices or midpoints of edges of the figures, through multiples of 90°. (C7.2)	• Construct given translations and enlargements (including positive, negative and fractional scale factors) of simple plane figures. (E7.2) • Recognise and describe reflections, rotations, translations and enlargements (including positive, negative and fractional scale factors). (E7.2) • Rotate simple plane figures through multiples of 90°. (E7.2)

Why this chapter matters

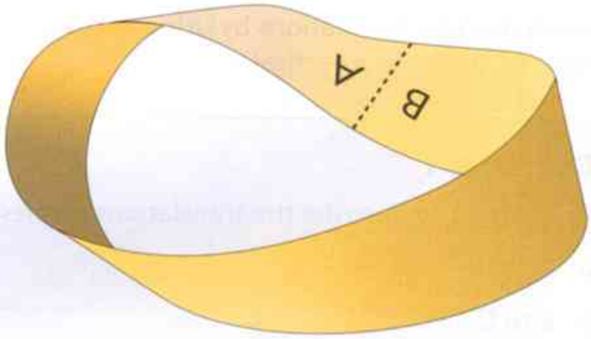
How many sides does a strip of paper have? Two or one?

Take a strip of paper about 20 cm by 2 cm.

How many sides does it have? Easy! You can see that this has two sides, a topside and an underside. If you were to draw a line along one side of the strip, you would have one side with a line 20 cm long on it and one side blank.



Now mark the ends A and B, put a single twist in the strip of paper and tape (or glue) the two ends together, as shown.



How many sides does this strip of paper have now?

Take a pen and draw a line on the paper, starting at any point you like. Continue the line along the length of the paper – you will eventually come back to your starting point. Your strip has only one side now! There is no blank side.

You have transformed a two-sided piece of paper into a one-sided piece of paper.

This shape is called a Möbius strip. It is named after August Ferdinand Möbius, a 19th-century German mathematician. Möbius caused a revolution in geometry.

Möbius strips have a number of applications that use its property of one-sidedness, including conveyor belts in industry and in vacuum cleaners.

The Möbius strip has become the universal symbol of recycling. The symbol was created in 1970 by Gary Anderson at the University of Southern California, as part of a contest sponsored by a paper company.

The Möbius strip is a form of transformation. In this chapter, you will look at some other transformations of shapes.

30.1 Translations

A **transformation** changes the position or the size of a shape.

There are four basic ways of changing the position and size of two-dimensional shapes: a **translation**, a reflection, a rotation or an enlargement. All of these transformations, except enlargement, keep shapes congruent.

A translation is the 'movement' of a shape from one place to another without reflecting it or rotating it. It is sometimes called a glide, since the shape appears to glide from one place to another. Every point in the shape moves in the same direction and through the same distance.

You can describe translations by using **vectors**. A vector is represented by the combination of a horizontal shift and a vertical shift.

Example 1

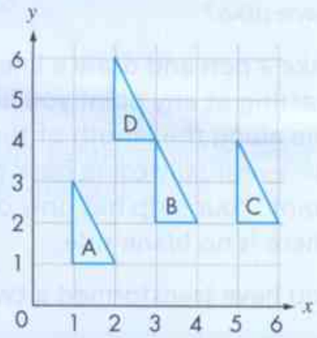
Use vectors to describe the translations of these triangles.

a A to B

b B to C

c C to D

d D to A



a The vector describing the translation from A to B is $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$.

b The vector describing the translation from B to C is $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$.

c The vector describing the translation from C to D is $\begin{pmatrix} -3 \\ 2 \end{pmatrix}$.

d The vector describing the translation from D to A is $\begin{pmatrix} -1 \\ -3 \end{pmatrix}$.

Note:

- The top number in the vector describes the horizontal movement. To the right +, to the left –.
- The bottom number in the vector describes the vertical movement. Upwards +, downwards –.
- These vectors are also called *direction vectors*.

EXERCISE 30A

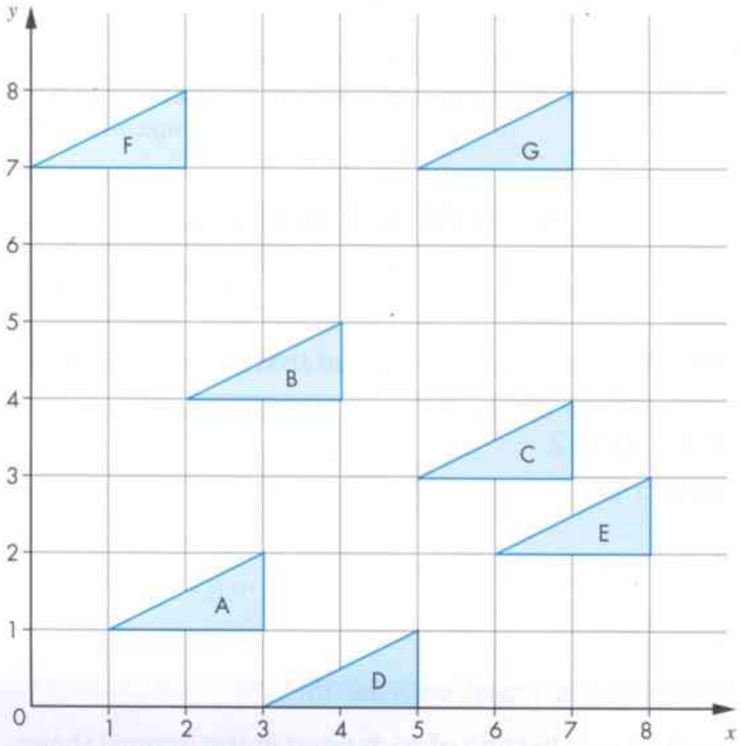
1 Use vectors to describe these translations of the shapes on the grid below.

a i A to B
ii A to C
iii A to D

b i B to E
ii B to F
iii B to G

c i C to A
ii C to E
iii C to G

d i G to D
ii F to G
iii G to E



2 **a** Draw a set of coordinate axes and on it the triangle with coordinates A(1, 1), B(2, 1) and C(1, 3).

b Draw the image of ABC after a translation with vector $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$. Label this triangle P.

c Draw the image of ABC after a translation with vector $\begin{pmatrix} -1 \\ 2 \end{pmatrix}$. Label this triangle Q.

d Draw the image of ABC after a translation with vector $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$. Label this triangle R.

e Draw the image of ABC after a translation with vector $\begin{pmatrix} -2 \\ -4 \end{pmatrix}$. Label this triangle S.

3 Using your diagram from question **2**, use vectors to describe the translation that will move

a P to Q **b** Q to R **c** R to S **d** S to P
e R to P **f** S to Q **g** R to Q **h** P to S.

4 If a translation is given by:

$$\begin{pmatrix} x \\ y \end{pmatrix}$$

describe the translation that would take the image back to the original position.

CORE

5 A boat travels between three jetties X, Y and Z on a lake. Its journeys are described by direction vectors, with distance in kilometres.

The direction vector from X to Y is $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$ and the direction vector from Y to Z is $\begin{pmatrix} -2 \\ -3 \end{pmatrix}$.

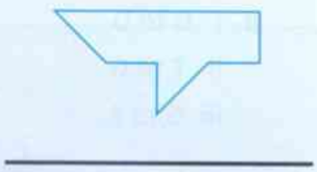
Using centimetre-squared paper and a scale of 1 cm : 1 km, draw a diagram to show journeys between X, Y and Z. Work out the direction vector for the journey from Z to X.

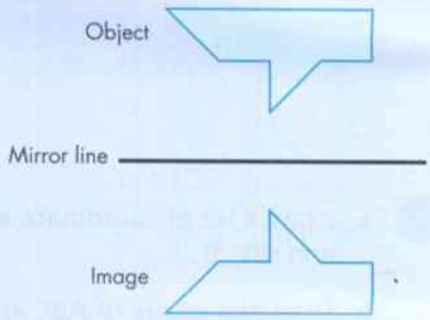
30.2 Reflections: 1

A **reflection** transforms a shape so that it becomes a mirror image of itself.

Example 2

Reflect this shape in the line provided.





The reflected shape looks like this.

Note: The reflection of each point in the original shape, called the **object**, is perpendicular to the mirror line. So if you 'fold' the whole diagram along the **mirror line**, the object will coincide with its reflection, called its **image**.

EXERCISE 30B

CORE

1 Copy the diagram.

On your copy, draw the reflection of the given triangle in each of these lines.

a $x = 2$

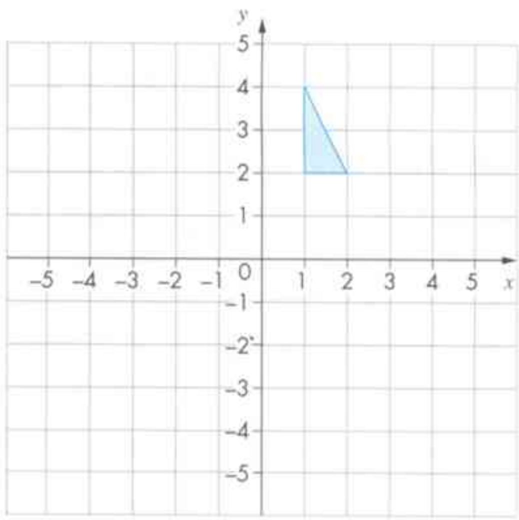
b $x = -1$

c $x = 3$

d $y = 2$

e $y = -1$

f y-axis



2 a Draw a pair of axes.
Label the x-axis from –5 to 5 and the y-axis from –5 to 5.

b Draw the triangle with coordinates A(1, 1), B(3, 1), C(4, 5).

c Reflect the triangle ABC in the x-axis. Label the image P.

d Reflect triangle P in the y-axis. Label the image Q.

e Reflect triangle Q in the x-axis. Label the image R.

f Describe the reflection that will move triangle ABC to triangle R.

3 a Draw a pair of axes.
Label the x-axis from –5 to +5 and the y-axis from –5 to +5.

b Reflect the points A(2, 1), B(5, 0), C(–3, 3), D(3, –2) in the x-axis.

c What do you notice about the values of the coordinates of the reflected points?

d What would the coordinates of the reflected point be if the point (a, b) were reflected in the x-axis?

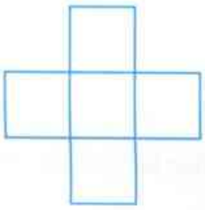
4 a Draw a pair of axes.
Label the x-axis from –5 to +5 and the y-axis from –5 to +5.

b Reflect the points A(2, 1), B(0, 5), C(3, –2), D(–4, –3) in the y-axis.

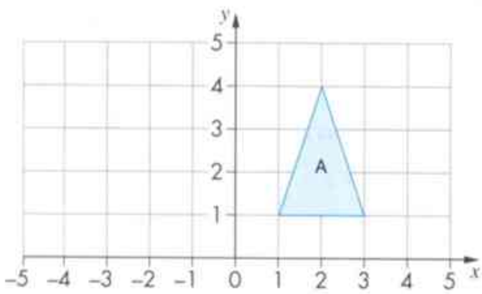
c What do you notice about the values of the coordinates of the reflected points?

d What would the coordinates of the reflected point be if the point (a, b) were reflected in the y-axis?

5 By using the middle square as a starting square called ABCD, describe how to keep reflecting the square to obtain the final shape in the diagram.



6 Triangle A is drawn on a grid.



Triangle A is reflected to form a new triangle B.
The coordinates of triangle B are (–4, 4), (–3, 1) and (–5, 1).
Work out the equation of the mirror line.

30.3 Reflections: 2

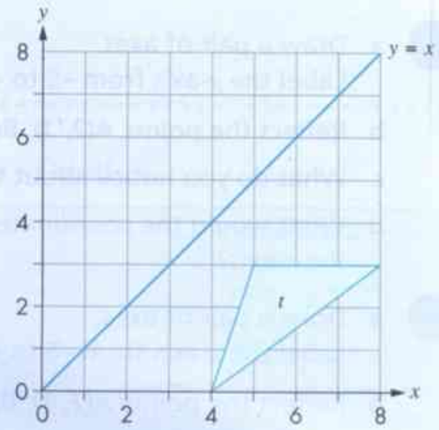
E

You have been learning about reflections in horizontal or vertical lines.

You can reflect a shape in any line.

Example 3

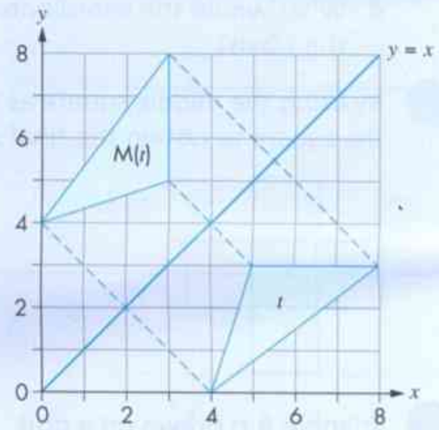
Draw the reflection of triangle t in the line with equation $y = x$.



To find the image of each vertex of the triangle, draw lines perpendicular to the mirror.

Each vertex and its image are the same distance from the mirror but on opposite sides. Use the grid to help you find the new vertices.

Join the new vertices to draw the reflection M(t) of the triangle t.



EXERCISE 30C

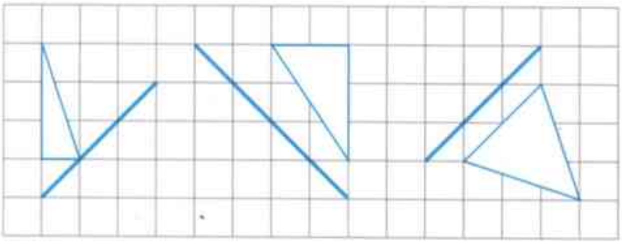
EXTENDED

1 A designer used these instructions to create a design.

- Start with any rectangle ABCD.
- Reflect the rectangle ABCD in the line AC.
- Reflect the rectangle ABCD in the line BD.

Draw a rectangle and use the above to create a design.

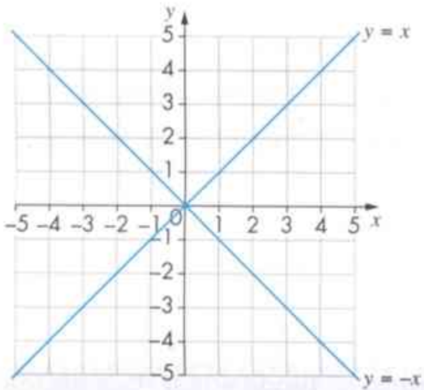
2 Draw each of these triangles on squared paper, leaving plenty of space on the opposite side of the given mirror line. Then draw the reflection of each triangle.



Advice and Tips

Turn the page around so that the mirror lines are vertical or horizontal.

3 **a** Draw a pair of axes and the lines $y = x$ and $y = -x$, as shown.



b Draw the triangle with coordinates A(2, 1), B(5, 1), C(5, 3).

c Draw the reflection of triangle ABC in the x-axis and label the image P.

d Draw the reflection of triangle P in the line $y = -x$ and label the image Q.

e Draw the reflection of triangle Q in the y-axis and label the image R.

f Draw the reflection of triangle R in the line $y = x$ and label the image S.

g Draw the reflection of triangle S in the x-axis and label the image T.

h Draw the reflection of triangle T in the line $y = -x$ and label the image U.

i Draw the reflection of triangle U in the y-axis and label the image W.

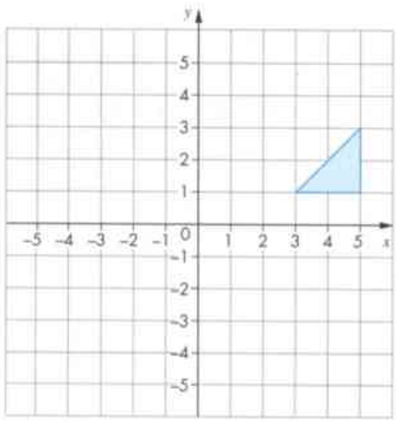
j What single reflection will move triangle W to triangle ABC?

4 Copy the diagram.

Reflect the triangle in these lines.

a $y = x$ **b** $x = 1$

c $y = -x$ **d** $y = -1$



EXTENDED

5 a Draw a pair of axes.
Label the x-axis from –5 to +5 and the y-axis from –5 to +5.

b Draw the line $y = x$.

c Reflect the points A(2, 1), B(5, 0), C(–3, 2), D(–2, –4) in the line $y = x$.

d What do you notice about the values of the coordinates of the reflected points?

e What would the coordinates of the reflected point be if the point (a, b) were reflected in the line $y = x$?

6 a Draw a pair of axes.
Label the x-axis from –5 to +5 and the y-axis from –5 to +5.

b Draw the line $y = -x$.

c Reflect the points A(2, 1), B(0, 5), C(3, –2), D(–4, –3) in the line $y = -x$.

d What do you notice about the values of the coordinates of the reflected points?

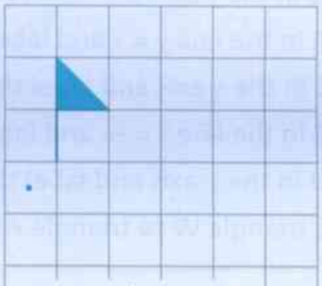
e What would the coordinates of the reflected point be if the point (a, b) were reflected in the line $y = -x$?

30.4 Rotations: 1

A **rotation** transforms a shape to a new position by turning it about a fixed point called the **centre of rotation**.

Example 4

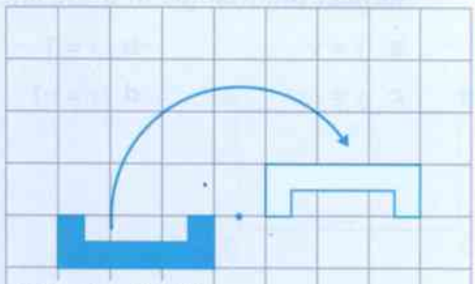
Rotate each of these shapes as described.


a Rotation of 90° clockwise


b Rotation of 180°

The rotations look like this.


a Rotation of 90° clockwise


b Rotation of 180°

Note:

- The direction of turn or the **angle of rotation** is expressed as **clockwise** or **anticlockwise**.
- The position of the centre of rotation is always specified.
- The rotations 180° clockwise and 180° anticlockwise are the same.

The rotations that appear most frequently are 90° and 180°.

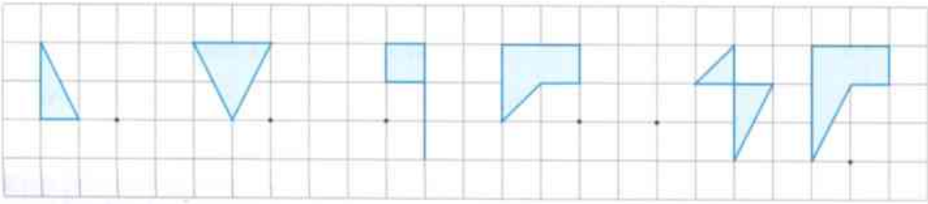
Advice and Tips

Use tracing paper to check rotations.

EXERCISE 30D

CORE

1 On squared paper, draw each of these shapes and its centre of rotation, leaving plenty of space all round the shape.



a Rotate each shape about its centre of rotation:

  i first by 90° clockwise (call the image A)

  ii then by 90° anticlockwise (call the image B).

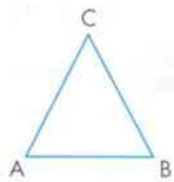
b Describe, in each case, the rotation that would take:

  i A back to its original position

  ii A to B.

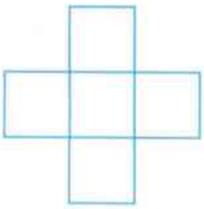
2 A graphics designer came up with this routine for creating a design.

- Start with a triangle ABC.
- Reflect the triangle in the line AB.
- Rotate the whole shape about point C clockwise 90°, then a further clockwise 90°, then a further clockwise 90°.

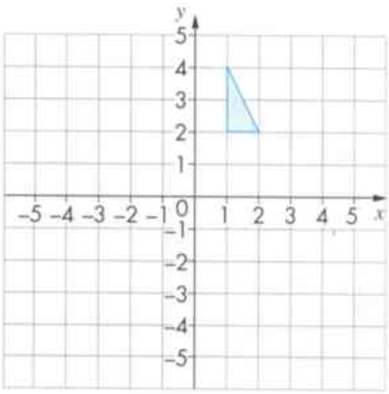


From any triangle of your choice, create a design using the above routine.

3 By using the middle square as a starting square, called ABCD, describe how to keep rotating the square to obtain the final shape in the diagram.



4 Copy the diagram. Rotate the given triangle by:



a 90° clockwise about (0, 0)

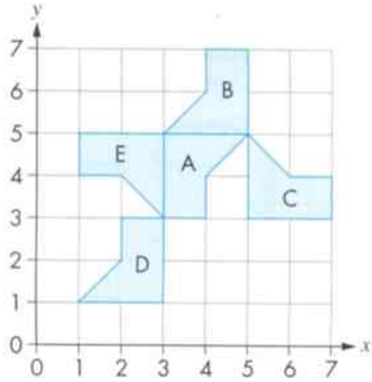
b 180° about (0, 0)

c 90° anticlockwise about (1, 4)

d 180° about (1, 3)

e 90° clockwise about (2, 2).

5 Give the centre and the angle for the rotations that will take:



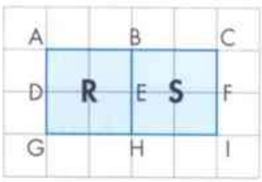
a A onto B

b A onto C

c A onto D

d A onto E

6



a A 180° rotation will take R onto S. Where is the centre of rotation?

b A 90° clockwise rotation will take R onto S. Where is the centre?

7 **a** Draw a pair of axes where both the x-values and y-values are from –5 to 5.

b Draw the triangle ABC, where A = (1, 2), B = (2, 4) and C = (4, 1).

c **i** Rotate triangle ABC 90° clockwise about the origin (0, 0) and label the image A′, B′, C′, where A′ is the image of A, etc.

ii Write down the coordinates of A′, B′, C′.

iii What connection is there between A, B, C and A′, B′, C′?

iv Will this connection always be so for a 90° clockwise rotation about the origin?

8 Repeat question **7**, but rotate triangle ABC through 180°.

9 Show that a reflection in the x-axis followed by a reflection in the y-axis is equivalent to a rotation of 180° about the origin.

30.5 Rotations: 2

E

EXTENDED

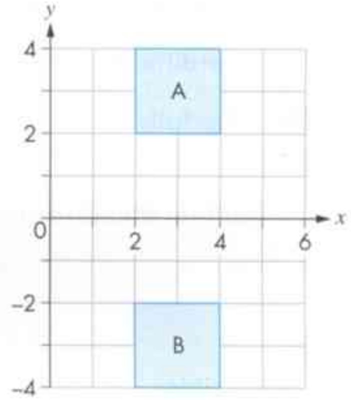
EXERCISE 30E

1 Draw a set of x and y axes and label them both from 0 to 12.

Draw the triangle with vertices at (5,5), (7,5) and (7,8). Label it T.

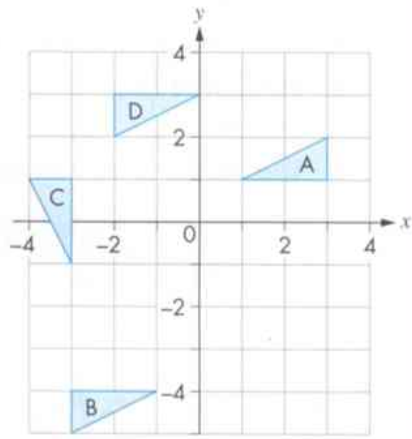
- **a** Rotate T 180° about (4,5). Label the new triangle A.
- **b** Rotate T 90° clockwise about (7,4). Label the new triangle B.
- **c** Rotate T 90° anticlockwise about (7,9). Label the new triangle C.
- **d** What rotation will take triangle B onto triangle C?

2
- **a** A 180° rotation will take square A onto square B. Where is the centre of the rotation?
- **b** A 90° *clockwise* rotation will take A onto B. Where is the centre of the rotation?
- **c** A 90° *anticlockwise* rotation will take A onto B. Where is the centre of the rotation?



3 Give the centre and angle for the rotation of:

- **a** A onto B
- **b** B onto C
- **c** C onto D
- **d** D onto A.



4 Show that a reflection in the line $y = x$ followed by a reflection in the line $y = -x$ is equivalent to a rotation of 180° about the origin.

EXTENDED

5 **a** Draw a regular hexagon ABCDEF with centre O. The letters should go round the hexagon clockwise.

b Using O as the centre of rotation, describe a transformation that will result in the following movements.

i Triangle AOB to triangle BOC

ii Triangle AOB to triangle COD

iii Triangle AOB to triangle DOE

iv Triangle AOB to triangle EOF

c Describe the transformations that will move the rhombus ABCO to these positions.

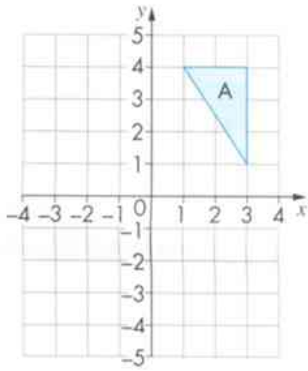
i Rhombus BCDO

ii Rhombus DEFO

6 Triangle A, as shown on the grid, is rotated to form a new triangle B.

The coordinates of the vertices of B are (0, −2), (−3, −2) and (−3, −4).

Describe fully the rotation that maps triangle A onto triangle B.



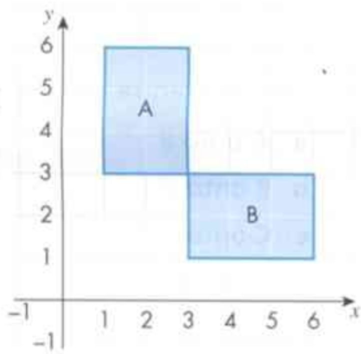
Find, if possible:

a The equation of a mirror line that will reflect A onto B

b The centre of a clockwise rotation of 90° that maps A onto B

c The centre of an anticlockwise rotation of 90° that maps A onto B

d The vector of a translation that maps A onto B



30.6 Enlargements: 1

An **enlargement** changes the size of a shape to give a similar image. It always has a **centre of enlargement** and a **scale factor**. Every length of the enlarged shape will be:

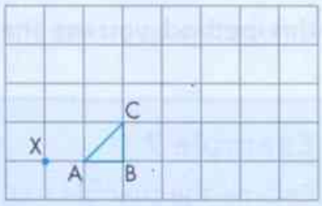
original length × scale factor

The distance of each image point on the enlargement from the centre of enlargement will be:

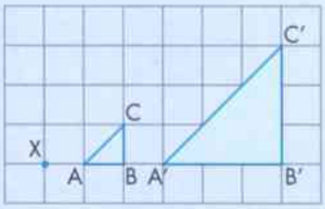
distance of original point from centre of enlargement × scale factor

Example 5

Enlarge triangle ABC by scale factor 3 about the centre of enlargement X.



This is the completed enlargement.



Note:

- Each length on the enlargement A′B′C′ is three times the corresponding length on the original shape.

  This means that the corresponding sides are in the same ratio:

  AB : A′B′ = AC : A′C′ = BC : B′C′ = 1 : 3
- The distance of any point on the enlargement from the centre of enlargement is three times the distance from the corresponding point on the original shape to the centre of enlargement.

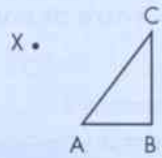
There are two distinct ways to enlarge a shape: the **ray method** and the **coordinate method** (counting squares).

Ray method

This is the *only* way to construct an enlargement when the diagram is not on a grid.

Example 6

Enlarge triangle ABC by scale factor 3 about the centre of enlargement X.

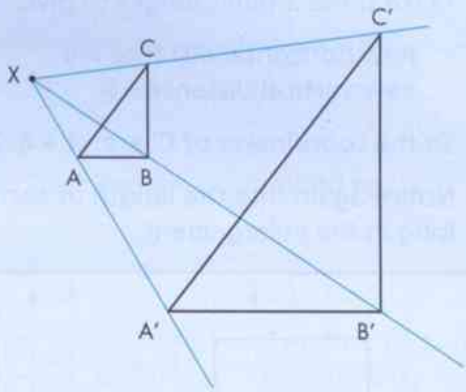


Notice that the rays have been drawn from the centre of enlargement to each vertex and beyond.

The distance from X to each vertex on triangle ABC is measured and multiplied by 3 to give the distance from X to each vertex A′, B′ and C′ for the enlarged triangle A′B′C′.

Once each image vertex has been found, the whole enlarged shape can then be drawn.

Check the measurements and see for yourself how the calculations have been done.



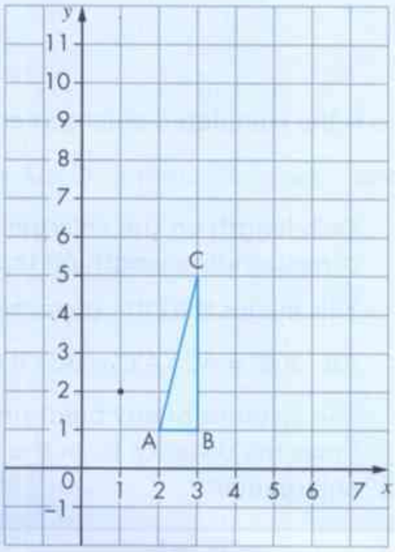
Notice again that the length of each side on the enlarged triangle is three times the length of the corresponding side on the original triangle.

Coordinate method

In this method, you use the coordinates of the vertices to 'count squares'.

Example 7

Enlarge the triangle ABC by scale factor 3 from the centre of enlargement (1, 2).



To find the coordinates of each image vertex, first work out the horizontal and vertical distances from each original vertex to the centre of enlargement.

Then multiply each of these distances by 3 to find the position of each image vertex.

For example, to find the coordinates of C′ work out the distance from the centre of enlargement (1, 2) to the point C(3, 5).

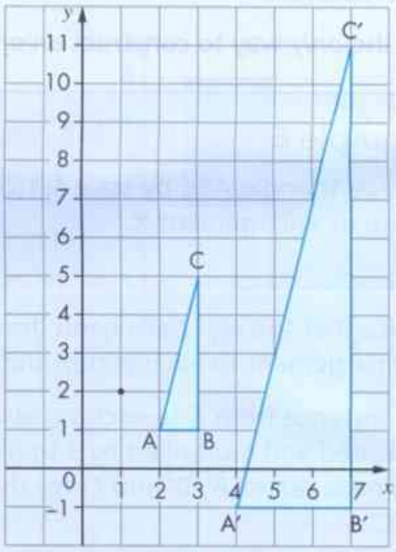
horizontal distance = 2
vertical distance = 3

Make these 3 times longer to give:

new horizontal distance = 6
new vertical distance = 9

So the coordinates of C′ are: (1 + 6, 2 + 9) = (7, 11)

Notice again that the length of each side is three times as long in the enlargement.

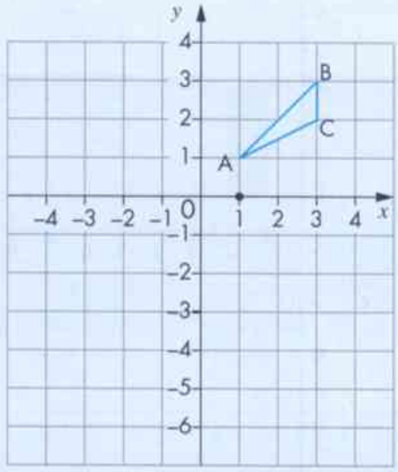


Negative enlargement

A **negative enlargement** produces an image shape on the opposite side of the centre of enlargement to the original shape.

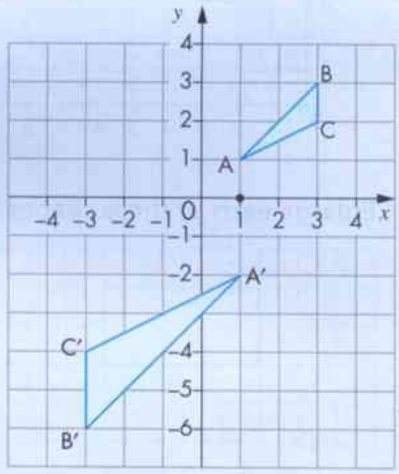
Example 8

Enlarge triangle ABC by scale factor –2, with the centre of enlargement at (1, 0).



You can enlarge triangle ABC to give triangle A′B′C′ by either the ray method or the coordinate method. You calculate the new lengths on the opposite side of the centre of enlargement to the original shape.

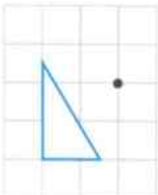
Notice how a negative scale factor also inverts the original shape.



EXERCISE 30F

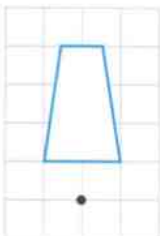
1 Copy each of these figures with its centre of enlargement. Then use the ray method to enlarge it by the given scale factor.

a



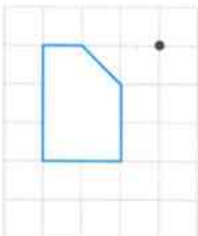
Scale factor 2

b



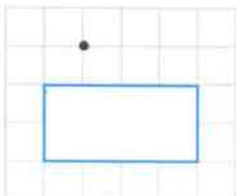
Scale factor 3

c



Scale factor 2

d



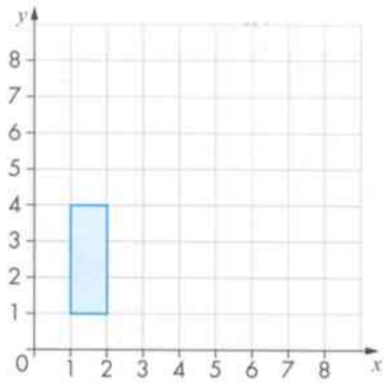
Scale factor 3

CORE

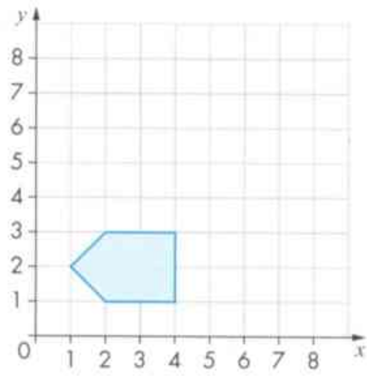
CORE

2 Copy each of these diagrams onto squared paper and enlarge it by scale factor 2, using the origin as the centre of enlargement.

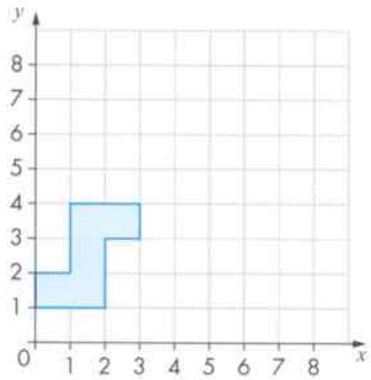
a



b



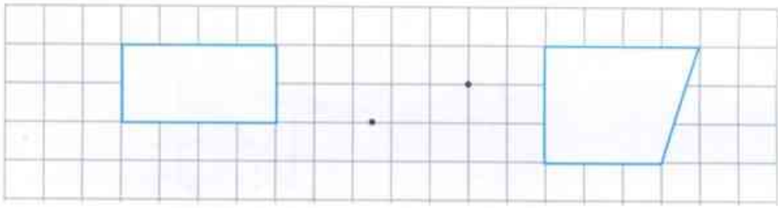
c



Advice and Tips

Even if you are using a counting square method, you can always check by using the ray method.

3 Enlarge each of these shapes by a scale factor of $\frac{1}{2}$ about its closest centre of enlargement.



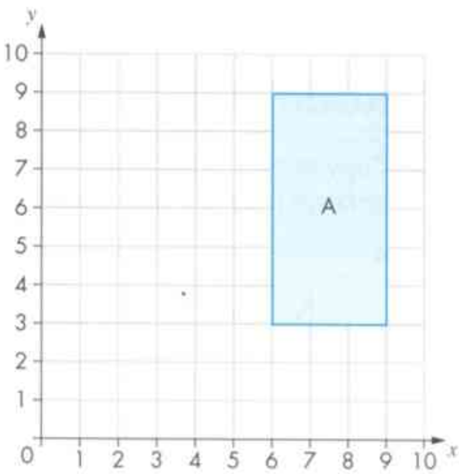
4 Copy this diagram onto squared paper.

a Enlarge the rectangle A by scale factor $\frac{1}{3}$ about the origin. Label the image B.

b Write down the ratio of the lengths of the sides of rectangle A to the lengths of the sides of rectangle B.

c Work out the ratio of the perimeter of rectangle A to the perimeter of rectangle B.

d Work out the ratio of the area of rectangle A to the area of rectangle B.



30.7 Enlargements: 2

E

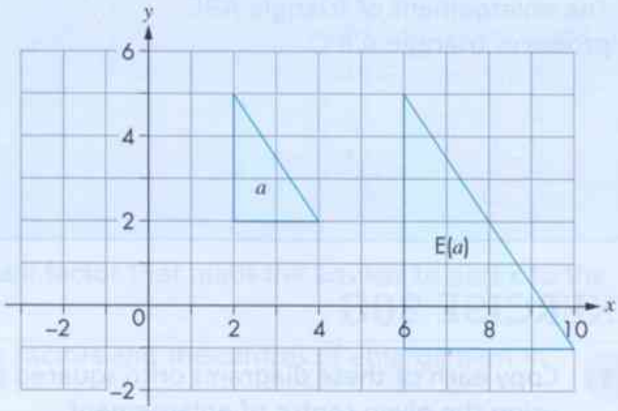
Finding the centre of enlargement

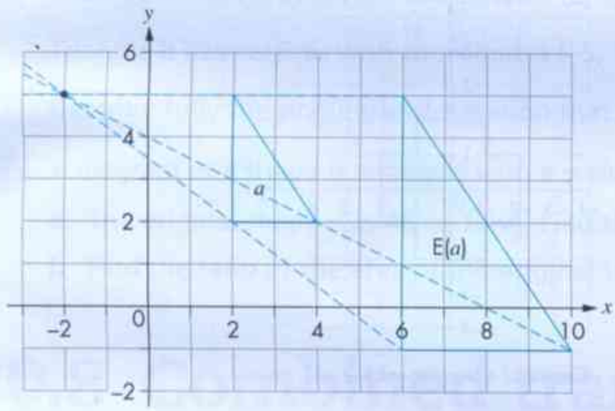
You can find the centre of an enlargement by drawing rays.

Example 9

E(a) is an enlargement of shape a.

Find the centre and the scale factor of the enlargement.





Draw rays through corresponding points on the object and image. They should all meet at one point. This is the centre of enlargement.

In this case the centre is (–2, 5).

To find the scale factor, find the ratio of corresponding sides.

$$\text{Scale factor} = \frac{\text{height of E}(a)}{\text{height of } a}$$

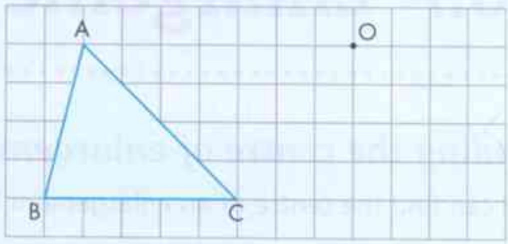
$$= \frac{6}{3} = 2$$

Fractional enlargement

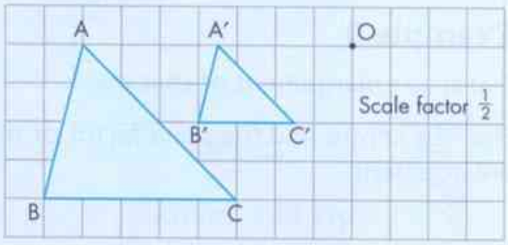
Strange but true ... you can have an enlargement in mathematics that is actually smaller than the original shape!

Example 10

Enlarge triangle ABC by a scale factor of $\frac{1}{2}$ about the centre of enlargement O.



The enlargement of triangle ABC produces triangle A′B′C′.

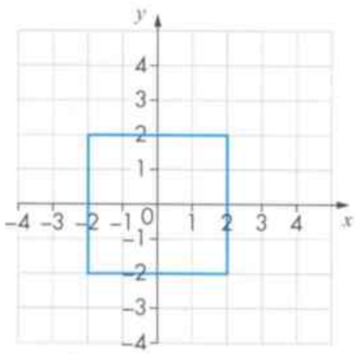


EXERCISE 30G

EXTENDED

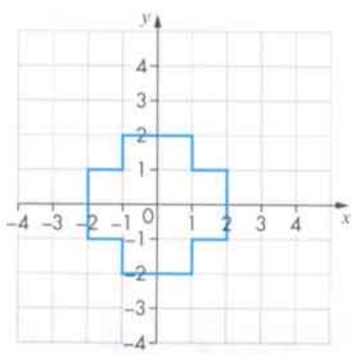
1 Copy each of these diagrams onto squared paper and enlarge it by scale factor 2, using the given centre of enlargement.

a



Centre of enlargement (–1, 1)

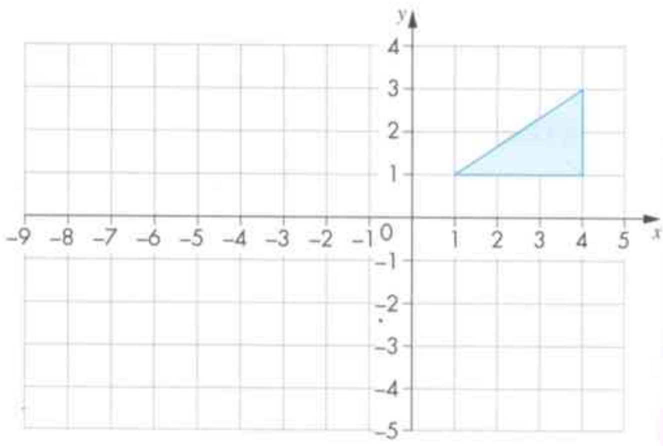
b



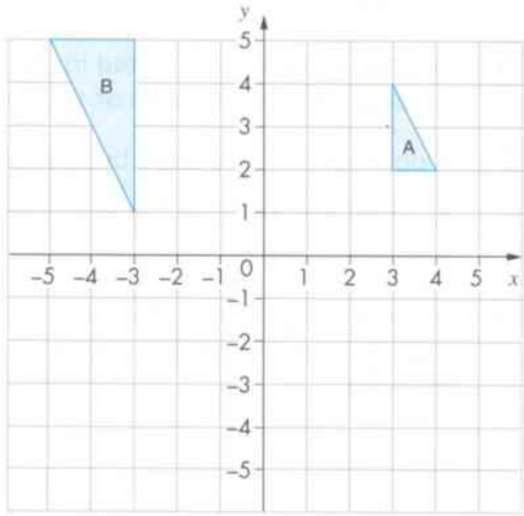
Centre of enlargement (–2, –3)

2 Copy this diagram onto squared paper.

Enlarge the triangle by scale factor –2 about the origin.



3 Copy this diagram onto squared paper.



a Enlarge A by a scale factor of 3 about a centre (4, 5).

b Enlarge B by a scale factor $\frac{1}{2}$ about a centre (−1, −3).

c Enlarge B by scale factor $-\frac{1}{2}$ about a centre (−3, −1).

d What is the cèntre of enlargement and scale factor that maps B onto A?

e What is the centre of enlargement and scale factor that maps A onto B?

f What is the centre of enlargement and scale factor that maps the answer to part **b** to the answer to part **c**?

g What is the centre of enlargement and scale factor that maps the answer to part **c** to the answer to part **b**?

h What is the connection between the scale factors and the centres of enlargement in parts **d** and **e**, and in parts **f** and **g**?

4 Triangle A has vertices with coordinates (2, 1), (4, 1) and (4, 4).

Triangle B has vertices with coordinates (−5, 1), (−5, 7) and (−1, 7).

Describe fully the single transformation that maps triangle A onto triangle B.

5 A diagram of a shape is enlarged with a scale factor of 0.2

a The original shape was 48 cm long. Find the length of the enlargement.

b Find the ratio of the area of the original shape to the area of the enlargement.

30.8 Combined transformations

E

Sometimes you will need to use more than one **transformation** to produce the required image from the given object. In this exercise, you will revise the transformations you have met so far.

Remember, to describe:

- a **translation** fully, you need to use a vector
- a **reflection** fully, you need to use a mirror line
- a **rotation** fully, you need a centre of rotation, an angle of rotation and the direction of turn
- an **enlargement** fully, you need a centre of enlargement and a scale factor.

EXERCISE 30H

EXTENDED

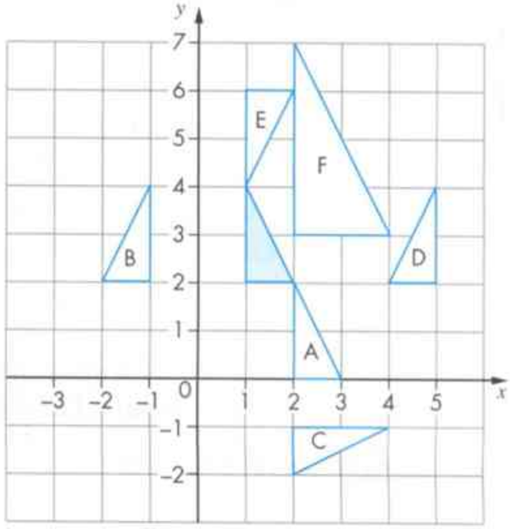
1 The point P(3, 4) is reflected in the x-axis, then rotated by 90° clockwise about the origin. What are the coordinates of the image of P?

2 A point Q(5, 2) is rotated by 180° about the origin, then reflected in the x-axis.

a What are the coordinates of the image of Q?

b What single transformation would have taken point Q directly to the image point?

3 Describe fully the transformations that will map the shaded triangle onto each of the triangles A–F.



4 Describe fully the transformations that will result in the movement of:

a T_1 to T_2

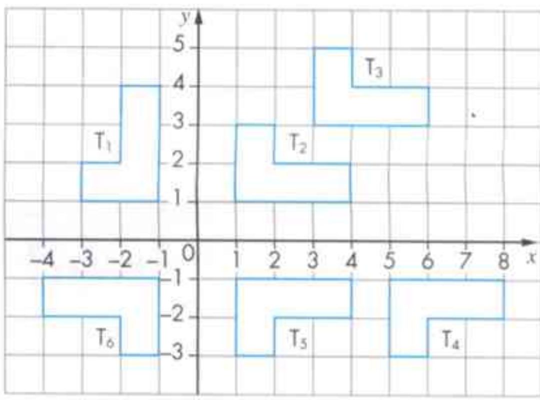
b T_1 to T_6

c T_2 to T_3

d T_6 to T_2

e T_6 to T_5

f T_5 to T_4.



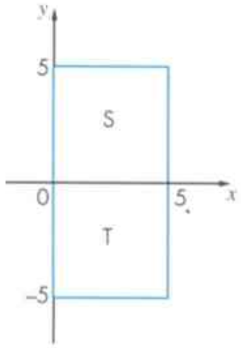
5 **a** Plot a triangle t with vertices (1, 1), (2, 1), (1, 3).

b Reflect triangle t in the y-axis and label the image M(t).

c Rotate triangle M(t) 90° anticlockwise about the origin and label the image RM(t).

d Describe fully the transformation that will move triangle RM(t) back to triangle t.

6 Describe fully at least three different transformations that could move the square labelled S to the square labelled T.



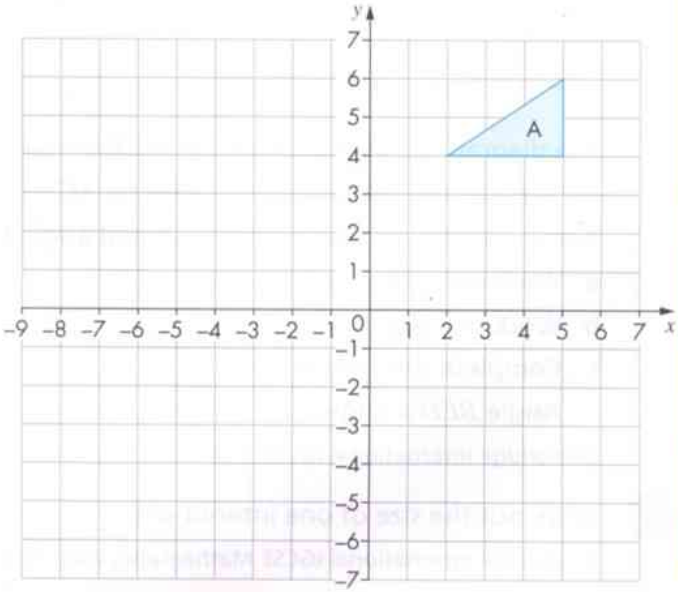
7 Copy the diagram onto squared paper.

a Triangle A is translated by the vector $\begin{pmatrix} -1.5 \\ -3 \end{pmatrix}$ to give triangle B.

Triangle B is then enlarged by a scale factor –2 about the origin to give triangle C.

Draw triangles B and C on the diagram.

b Describe fully the single transformation that maps triangle C onto triangle A.



Check your progress

Core

- I can reflect a simple plane figure in a horizontal or vertical line
- I can rotate a simple plane figure through a multiple of 90° about the origin or a vertex or the midpoint of an edge
- I can translate simple plane figures
- I can enlarge simple plain figures with a positive scale factor, including fractions
- I can recognise and describe reflections, rotations, translations and enlargements

Extended

- I can reflect a simple plane figure
- I can rotate a simple plane figure
- I can enlarge simple plain figures with a negative scale factor

Examination questions: Geometry

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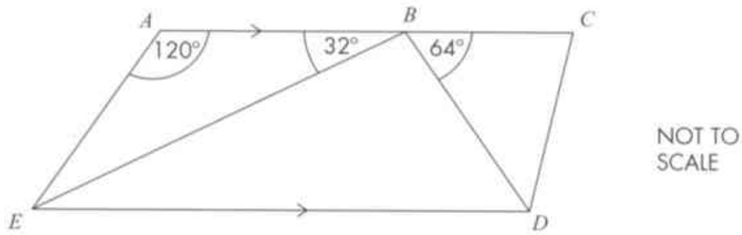
PAPER 1

CORE

1



The diagram shows quadrilateral $ACDE$.

AC is parallel to ED and B is a point on AC.

Angle $EAB = 120°$, angle $ABE = 32°$ and angle $CBD = 64°$.

a Work out angle EBD. [1]

b Work out angle AEB. [1]

c Complete this statement.

Angle BED = angle ABE because they are .... angles. [1]

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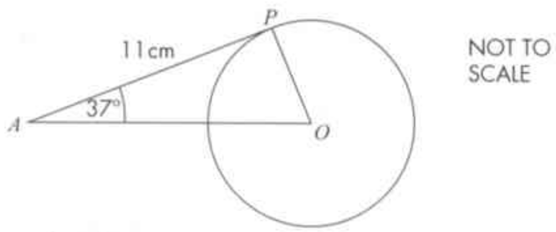
2 Work out the size of one interior angle of a regular 15-sided polygon. [3]

Cambridge International IGCSE Mathematics 0580 *Paper 11 Q14 Oct/Nov 2015*

3



In the diagram, AP is a tangent to the circle at P.

O is the centre of the circle, angle $PAO = 37°$ and $AP = 11$ cm.

a Write down the size of angle OPA. [1]

b Work out the radius of the circle. [2]

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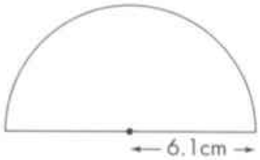
4



NOT TO SCALE

A protractor is a semi-circle of radius 6.1 cm.

Calculate the **perimeter** of the protractor. [3]

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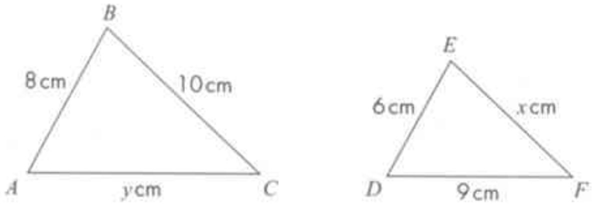
5



NOT TO SCALE

Triangle ABC is similar to triangle DEF.

Calculate the value of

a x, [2]

b y. [2]

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6 Six donkeys are **each** given two 5 ml spoons of medicine three times each day.

Calculate the number of whole days a 2 litre bottle of medicine will last. [3]

Cambridge International IGCSE Mathematics 0580 *Paper 11 Q14 May/June 2015*

7 A cuboid has volume 288 cm^3.

a The cuboid has length 12 cm and width 5 cm.

Calculate the height of the cuboid. [2]

b 1 cm^3 of the cuboid has a mass of 4 g.

Work out the mass of the cuboid. [1]

Cambridge International IGCSE Mathematics 0580 *Paper 11 Q15 May/June 2015*

9 Write each of the following as a single vector.

a $\begin{pmatrix} 6 \\ 1 \end{pmatrix} + \begin{pmatrix} -4 \\ 2 \end{pmatrix}$ [1]

b $4\begin{pmatrix} 2 \\ -3 \end{pmatrix}$ [1]

Cambridge International IGCSE Mathematics 0580 *Paper 11 Q5 May/June 2013*

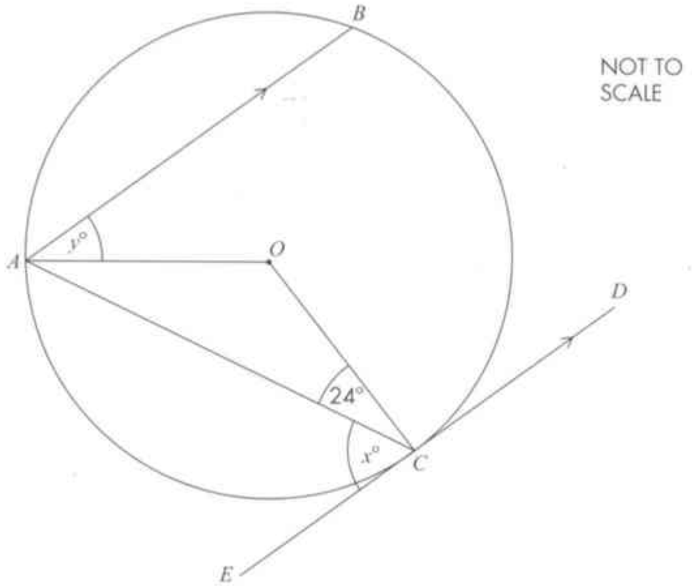
9 A cylinder has radius 3.6 cm and height 16 cm.

Calculate the volume of the cylinder. [2]

Cambridge International IGCSE Mathematics 0580 *Paper 11 Q9 Oct/Nov 2014*

CORE

10



The diagram shows a circle with centre O.

ED is a tangent to the circle at C.

AB is parallel to ED and angle $ACO = 24°$.

Find the value of

a x, [1]

b y. [2]

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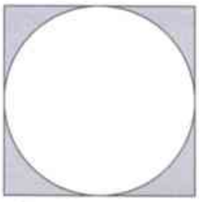
11



The diagram shows a circle inside a square.

The circumference of the circle touches all four sides of the square.

a Calculate the area of the circle when the side of the square is 15 cm. [2]

b Draw all the lines of symmetry on the diagram. [2]

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12

The diagram shows the route of a ship that leaves a port, P.

It travels due west for 16 km and then changes course to due south for 9 km.

a Calculate the straight line distance PQ. [2]

b Use trigonometry to calculate the bearing of P from Q. [2]

Cambridge International IGCSE Mathematics 0580 *Paper 11 Q22 Oct/Nov 2015*

PAPER 3

1 Irina has some solid building blocks.

a Write down the mathematical name of this solid. [1]

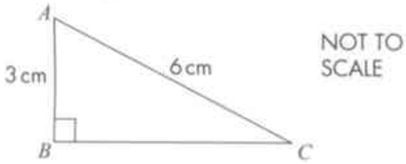
b Irina describes the shape of a different block.

She says:

It has 12 edges and 8 vertices. All the faces are the same shape.

Write down the mathematical name of this solid. [1]

c The diagram shows the end face of another block.



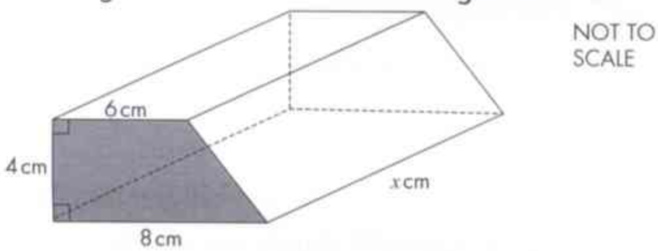
i Show that $BC = 5.2$ cm, correct to 1 decimal place. [3]

ii Find the area of triangle ABC. [2]

iii This block is a triangular prism with length 8 cm.

Calculate the volume of the block. [1]

d The diagram shows another building block.



i Calculate the area of the end face of this block. [2]

ii The volume of this block is 336 cm^3.

Find the value of x. [1]

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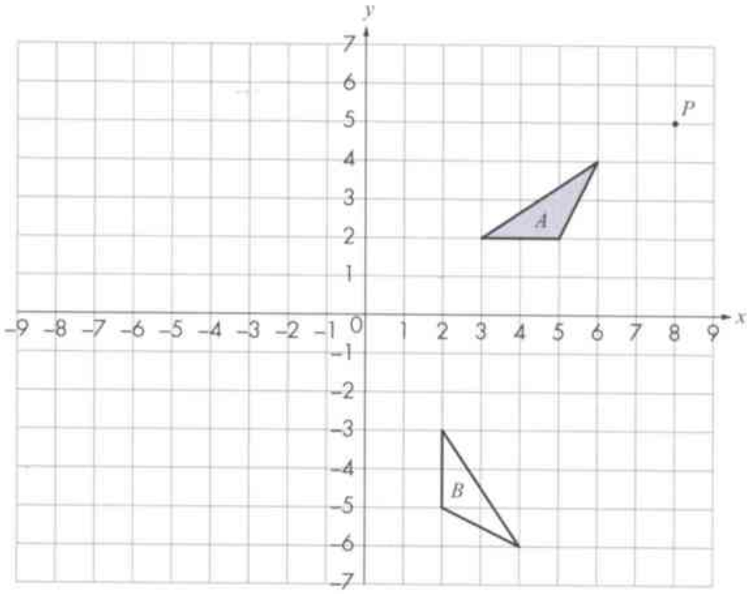
2 a

i Write down the order of rotational symmetry of this shape. [1]

ii Draw the lines of symmetry on the shape. [2]

CORE

b



i On the grid, reflect triangle A in the **line** $x = -1$. [2]

ii On the grid, enlarge triangle A with centre P and scale factor 3. [2]

iii Describe fully the **single** transformation that maps triangle A onto triangle B. [3]

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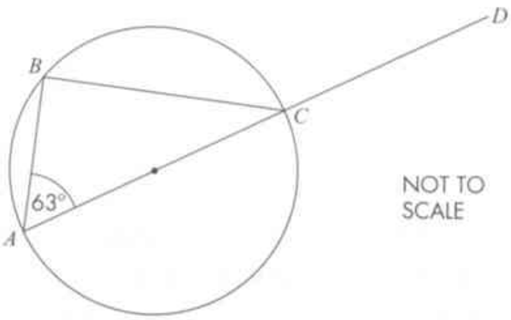
3 **a**



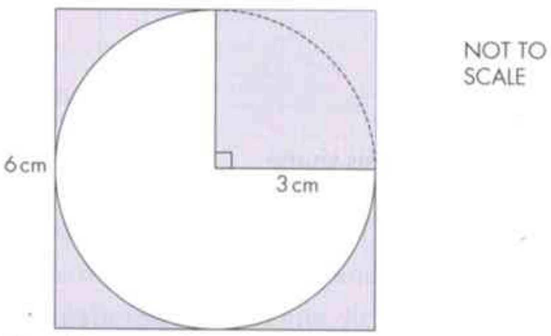
A, B and C lie on a circle with diameter AC.

AC is extended to D and angle $BAC = 63°$.

Work out angle BCD.

Give reasons to explain your answer. [4]

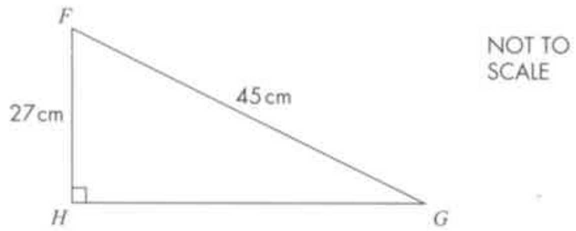
b



The diagram shows a circle with radius 3 cm inside a square of side 6 cm.

Calculate the shaded area. [5]

c



FGH is a right-angled triangle.

Calculate

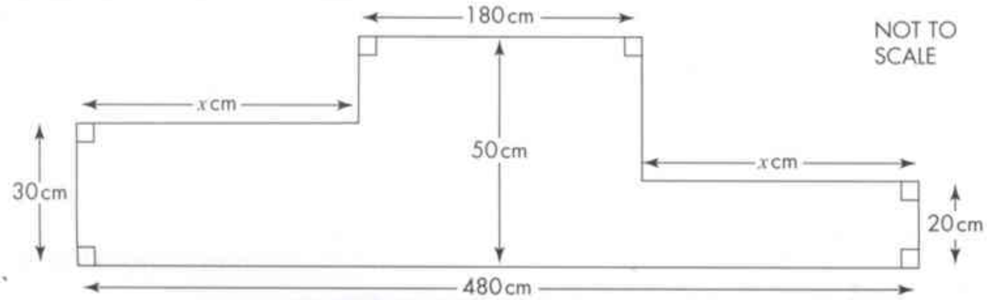
i *GH*, [3]

ii the perimeter of the triangle, [1]

iii the area of the triangle. [2]

Cambridge International IGCSE Mathematics 0580 *Paper 31 Q8 May/June 2015*

4



The diagram shows the cross section of a medal presentation platform.

a Show that $x = 150$. [2]

b Work out the perimeter of the cross section. [2]

c i Calculate the area of the cross section. [2]

ii The platform is a prism, 170 cm deep.

Find the volume of the platform. [1]

iii The prism is completely filled with a light material.

1 **cubic metre** of this material has mass 16 kg.

Calculate the mass of the material used. [2]

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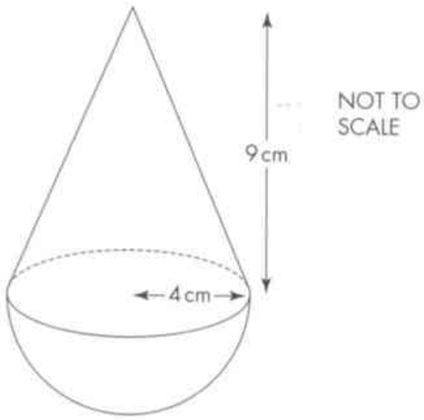
CORE

5



The diagram shows a toy.

The shape of the toy is a cone, with radius 4 cm and height 9 cm, on top of a hemisphere with radius 4 cm.

Calculate the volume of the toy.

Give your answer correct to the nearest cubic centimetre. [4]

[The volume, V, of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.]

[The volume, V, of a sphere with radius r is $\frac{4}{3}\pi r^3$.]

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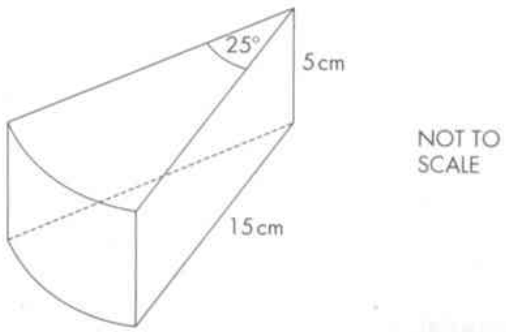
6



The diagram shows a wooden prism of height 5 cm.

The cross section of the prism is a sector of a circle with sector angle 25°.

The radius of the sector is 15 cm.

Calculate the total surface area of the prism. [5]

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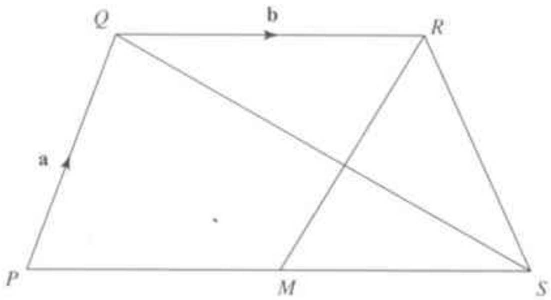
PAPER 2

1



NOT TO SCALE

$PQRS$ is a quadrilateral and M is the midpoint of PS.

$\overrightarrow{PQ} = \mathbf{a}$, $\overrightarrow{QR} = \mathbf{b}$ and $\overrightarrow{SQ} = \mathbf{a} - 2\mathbf{b}$.

a Show that $\overrightarrow{PS} = 2\mathbf{b}$. [1]

b Write down the mathematical name for the quadrilateral $PQRM$, giving reasons for your answer. [2]

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2 ZEBRA

Write down the letters in the word above that have

a exactly one line of symmetry, [1]

b rotational symmetry of order 2. [1]

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3 A triangle has sides of length 2 cm, 8 cm and 9 cm.

Calculate the value of the largest angle in this triangle. [4]

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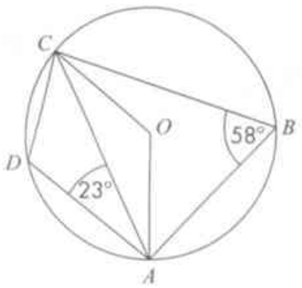
4



NOT TO SCALE

A, B, C and D lie on a circle centre O.

Angle $ABC = 58°$ and angle $CAD = 23°$.

Calculate

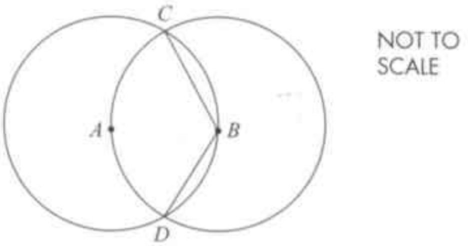
a angle OCA, [2]

b angle DCA. [2]

Cambridge International IGCSE Mathematics 0580 *Paper 21 Q13 May/June 2014*

EXTENDED

5

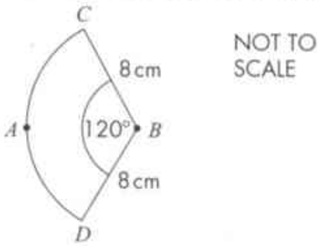


Two circles, centres A and B, are each of radius 8 cm and intersect at C and D.

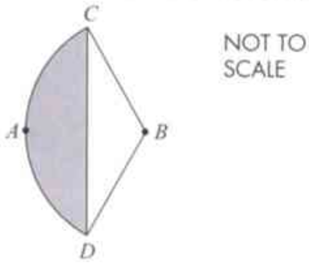
Each circle passes through the centre of the other circle.

a Explain why angle CBD is 120°. [1]

b For the circle, centre B, find the area of the sector BCD. [2]



c **i** Find the area of the shaded segment CAD. [3]



ii Find the area of overlap of the two circles. [1]

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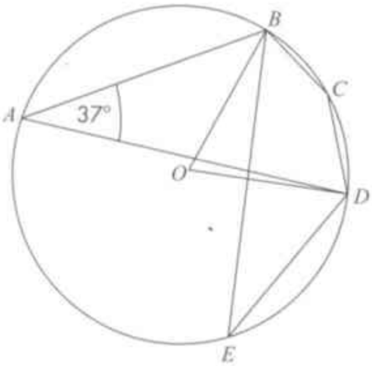
PAPER 4

1



NOT TO SCALE

A, B, C, D and E are points on the circle, centre O.

Angle $BAD = 37°$.

Complete the following statements.

a Angle BED = ... because .... [2]

b Angle BOD = ... because .... [2]

c Angle BCD = ... because .... [2]

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2 a Andrei stands on level horizontal ground, 294 m from the foot of a vertical tower which is 55 m high.

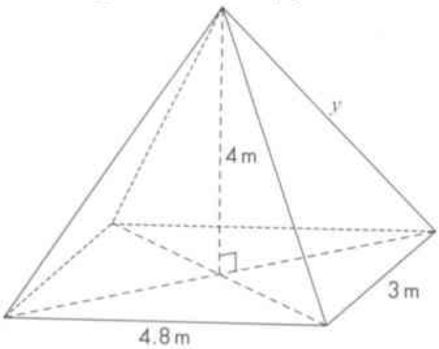
i Calculate the angle of elevation of the top of the tower. [2]

ii Andrei walks a distance x metres directly towards the tower.

The angle of elevation of the top of the tower is now 24.8°.

Calculate the value of x. [4]

b The diagram shows a pyramid with a horizontal rectangular base.



NOT TO SCALE

The rectangular base has length 4.8 m and width 3 m and the height of the pyramid is 4 m.

Calculate

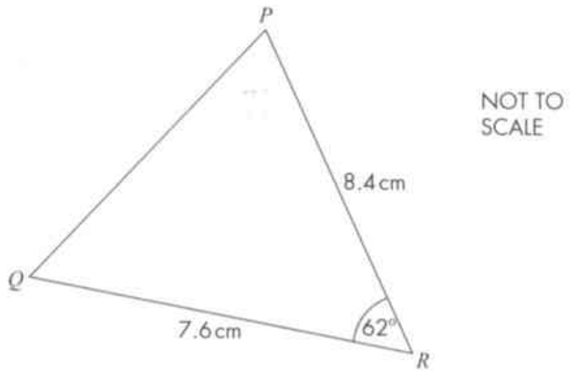
i y, the length of a sloping edge of the pyramid, [4]

ii the angle between a sloping edge and the rectangular base of the pyramid. [2]

Cambridge International IGCSE Mathematics 0580 *Paper 41 Q5 May/June 2015*

EXTENDED

3 a



In the triangle PQR, QR = 7.6 cm and PR = 8.4 cm.

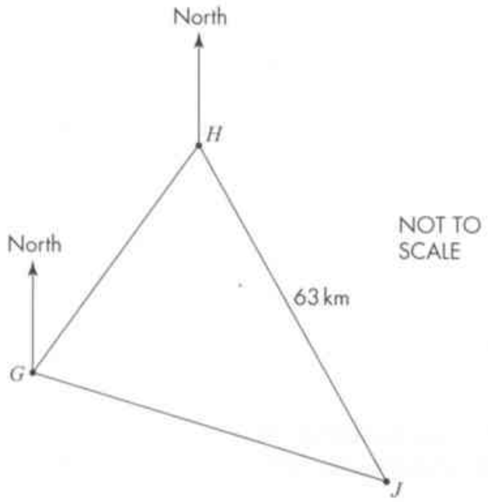
Angle QRP = 62°.

Calculate

i PQ, [4]

ii the area of triangle PQR. [2]

b



The diagram shows the positions of three small islands G, H and J.

The bearing of H from G is 045°.

The bearing of J from G is 126°.

The bearing of J from H is 164°.

The distance HJ is 63 km.

Calculate the distance GJ. [5]

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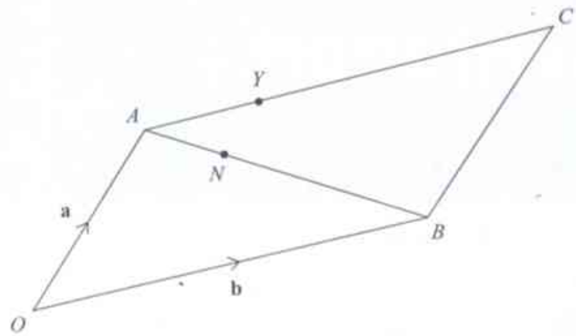
4 a $\overrightarrow{PQ} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$

i P is the point (−2, 3).

Work out the co-ordinates of Q. [1]

ii Work out $|\overrightarrow{PQ}|$, the magnitude of $\overrightarrow{PQ}$. [2]

b



$OACB$ is a parallelogram.

$\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.

$AN{:}NB = 2{:}3$ and $AY = \frac{2}{5}AC$.

i Write each of the following in terms of **a** and/or **b**.

Give your answers in their simplest form.

a $\overrightarrow{ON}$ [2]

b $\overrightarrow{NY}$ [2]

ii Write down two conclusions you can make about the line segments NY and BC. [2]

Cambridge International IGCSE Mathematics 0580 *Paper 41 Q11 May/June 2014*

Chapter 31

Statistical representation

Topics	Level	Key words
1 Frequency tables	CORE	tally chart, frequency, frequency table, classes, class interval, grouped frequency table
2 Pictograms	CORE	pictogram, symbol, key
3 Bar charts	CORE	bar chart, axis
4 Pie charts	CORE	pie chart, angle, sector
5 Scatter diagrams	CORE	scatter diagram, variables, correlation, positive correlation, negative correlation, zero correlation, line of best fit
6 Histograms	CORE	histogram
7 Histograms with bars of unequal width	EXTENDED	class frequency, frequency density

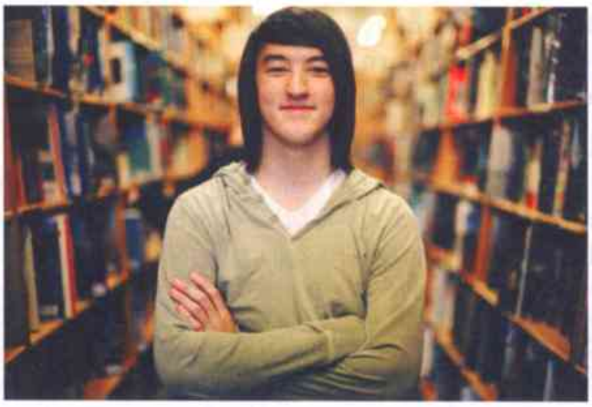
In this chapter you will learn how to:

CORE	EXTENDED
• Collect, classify and tabulate statistical data. (C9.1 and E9.1) • Read, interpret and draw simple inferences from tables and statistical diagrams. Compare sets of data using tables, graphs and statistical measures. Appreciate restrictions on drawing conclusions from given data. (C9.2 and E9.2) • Construct and interpret bar charts, pie charts, pictograms, simple frequency distributions, histograms with equal intervals and scatter diagrams. (C9.3 and E9.3) • Understand what is meant by positive, negative and zero correlation with reference to a scatter diagram. (C9.7 and E9.7) • Draw, interpret and use lines of best fit by eye. (C9.8 and E9.8)	• Construct and read histograms with equal and unequal intervals (areas proportional to frequencies and vertical axis labelled 'frequency density'). (E9.3)

Why this chapter matters

Statistical graphs such as bar charts and line graphs are used in many areas of life from science to politics. They help us to analyse and interpret information.

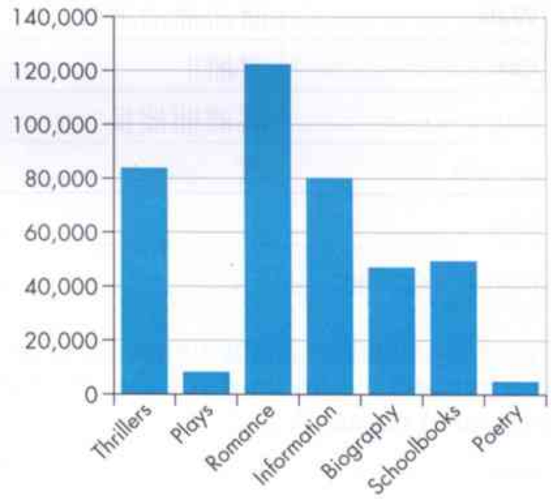
One of the best ways to analyse information is to present it in a visual form. Some of the earliest types of statistical diagram were line graphs, bar charts and pie charts. They all show information in different ways.

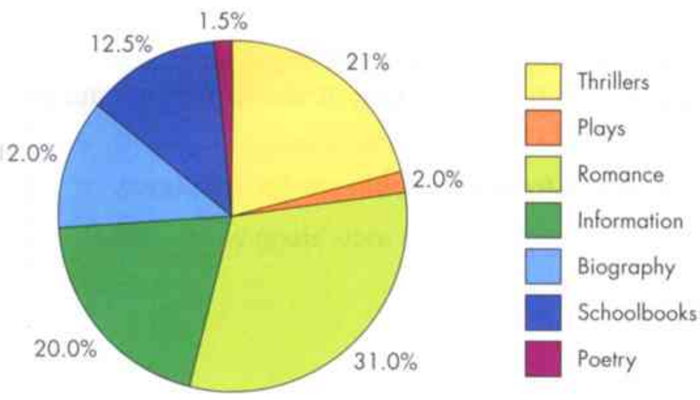


Think about the owner of a bookshop.

He might use a graph like the one below to show how his sales go up and down over the year. Graphs like these are particularly good at showing trends in figures over time (see Chapter 14).

He might use a bar chart like the one on the right to show how many books they sell in different categories. Bar charts are very good at showing actual numbers.





And he can get an idea of the percentage of different types of book he sells out of total sales by using a pie chart like the one on the left. Pie charts are good for analysing a whole (100%) by its parts.

This chapter introduces you to some of the most common forms of statistical representation. They fall into two groups – graphical diagrams such as bar charts and pie charts and quantitative diagrams such as frequency tables.

Chapter 31 . Topic 1

31.1 Frequency tables

Statistics is concerned with the collection and organisation of data, the representation of data on diagrams and the interpretation of data.

When you are collecting data for simple surveys, it is usual to use a **tally chart**. For example, data collection sheets are used to gather information about how people travel to work, how students spend their free time and the amount of time people spend watching TV.

It is easy to record the data by using tally marks, as shown in Example 1. Counting up the tally marks in each row of the chart gives the **frequency** of each category. By listing the frequencies in a column on the right-hand side of the chart, you can make a **frequency table** (see Example 1). Frequency tables are an important part of making statistical calculations.

Example 1

Sandra wanted to find out about the ways in which students travelled to school. She carried out a survey. Her frequency table looked like this.

Method of travel	Tally	Frequency
Walk	卌 卌 卌 卌 卌 |||	28
Car	卌 卌 ||	12
Bus	卌 卌 卌 卌 |||	23
Bicycle	卌	5
Taxi	||	2

What does it tell you?

By adding together all the frequencies, you can see that 70 students took part in the survey. The frequencies also show you that more students travelled to school on foot than by any other method of transport.

Grouped data

Many surveys produce a lot of data that covers a wide range of values. In these cases, it is sensible to put the data into groups before attempting to compile a frequency table. These groups of data are called **classes** or **class intervals**.

Once the data has been grouped into classes, a **grouped frequency table** can be completed. The method is shown in Example 2.

Example 2

These marks are for 36 students in a Year 10 mathematics examination.

31	49	52	79	40	29	66	71	73	19	51	47
81	67	40	52	20	84	65	73	60	54	60	59
25	89	21	91	84	77	18	37	55	41	72	38

a Construct a frequency table, using classes of 1–20, 21–40 and so on.

b What was the most frequent interval of marks?

a Draw the grid of the table shown below and put in the headings.

Next, list the classes, in order, in the column headed 'Marks'.

Using tally marks, indicate each student's score against the class to which it belongs. For example, 81, 84, 89 and 91 belong to the class 81–100, giving five tally marks, as shown below.

Finally, count the tally marks for each class and enter the result in the column headed 'Frequency'. The table is now complete.

Marks	Tally	Frequency
1–20	|||	3
21–40	~~||||~~ |||	8
41–60	~~||||~~ ~~||||~~ |	11
61–80	~~||||~~ ||||	9
81–100	~~||||~~	5

b From the grouped frequency table, you can see that the highest number of students obtained a mark in the 41–60 interval.

EXERCISE 31A

1 Kurt kept a record of the number of goals scored by his local team in the last 20 matches. These are his results.

0 1 1 0 2 0 1 3 2 1

0 1 0 3 2 1 0 2 1 1

a Draw a frequency table for his data.

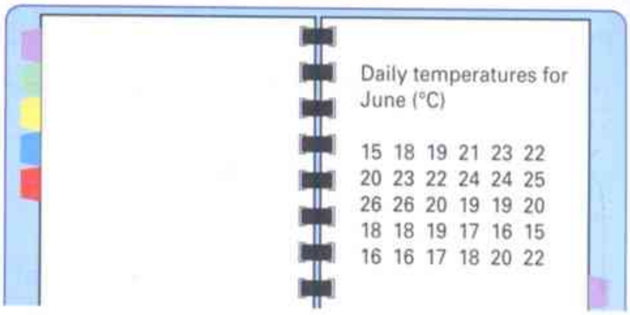
b Which was the most frequent score?

c How many goals were scored in total for the 20 matches?

CORE

CORE

2 Monique was doing a geography project on the weather. As part of her work, she kept a record of the daily midday temperatures in June.



a Copy and complete the grouped frequency table for her data.

b In which interval do the most temperatures lie?

c Describe what the weather was probably like throughout the month.

Temperature (°C)	Tally	Frequency
14–16		
17–19		
20–22		
23–25		
26–28		

3 In a game, Mitesh used a six-sided dice. He decided to keep a record of his scores to see whether the dice was fair. These are his scores.

2 4 2 6 1 5 4 3 3 2 3 6 2 1 3
5 4 3 4 2 1 6 5 1 6 4 1 2 3 4

a Draw a frequency table for his data.

b How many throws did Mitesh have during the game?

c Do you think the dice was a fair one? Explain why.

4 The data shows the heights, in centimetres, of a sample of 32 students.

172	158	160	175	180	167	159	180
167	166	178	184	179	156	165	166
184	175	170	165	164	172	154	186
167	172	170	181	157	165	152	164

a Draw a grouped frequency table for the data, using class intervals 151–155, 156–160, …

b In which interval do the most heights lie?

c Does this agree with a survey of the students in your class?

5 A student used a stopwatch to time how long it took her rabbit to find food left in its hutch.

This is her record, in seconds.

7	30	14	27	8	31	8	28	10	41	51	37	15	21	37	16	38
23	20	9	11	55	9	33	8	35	45	35	25	25	49	23	43	55
45	8	13	9	39	12	57	16	37	26	32	19	48	29	37		

Find the best way to put this data into a frequency chart to illustrate the length of time it took the rabbit to find the food.

6 A student was doing a survey to find the ages of people at a football competition.

He said that he would make a frequency table with the regions 15–20, 20–25, 25–30.

Explain what difficulty he could have with these class divisions.

31.2 Pictograms

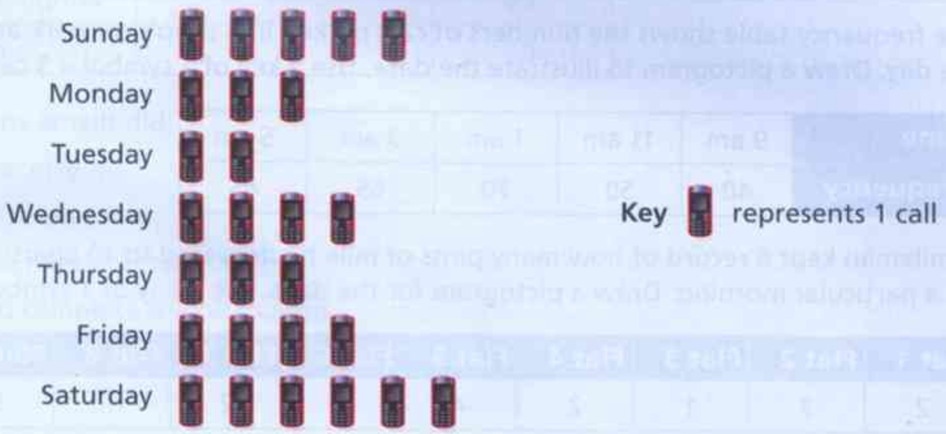
Data collected from a survey can be presented in pictorial or diagrammatic form to help people to understand it more quickly. You see plenty of examples of this in newspapers and magazines and on TV, where every type of visual aid is used to communicate statistical information.

Pictograms

A **pictogram** is a frequency table in which frequency is represented by a repeated **symbol**. The symbol itself usually represents a number of items, as Example 4 on the next page shows. However, sometimes it is more sensible to let a symbol represent just a single unit, as in Example 3 below. The **key** tells you how many items are represented by a symbol.

Example 3

The pictogram shows the number of phone calls made by Nurul from her mobile phone during a week.



How many calls did Nurul make in the week?

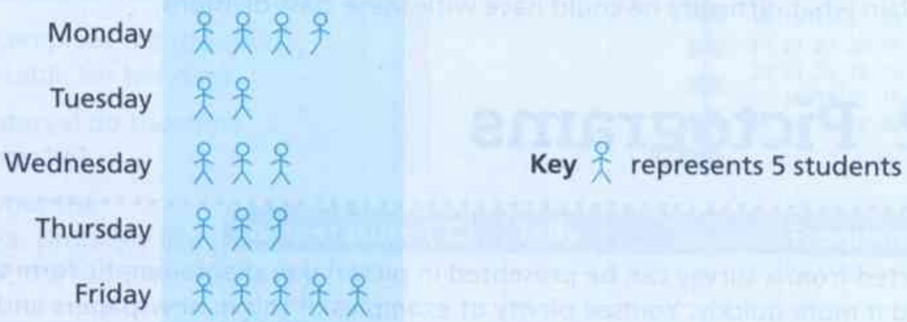
From the pictogram, you can see that Nurul made a total of 27 calls.

Although pictograms can have great visual impact (particularly as used in advertising) and are easy to understand, they have a serious drawback. Apart from a half, fractions of a symbol cannot usually be drawn accurately and so frequencies are often represented only approximately by symbols.

Example 4 on the next page highlights this difficulty.

Example 4

The pictogram shows the numbers of students who were late for school during a week.



How many students were late on:

a Monday **b** Thursday?

Precisely how many students were late on Monday and Thursday respectively?

If you assume that each 'limb' of the symbol represents one student and its 'body' also represents one student, then the answers are:

a 19 students were late on Monday. **b** 13 on Thursday.

EXERCISE 31B

CORE

1 The frequency table shows the numbers of cars parked in a shop's car park at various times of the day. Draw a pictogram to illustrate the data. Use a key of 1 symbol = 5 cars.

Time	9 am	11 am	1 am	3 am	5 am
Frequency	40	50	70	65	45

2 A milkman kept a record of how many pints of milk he delivered to 10 apartments on a particular morning. Draw a pictogram for the data. Use a key of 1 symbol = 1 pint.

Flat 1	Flat 2	Flat 3	Flat 4	Flat 5	Flat 6	Flat 7	Flat 8	Flat 9	Flat 10
2	3	1	2	4	3	2	1	5	1

3 The pictogram, taken from a Suntours brochure, shows the average daily hours of sunshine for five months in Tenerife.

a Write down the average daily hours of sunshine for each month.

b Give a reason why pictograms are useful in holiday brochures.

CORE

4 The pictogram shows the amounts of money collected by six students after they had completed a sponsored walk for charity.

Anthony	$ $ $ $ $
Ben	$ $ $ $ $ $
Emma	$ $ $ $ $
Leanne	$ $ $ $
Reena	$ $ $ $ $ $
Simon	$ $ $ $ $ $ $

Key $ represents $5

a Who raised the most money?

b How much money was raised altogether by the six students?

c Robert also took part in the walk and raised $32. Why would it be difficult to include him on the pictogram?

5 A newspaper showed this pictogram about a family and the numbers of emails each family member received during one Sunday.

		Frequency
Dad		
Mum		
Teenage son		
Teenage daughter		23
Young son		9

Key  represents 4 emails

a How many emails did:

i Dad receive

ii Mum receive

iii the teenage son receive?

b Copy and complete the pictogram.

c How many emails were received altogether?

31.3 Bar charts

A **bar chart** consists of a series of bars or blocks of the *same* width, drawn either vertically or horizontally from an **axis**.

The heights or lengths of the bars always represent *frequencies*.

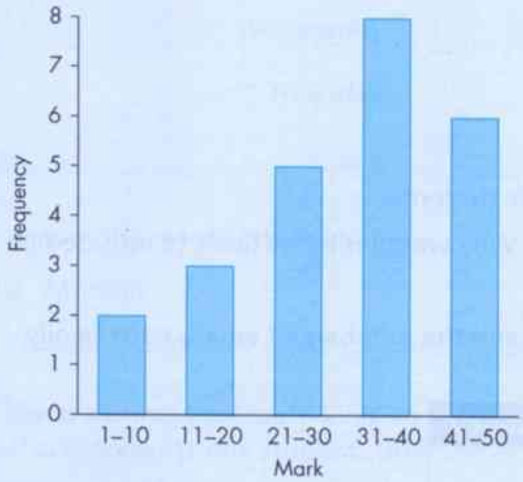
Sometimes, the bars are separated by narrow gaps of equal width, which makes the chart easier to read.

Example 5

The grouped frequency table shows the marks of 24 students in a test.

Marks	1–10	11–20	21–30	31–40	41–50
Frequency	2	3	5	8	6

Draw a bar chart for the data.



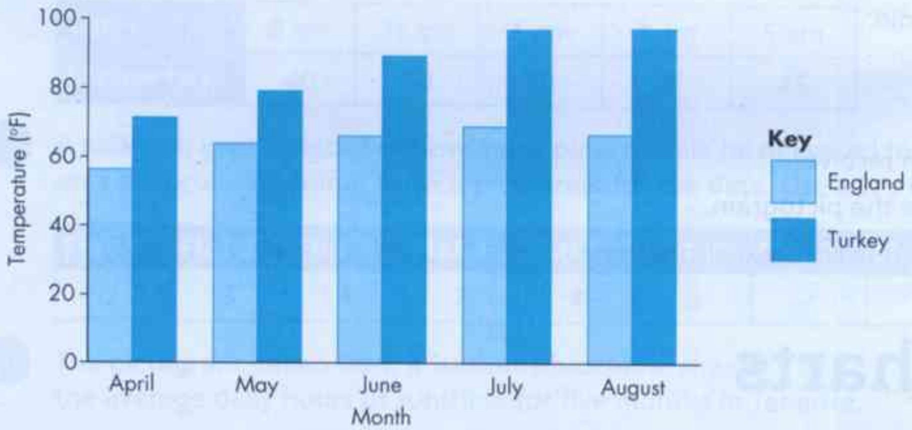
Note:

- Both axes are labelled.
- The class intervals are written under the middle of each bar.
- The bars are separated by equal spaces.

If you use a **dual bar chart**, it is easy to compare two sets of related data, as in Example 6.

Example 6

This dual bar chart shows the average daily maximum temperatures for England and Turkey over a five-month period.



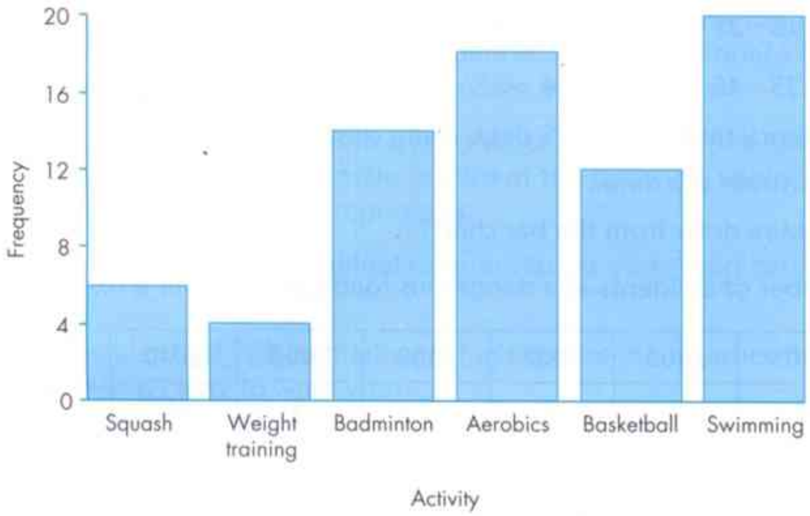
In which month was the *difference* between temperatures in England and Turkey the greatest?

The largest difference can be seen in August.

Note: You must always include a key to identify the two different sets of data.

EXERCISE 31C

1 For her survey on fitness, Samina asked a sample of people, as they left a sports centre, which activity they had taken part in. She then drew a bar chart to show her data.



a Which was the most popular activity?

b How many people took part in Samina's survey?

2 The frequency table below shows the levels achieved by 100 students in their practice IGCSE examinations end of year tests.

Grade	F	E	D	C	B	A
Frequency	12	22	24	25	15	2

a Draw a suitable bar chart to illustrate the data.

b What fraction of the students achieved a grade C or grade B?

c Give one advantage of drawing a bar chart rather than a pictogram for this data.

3 This table shows the number of points Amir and Hasrul were each awarded in eight rounds of a general knowledge quiz.

Round	1	2	3	4	5	6	7	8
Amir	7	8	7	6	8	6	9	4
Hasrul	6	7	6	9	6	8	5	6

a Draw a dual bar chart to illustrate the data.

b Comment on how well each of them did in the quiz.

CORE

4 Mira did a survey about the time it took students in her class to get to school on a particular morning. She wrote down their times, correct to the nearest minute.

15	23	36	45	8	20	34	15	27	49
10	60	5	48	30	18	21	2	12	56
49	33	17	44	50	35	46	24	11	34

a Draw a grouped frequency table for Mira's data, using class intervals 1–10, 11–20, ...

b Draw a bar chart to illustrate the data.

c What conclusions can Mira draw from the bar chart?

5 This table shows the number of accidents at a dangerous road junction over a six-year period.

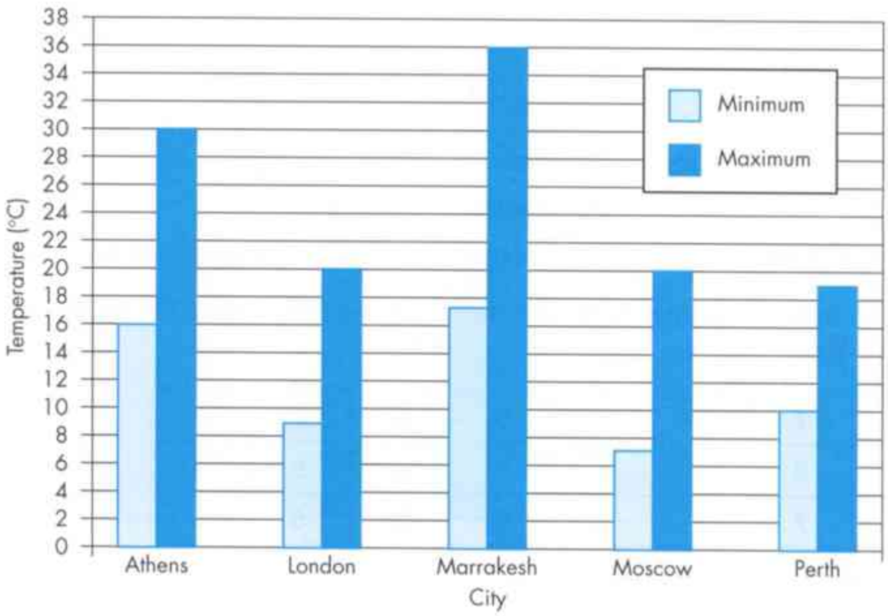
Year	2005	2006	2007	2008	2009	2010
No. of accidents	6	8	7	9	6	4

a Draw a pictogram for the data.

b Draw a bar chart for the data.

c Which diagram would you use if you were going to argue that traffic lights should be installed at the junction? Explain why.

6 The diagram below shows the minimum and maximum temperatures for one day in August in five cities.



Lee says that the minimum temperature is always about half the maximum temperature for most cities.

Is Lee correct?

Give reasons to justify your answer.

31.4 Pie charts

Pictograms, bar charts and line graphs are easy to draw but they can be difficult to interpret when there is a big difference between the frequencies or there are only a few categories. In these cases, it is often more convenient to illustrate the data on a **pie chart**.

In a pie chart, the whole of the data is represented by a circle (the 'pie') and each category of it is represented by a **sector** of the circle (a 'slice of the pie'). The **angle** of each sector is proportional to the frequency of the category it represents.

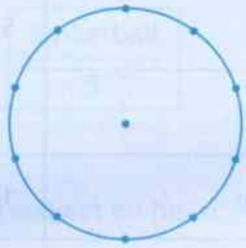
So, a pie chart cannot show individual frequencies, as a bar chart can, for example. It can only show proportions.

Sometimes the pie chart will be marked off in equal sections rather than angles. In these cases, the numbers are always easy to work with.

Example 7

20 people were surveyed about their preferred drink. Their replies are shown in the table.

Drink	Tea	Coffee	Milk	Cola
Frequency	6	7	4	3



Show the results on the pie chart given.

You can see that the pie chart has 10 equally-spaced divisions.

As there are 20 people, each division is worth two people. So the sector for tea will take up 3 of these divisions. In the same way, coffee will take up $3\frac{1}{2}$ divisions, milk will take up 2 divisions and cola will take up $1\frac{1}{2}$ divisions.

The finished pie chart will look like this.

Note:

- You should always label the sectors of the pie chart (use shading and a separate key if there is not enough space to write on the pie chart).
- Give your chart a title.

Example 8

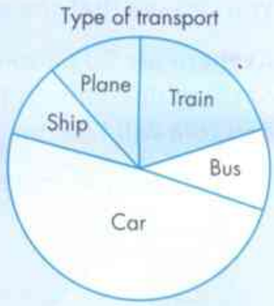
In a survey about holidays, 120 people were asked to state which type of transport they used on their last holiday. This table shows the results of the survey. Draw a pie chart to illustrate the data.

Type of transport	Train	Bus	Car	Ship	Plane
Frequency	24	12	59	11	14

You need to find the angle for the fraction of 360° that represents each type of transport. This is usually done in a table, as shown below.

Type of transport	Frequency	Calculation	Angle
Train	24	$\frac{24}{120} \times 360° = 72°$	72°
Bus	12	$\frac{12}{120} \times 360° = 36°$	36°
Car	59	$\frac{59}{120} \times 360° = 177°$	177°
Ship	11	$\frac{11}{120} \times 360° = 33°$	33°
Plane	14	$\frac{14}{120} \times 360° = 42°$	42°
Totals	120		360°

Draw the pie chart, using the calculated angle for each sector.



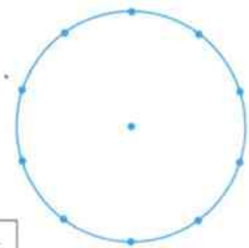
Note:

- Use the frequency total (120 in this case) to calculate each fraction.
- Check that the sum of all the angles is 360°.
- Label each sector.
- The angles or frequencies do not have to be shown on the pie chart.

EXERCISE 31D

CORE

1 Copy the diagram on the right and draw a pie chart to show each set of data.



a The favourite pets of 10 children

Pet	Bird	Cat	Rabbit
Frequency	4	5	1

b The makes of cars of 20 teachers

Make of car	Ford	Toyota	BMW	Nissan	Peugeot
Frequency	4	5	2	3	6

c The newspaper read by 40 office workers

Newspaper	*The Post*	*Today*	*The Mail*	*The Times*
Frequency	14	8	6	12

2 Draw a pie chart to represent each set of data.

a The numbers of children in 40 families

No. of children	0	1	2	3	4
Frequency	4	10	14	9	3

b How 90 students get to school

Journey to school	Walk	Car	Bus	Cycle
Frequency	42	13	25	10

Advice and Tips

Remember to complete a table as shown in the examples. Check that all angles add up to 360°.

3 Mariam asked 24 of her friends which sport they preferred to play. Her data is shown in this frequency table.

Sport	Rugby	Football	Tennis	Baseball	Basketball
Frequency	4	11	3	1	5

Illustrate her data in a pie chart.

4 Ameer wrote down the number of lessons he had per week in each subject on his school timetable.

Mathematics 5
English 5
Science 8
History 6
Geography 6
Arts 4
Sport 2

a How many lessons did Ameer have on his timetable?

b Draw a pie chart to show the data.

c Draw a bar chart to show the data.

d Which diagram better illustrates the data? Give a reason for your answer.

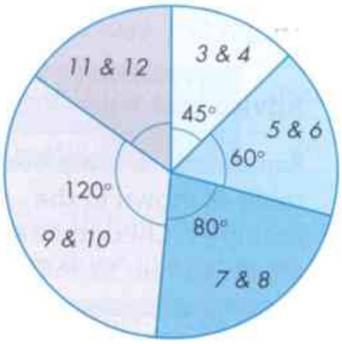
5 A market researcher asked 720 people which new brand of tinned beans they preferred. The results are given in the table.

A	248
B	264
C	152
D	56

a Draw a pie chart to illustrate the data.

b Why do you think pie charts are used to show this sort of information?

6 This pie chart shows the proportions of the different shoe sizes worn by 144 students in one year group in a school.



a What is the angle of the sector representing shoe sizes 11 and 12?

b How many students had a shoe size of 11 or 12?

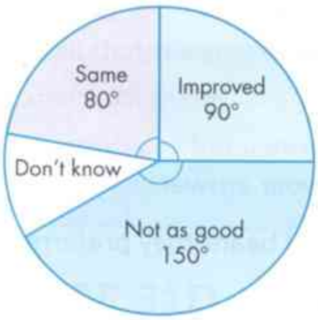
7 The table below shows the numbers of candidates, at each grade, taking music examinations in Strings and Brass.

	Grades					Total number of candidates
	3	4	5	6	7	
Strings	300	980	1050	600	70	3000
Brass	250	360	300	120	70	1100

a Draw a pie chart to represent each of the two examinations.

b Compare the pie charts to decide which group of candidates, Strings or Brass, did better overall. Give reasons to justify your answer.

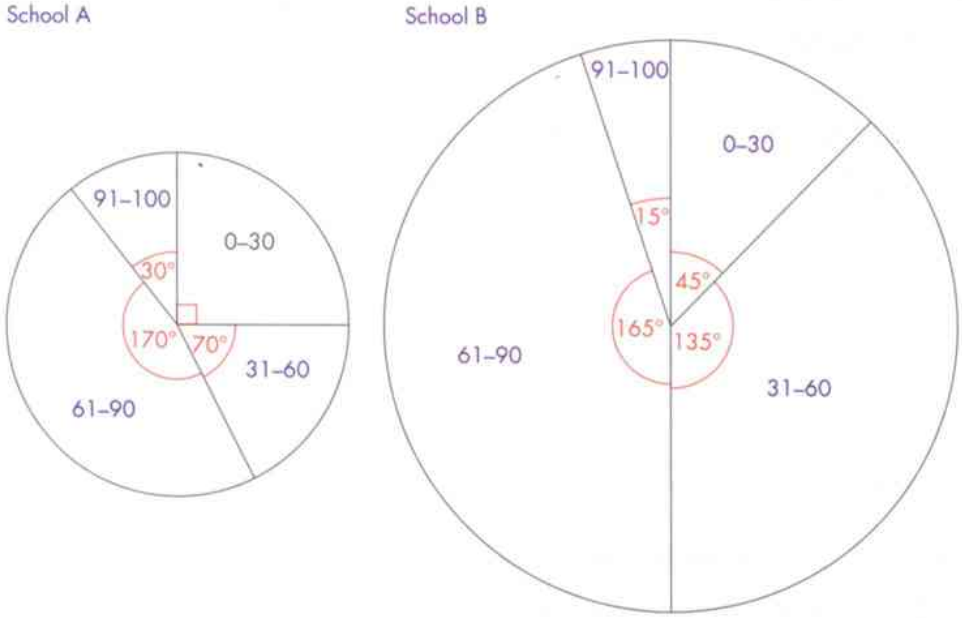
8 In a survey, a rail company asked passengers whether their service had improved.



What is the probability that a person picked at random from this survey answered "Don't know"?

9 Two classes of 13-year-old students from different schools were give the same test.

The pie charts show the results of the test. Each pie chart was drawn by a teacher at the students' school.



a Write a short report comparing the test results.

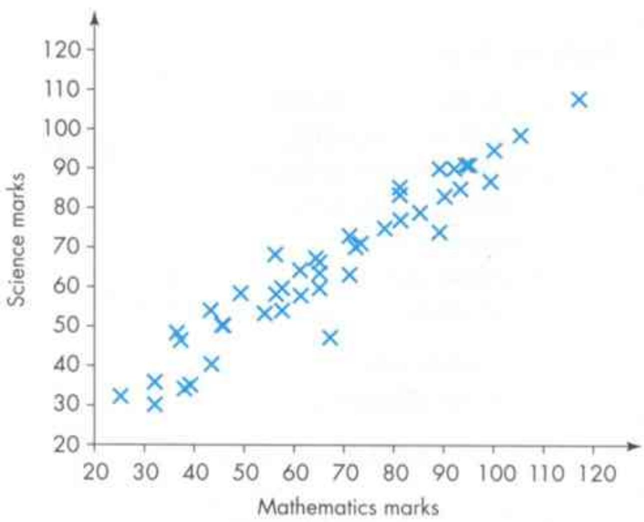
b State a reason why comparing pie charts like this can be unsatisfactory.

31.5 Scatter diagrams

A **scatter diagram** (also called a scattergraph or scattergram) is a method of comparing two **variables** by plotting their corresponding values on a graph. These values are usually taken from a table.

The variables are treated just like a set of (x, y) coordinates. This is shown in the scatter diagram on the right, in which the marks scored in a science test are plotted against the marks scored in a mathematics test.

This graph shows **positive correlation**. This means that students who get high marks in mathematics tests also tend to get high marks in science tests.

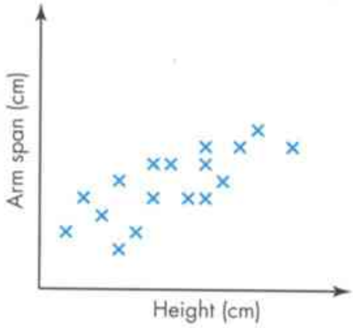


Correlation

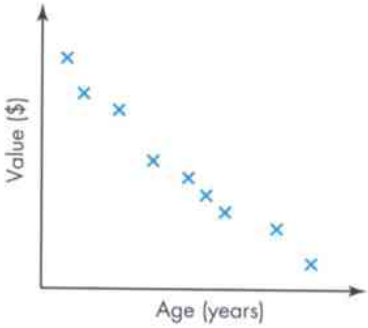
There are different types of **correlation**. Here are three statements that may or may not be true.

- The taller people are, the wider their arm span is.
- The older a car is, the lower its value will be.
- The distance you live from your place of work will affect how much you earn.

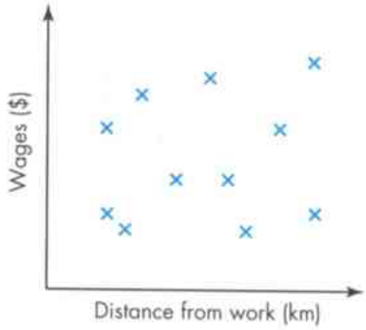
These relationships could be tested by collecting data and plotting the points on a scatter diagram. For example, the first statement may give a scatter diagram like the one on the left below.



Positive correlation



Negative correlation



Zero correlation

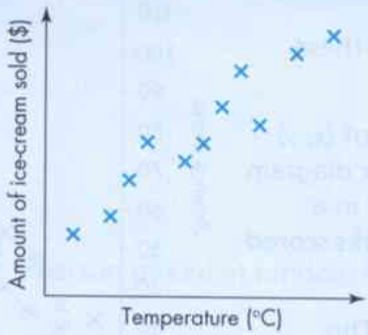
This first diagram has **positive correlation** because, as one quantity increases, so does the other. From such a scatter diagram, you could say that the taller someone is, the wider their arm span.

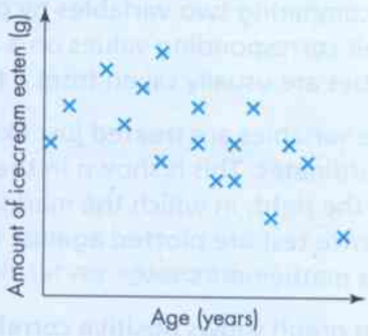
Testing the second statement may give a scatter diagram like the middle one above. This has **negative correlation** because, as one quantity increases, the other quantity decreases. From such a scatter diagram, you could say that, as a car gets older, its value decreases.

Testing the third statement may give a scatter diagram like the one on the right, above. This scatter diagram has **zero correlation**. There is no relationship between the distance a person lives from their work and how much they earn.

Example 9

The graphs below show the relationship between the temperature and the amount of ice-cream sold, and the relationship between the age of people and the amount of ice-cream they eat.





a Comment on the correlation of each graph.

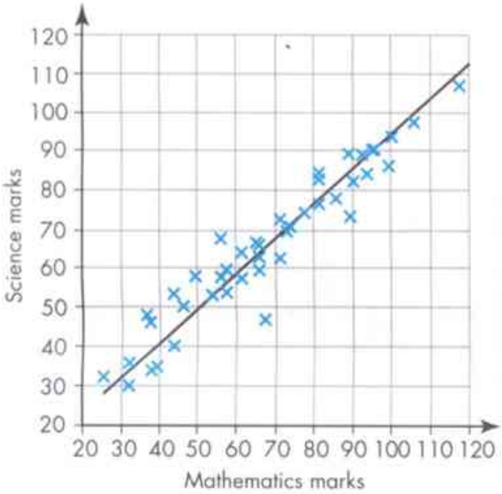
b What does each graph tell you?

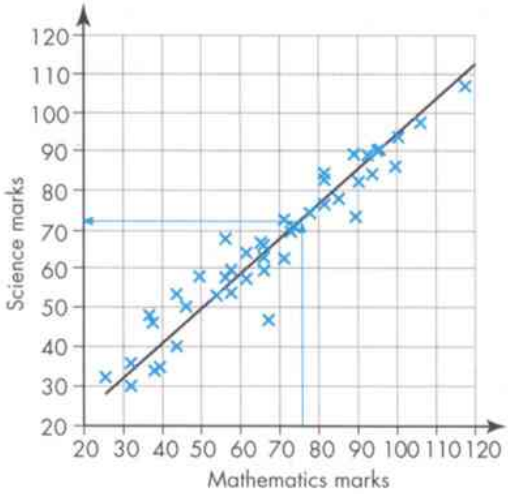
a The first graph has positive correlation and shows that, as the temperature increases, the amount of ice-cream sold increases.

b The second graph has negative correlation and shows that, as people get older, they eat less ice-cream.

Line of best fit

A **line of best fit** is a straight line that goes between all the points on a scatter diagram, passing as close as possible to all of them. You should try to have the same number of points on both sides of the line. Because you are drawing this line by eye, generous allowances are made around the correct answer. The line of best fit for the scatter diagram at the start of this section is shown below, left.





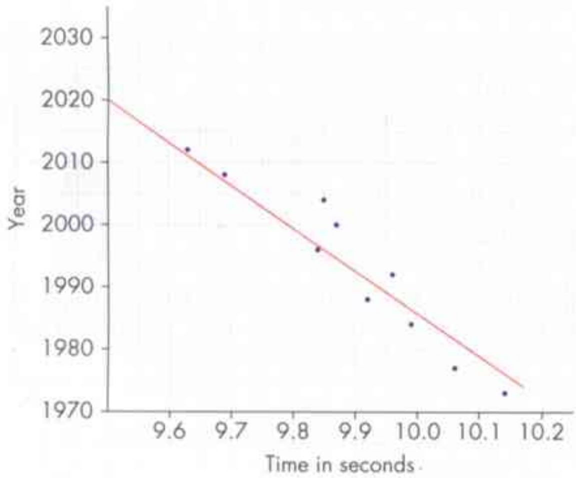
The line of best fit can be used to answer questions such as: 'A girl took the mathematics test and scored 75 marks but was ill for the science test. How many marks was she likely to have scored?'

You can find the answer by drawing a line up from 75 on the mathematics axis to the line of best fit and then drawing a line across to the science axis, as shown in the graph to the right of the graph showing the line of best fit. This gives 73, which is the mark she is likely to have scored in the science test.

Restrictions on the use of scatter diagrams

Scatter diagrams are very useful for estimating one value when given another. However, you need to be aware that there are limitations.

For example, here is a scatter diagram of the Olympic Men's 100 m winning time.



If you use the line of best fit, then the graph predicts that the time in the 2020 race should be about 9.5 seconds. However, this cannot continue forever, because it would imply that at some point the race could be won in 9 seconds, then 8 seconds, and eventually in no time at all!

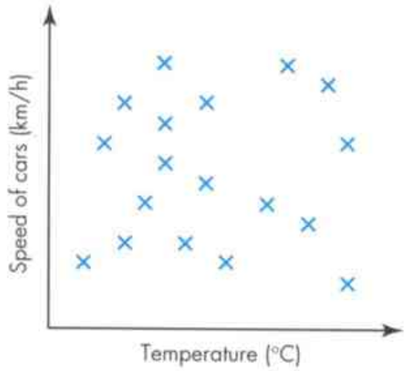
Scatter diagrams and lines of best fit are very useful for predicting data within the range of values given, but are not so reliable when predicting outside of those ranges.

EXERCISE 31E

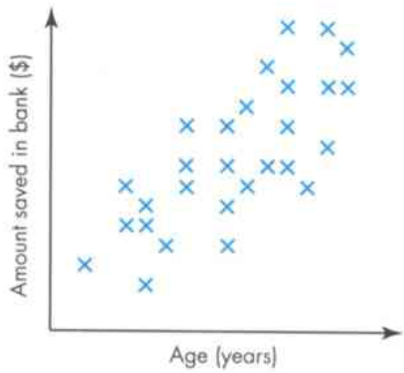
CORE

1 Describe the correlation of each of these two graphs.

a



b



2 Explain what each graph in question **1** tells you.

3 The table below shows the results of a science experiment in which a ball is rolled along a desk top. The speed of the ball is measured at various points.

Distance from start (cm)	10	20	30	40	50	60	70	80
Speed (cm/s)	18	16	13	10	7	5	3	0

a Plot the data on a scatter diagram.

b Draw the line of best fit.

c If the ball's speed had been measured at 5 cm from the start, what is it likely to have been?

d Estimate how far the ball was from the start when its speed was 12 cm/s.

4 The heights, in centimetres, of 20 mothers and their 15-year-old daughters were measured. These are the results.

Mother	153	162	147	183	174	169	152	164	186	178
Daughter	145	155	142	167	167	151	145	152	163	168
Mother	175	173	158	168	181	173	166	162	180	156
Daughter	172	167	160	154	170	164	156	150	160	152

a Plot these results on a scatter diagram. Take the horizontal axis (the x-axis) for the mothers' heights from 140 to 200. Take the vertical axis (the y-axis) for the daughters' heights from 140 to 200.

b Is it true that the tall mothers have tall daughters?

5 The table shows the marks for ten students in their mathematics and geography examinations.

Student	Anna	Becky	Cath	Dema	Emma	Fatima	Greta	Hannah	Imogen	Sitara
Maths	145	155	142	167	167	151	145	152	163	168
Geog	175	173	158	168	181	173	166	162	180	156

a Plot the data on a scatter diagram. Take the x-axis for the mathematics scores and the y-axis for the geography scores.

b Draw the line of best fit.

c One of the students was ill when she took the geography examination. Which student was it most likely to be?

d If another student, Kate, was absent for the geography examination but scored 75 in mathematics, what mark would you expect her to have scored in geography?

e If another student, Lina, was absent for the mathematics examination but scored 65 in geography, what mark would you expect her to have scored in mathematics?

6 A teacher carried out a survey of 20 students from his class and asked them to say how many hours per week they spent playing sport and how many hours per week they spent watching TV. This table shows the results of the survey.

Student	1	2	3	4	5	6	7	8	9	10
Hours playing sport	12	3	5	15	11	0	9	7	6	12
Hours watching TV	18	26	24	16	19	27	12	13	17	14

Student	11	12	13	14	15	16	17	18	19	20
Hours playing sport	12	10	7	6	7	3	1	2	0	12
Hours watching TV	22	16	18	22	12	28	18	20	25	13

a Plot these results on a scatter diagram. Take the x-axis as the number of hours playing sport and the y-axis as the number of hours watching TV.

b If you knew that another student from the form watched 8 hours of TV a week, would you be able to predict how long they spent playing sport? Explain why.

7 The table shows the times taken and distances travelled by a taxi driver in 10 journeys on one day.

Distance (km)	1.6	8.3	5.2	6.6	4.8	7.2	3.9	5.8	8.8	5.4
Time (minutes)	3	17	11	13	9	15	8	11	16	10

a Draw a scatter diagram of this information, with time on the horizontal axis.

b Draw a line of best fit on your diagram.

c If a taxi journey takes 5 minutes, how far, in kilometres, would you expect the journey to have been?

d How much time would you expect a journey of 4 km to take?

CORE

8 Omar records the time taken, in hours, and the average speed, in kilometres per hour (km/h), for several different journeys.

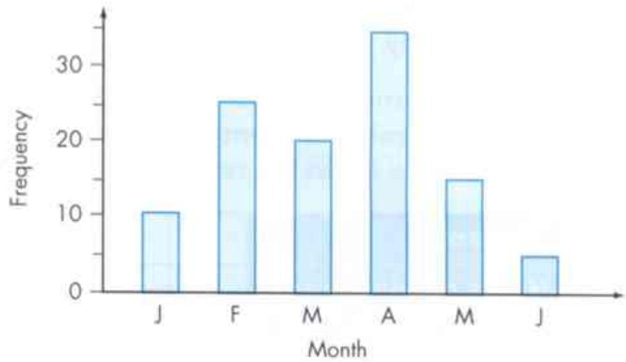
Time (h)	0.5	0.8	1.1	1.3	1.6	1.75	2	2.4	2.6
Speed (km/h)	42	38	27	30	22	23	21	9	8

Estimate the average speed for a journey of 90 minutes.

9 Describe what you would expect the scatter graph to look like if someone said that it showed negative correlation.

31.6 Histograms

You should already be familiar with bar charts like the one on the right, in which the vertical axis represents frequency, and each bar has a label to show what it represents. (Sometimes it is more convenient to have the axes the other way round.)



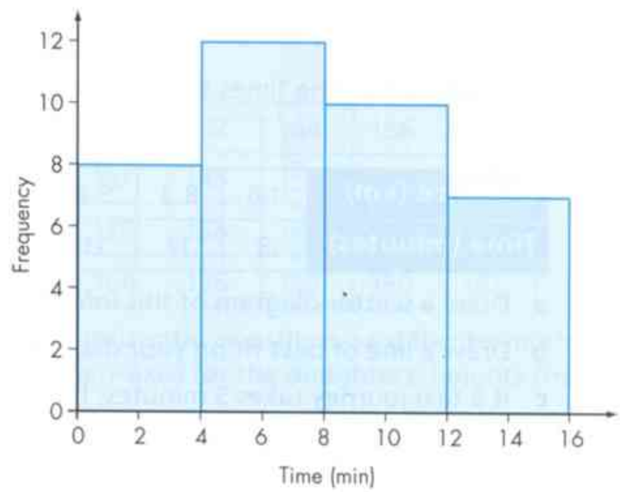
A **histogram** looks similar to a bar chart, but there are **three** fundamental differences.

- There are no gaps between the bars.
- The horizontal axis has a continuous scale.
- The area of each bar represents the class or group frequency of the bar.

This table shows times it takes people to walk to work.

Times, t minutes	$0 < t \leqslant 4$	$4 < t \leqslant 8$	$8 < t \leqslant 12$	$12 < t \leqslant 16$
Frequency	8	12	10	7

This **histogram** has been drawn from the table. The columns are *not* labelled $0 \leqslant t < 4$ and so on, as they would be on a bar chart. Instead there is a scale on the horizontal (time) axis. The first column is drawn between 0 and 4, the second between 4 and 8 and so on.



EXERCISE 31F

1 The table shows the range of heights of the girls in one year group in a school.

Height, h (cm)	$120 < h \leqslant 130$	$130 < h \leqslant 140$	$140 < h \leqslant 150$	$150 < h \leqslant 160$	$160 < h \leqslant 170$
Frequency	8	12	10	7	−2

Draw a histogram for this data.

2 A doctor was concerned at the length of time her patients had to wait to see her when they came to the morning clinic. Her survey gave these results.

Time, m (minutes)	$0 < m \leqslant 10$	$10 < m \leqslant 20$	$20 < m \leqslant 30$	$30 < m \leqslant 40$	$40 < m \leqslant 50$	$50 < m \leqslant 60$
Monday	5	8	17	9	7	4
Tuesday	9	8	16	3	2	1
Wednesday	7	6	18	2	1	1

a Draw a separate histogram for each day.

b On which day did patients tend to have to wait longer?

3 These are the prices of twenty second-hand cars, in dollars.

2590, 2650, 2650, 2790, 2850, 2925, 3090, 3125, 3125, 3420, 3595, 3740, 3750, 3920, 3945, 4095, 4150, 4200, 4750, 4785

a Copy and complete the frequency table to show the prices.

Prices (dollars)	$2500 < p \leqslant 3000$	$3000 < p \leqslant 3500$	$3500 < p \leqslant 4000$	$4000 < p \leqslant 4500$	$4500 < p \leqslant 5000$
Frequency					

b Illustrate the data on a histogram.

4 Boys and girls were given a simple task to complete. The times taken are shown below.

Times, t (minutes)	$0 < t \leqslant 4$	$4 < t \leqslant 8$	$8 < t \leqslant 12$	$12 < t \leqslant 16$	$16 < t \leqslant 20$
Boys	3	7	21	26	15
Girls	4	8	17	23	20

a Draw separate histograms to show the boys' times and the girls' times.

b How many boys and how many girls completed the task?

c What can you say about the longest time taken?

d Look at the two histograms and say whether you think boys or girls were better at the task. Give a reason for your answer.

5 These are the results of a survey of the masses of 50 girls and 50 boys of the same age.

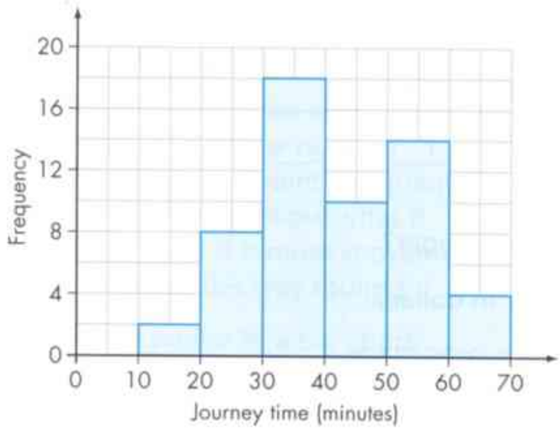
Mass, k (kg)	$15 \leqslant k < 20$	$20 \leqslant k < 25$	$25 \leqslant k < 30$	$30 \leqslant k < 35$	$35 \leqslant k < 40$	$40 \leqslant k < 45$
Girls	4	6	12	14	6	8
Boys	1	4	16	10	15	4

a Show this data on two separate histograms.

b Children of this age who weigh less than 25kg are underweight. Shade the corresponding part of each histogram.

c Were more boys or girls underweight?

6 This histogram shows the times a group of employees took to travel to work one day.



a How many took more than 50 minutes?

b How many took between 20 and 50 minutes?

c What can you say about the shortest and longest journey times?

The company introduced flexible working so that employees could start work at different times. Here are the results of a survey of journey times after the change.

Journey time, t (minutes)	$0 \leqslant t < 10$	$10 \leqslant t < 20$	$20 \leqslant t < 30$	$30 \leqslant t < 40$	$40 \leqslant t < 50$	$50 \leqslant t < 50$	$60 \leqslant t < 70$
Frequency	2	4	14	12	14	5	5

d Show the new journey times on a histogram.

e Compare the two histograms. Have journey times been shortened? Justify your answer.

31.7 Histograms with bars of unequal width

E

Sometimes the data in a frequency distribution are grouped into classes with intervals that are different. In this case, the resulting histogram has bars of unequal widths.

The key fact to remember is that the area of a bar in a histogram represents the **class frequency** of the bar. So, in the case of an unequal-width histogram, you find the height to draw each bar by dividing its class frequency by its **class interval** width (bar width), which is the difference between the lower and upper bounds for each interval.

Conversely, given a histogram, you can find any of its class frequencies by multiplying the height of the corresponding bar by its width.

It is for this reason that the scale on the vertical axes of these histograms is always labelled 'frequency density', where:

$$\text{frequency density} = \frac{\text{frequency of class interval}}{\text{width of class interval}}$$

Example 10

The heights of a group of girls were measured. The results were classified as shown in the table.

Height, h (cm)	$151 \leqslant h < 153$	$153 \leqslant h < 154$	$154 \leqslant h < 155$	$155 \leqslant h < 159$	$159 \leqslant h < 160$
Frequency	64	43	47	96	12

Draw a histogram to show the data.

It is convenient to write the table vertically and add two columns for class width and frequency density.

Calculate the class width by subtracting the lower class boundary from the upper class boundary. Calculate the frequency density by dividing the frequency by the class width.

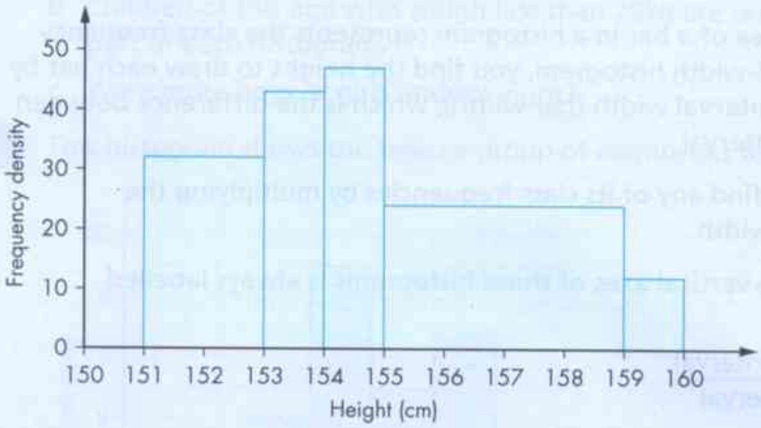
Height, h (cm)	Frequency	Class width	Frequency density
$151 \leqslant h < 153$	64	2	32
$153 \leqslant h < 154$	43	1	43
$154 \leqslant h < 155$	47	1	47
$155 \leqslant h < 159$	96	4	24
$159 \leqslant h < 160$	12	1	12

The histogram can now be drawn.

The horizontal scale should be marked off as normal, from a value below the lowest value in the table to a value above the largest value in the table. In this case, mark the scale from 150 cm to 160 cm.

The vertical scale is always frequency density and is marked up to at least the largest frequency density in the table. In this case, 50 is a sensible value.

Each bar is drawn between the lower class interval and the upper class interval horizontally, and up to the frequency density vertically.



Now check that the area of each column is equal to the frequency.

151–153 is 2 × 32 = 64

153–154 is 1 × 43 = 43

and so on.

If the bars are of equal width, the frequency density and the frequency will be proportional. In that case you can use frequency on the vertical axis, as in section 31.6.

EXERCISE 31G

EXTENDED

1 Draw histograms for these grouped frequency distributions.

a

Temperature, t (°C)	$8 \leqslant t < 10$	$10 \leqslant t < 12$	$12 \leqslant t < 15$	$15 \leqslant t < 17$	$17 \leqslant t < 20$	$20 \leqslant t < 24$
Frequency	5	13	18	4	3	6

b

Wage, w (\$1000)	$6 \leqslant w < 10$	$10 \leqslant w < 12$	$12 \leqslant w < 16$	$16 \leqslant w < 24$
Frequency	16	54	60	24

c

Age, a (year)	$11 \leqslant a < 14$	$14 \leqslant a < 16$	$16 \leqslant a < 17$	$17 \leqslant w < 20$
Frequency	51	36	12	20

d

Pressure, p (mm)	$745 \leqslant p < 755$	$755 \leqslant p < 760$	$760 \leqslant p < 765$	$765 \leqslant p < 775$
Frequency	4	6	14	10

e

Time, t (min)	$0 \leqslant t < 8$	$8 \leqslant t < 12$	$12 \leqslant t < 16$	$16 \leqslant t < 20$
Frequency	72	84	54	36

2 This information was gathered about the weekly pocket money given to 14-year-olds.

Pocket money, p (\$)	$0 \leqslant p < 2$	$2 \leqslant p < 4$	$4 \leqslant p < 5$	$5 \leqslant p < 8$	$8 \leqslant p < 10$
Girls	8	15	22	12	4
Boys	6	11	25	15	6

Represent the information about the boys and girls on separate histograms.

3 The sales of the *Star* newspaper over 70 years are recorded in this table.

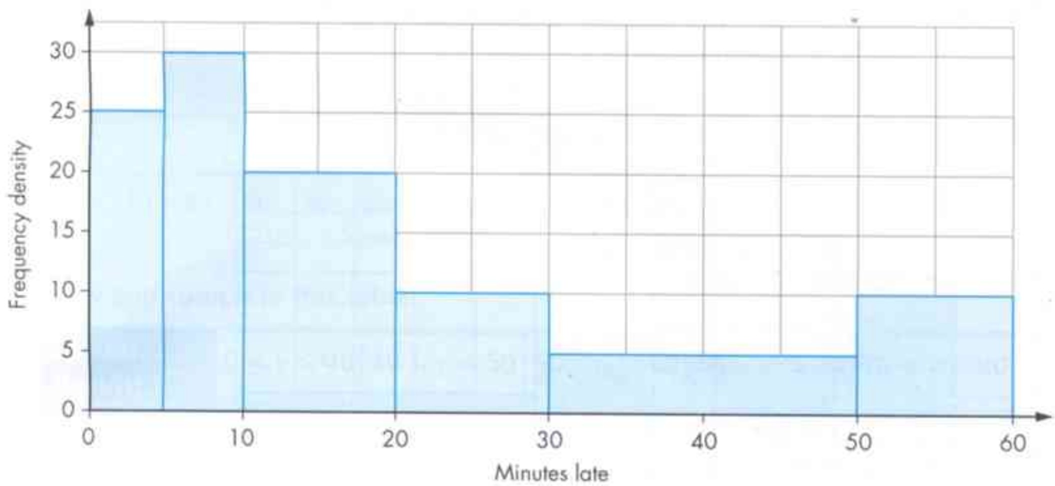
Years	1940–60	1961–80	1981–90	1991–2000	2001–05	2006–2010
Copies	62 000	68 000	71 000	75 000	63 000	52 000

Illustrate this information on a histogram.

Take the class boundaries as 1940, 1960, 1980, 1990, 2000, 2005, 2010.

4 The Madrid trains were always late, so one month a survey was undertaken to find how many trains were late, and by how many minutes.

The results are illustrated by this histogram.



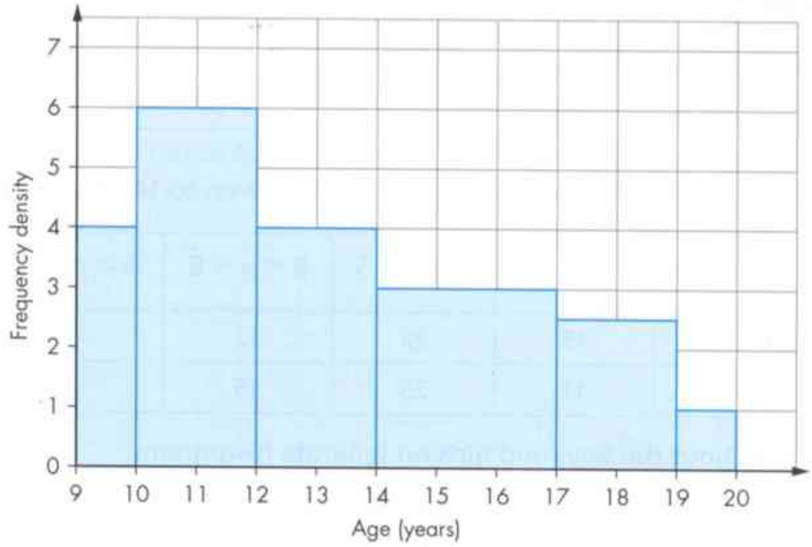
a How many trains were in the survey?

b How many trains were delayed for longer than 15 minutes?

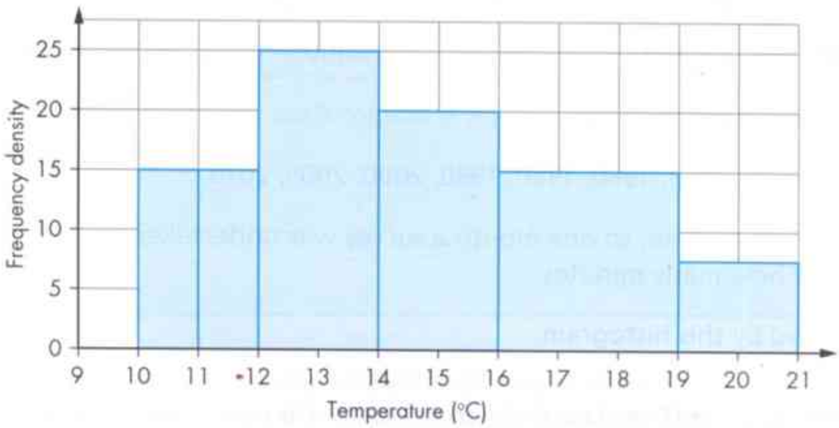
EXTENDED

5 For each of the frequency distributions illustrated in the histograms draw up the grouped frequency table.

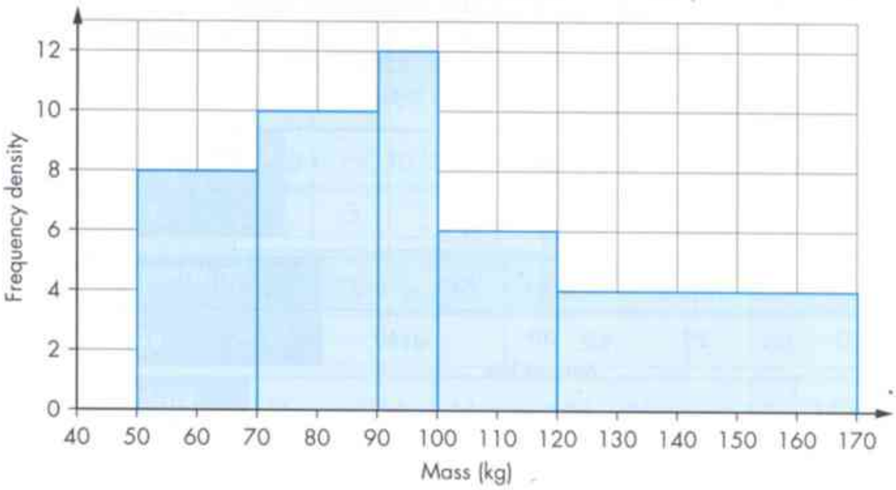
a



b



c



6 All the patients in a hospital were asked how long it was since they last saw a doctor. The results are shown in the table.

Hours, h	$0 \leqslant h < 2$	$2 \leqslant h < 4$	$4 \leqslant h < 6$	$6 \leqslant h < 10$	$10 \leqslant h < 16$	$16 \leqslant h < 24$
Frequency	8	12	20	30	20	10

a Draw a histogram to illustrate the data.

b Estimate how many people waited more than 8 hours.

Advice and Tips

Find the area to the right of $h = 8$.

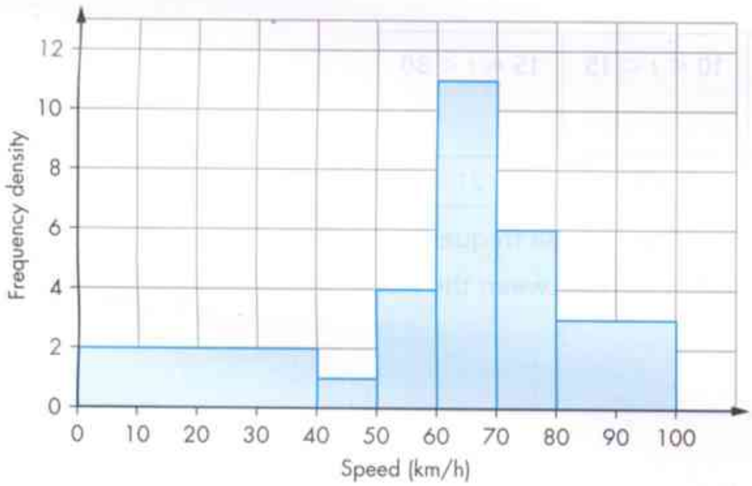
7 One summer, Albert monitored the mass of the tomatoes grown on each of his plants. His results are summarised in this table.

Mass, m (kg)	$6 \leqslant m < 10$	$10 \leqslant m < 12$	$12 \leqslant m < 16$	$16 \leqslant m < 20$	$20 \leqslant m < 25$
Frequency	8	15	28	16	10

a Draw a histogram for this distribution.

b Estimate how many plants produced more than 15 kg.

8 A survey was carried out to find the speeds of cars passing a particular point on a road. The histogram illustrates the results of the survey.



a Copy and complete this table.

Speed, v (km/h)	$0 < v \leqslant 40$	$40 < v \leqslant 50$	$50 < v \leqslant 60$	$60 < v \leqslant 70$	$70 < v \leqslant 80$	$80 < v \leqslant 100$
Frequency		10	40	110		

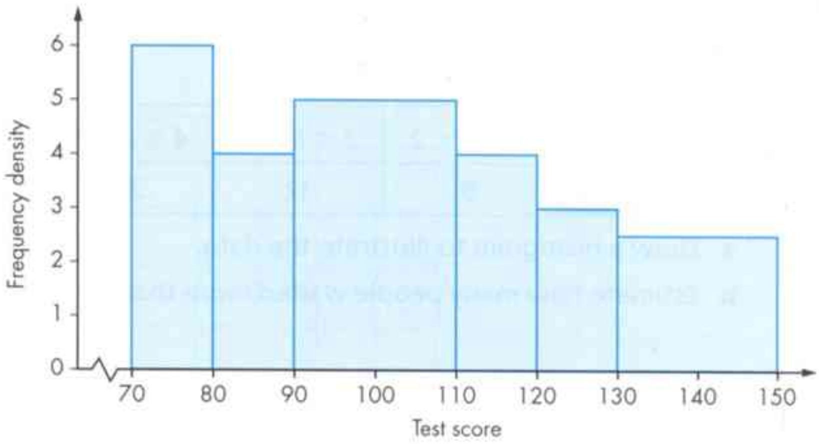
b Find the number of cars included in the survey.

EXTENDED

9 The histogram shows the test scores for 320 students in a school.

a How many students scored more than 120?

b The pass mark was 90. What percentage of students failed the test?



10 Adrienne and Bernice collected the same data about journey times but grouped it differently.

Here are Adrienne's figures.

Journey time (t minutes)	$0 \leqslant t < 5$	$5 \leqslant t < 10$	$10 \leqslant t < 15$	$15 \leqslant t < 20$	$20 \leqslant t < 25$	$25 \leqslant t < 30$
Frequency	10	15	30	12	6	3

Here are Bernice's figures.

Journey time (t minutes)	$0 \leqslant t < 10$	$10 \leqslant t < 15$	$15 \leqslant t < 30$
Frequency	25	30	21

a Draw a histogram for each set of figures. Use frequency density on the vertical axis each time.

b Describe any similarities or differences between the histograms.

Check your progress

Core

- I can collect, classify and tabulate data
- I can read and interpret tables and statistical diagrams
- I can compare sets of data using tables and graphs
- I appreciate restrictions on drawing conclusions from data
- I can construct and interpret bar charts, pie charts, pictograms and simple frequency distributions
- I can construct and interpret histograms with equal intervals
- I can construct and interpret scatter diagrams and understand what is meant by positive, negative and zero correlation
- I can draw and interpret lines of best fit

Extended

- I can construct and interpret histograms with unequal intervals

Chapter 32

Statistical measures

Topics	Level	Key words
1 The mode	CORE	average, mode, frequency, modal value
2 The median	CORE	median, middle value
3 The mean	CORE	average, mean
4 The range	CORE	range, spread, consistency
5 Which average to use	CORE	representative, appropriate, extreme values
6 Stem-and-leaf diagrams	CORE	frequency table
7 Using frequency tables	CORE	grouped data, estimated, modal class, continuous data, discrete data
8 Grouped data	EXTENDED	quartile, inter-quartile range, dispersion, cumulative frequency, lower quartile, upper quartile
9 Cumulative frequency diagrams	EXTENDED	
10 Box-and-whisker plots	EXTENDED	

In this chapter you will learn how to:

CORE	EXTENDED
• Calculate the mean, median, mode and range for individual and discrete data and distinguish between the purposes for which they are used. (C9.4 and E9.4) • Construct and interpret stem-and-leaf diagrams. (C9.3 and E9.3)	• Calculate an estimate of the mean for grouped and continuous data. (E9.5) • Identify the modal class from a grouped frequency distribution. (E9.5) • Construct and use cumulative frequency diagrams. (E9.6) • Estimate and interpret the median, percentiles, quartiles and inter-quartile range. (E9.6) • Construct and interpret box-and-whisker plots. (E9.6)

Why this chapter matters

The idea of 'average' is important in statistics. But there are several ways of working out an average, which have been developed over a long period of time.

Mean in Ancient India

There is a story about Rtuparna who was born in India around 5000 BCE. He wanted to estimate the amount of fruit on a tree:

- He counted how much fruit there was on one branch, then estimated the number of branches on the tree.
- He multiplied the estimated number of branches by the counted fruit on one branch.

He was amazed that the total was very close to the actual counted number of fruit when it was picked.

Rtuparna was one of the first to use arithmetic **mean**.

The branch he chose was an average one representing all the branches. So the number of fruit on that branch would have been in the middle of the smallest and largest number of fruit on other branches on the tree.

Mode in Ancient Greece

This story comes from a war in Ancient Greece (431–404 BCE). It is about a battle between the Spartans and the Athenians.

The Athenians had to get over the Spartan Wall so they needed to work out its height. They started by counting the layers of bricks. This was done by hundreds of soldiers at the same time because many of them would get it wrong – but the majority would get it about right.

This is seen as an early use of the **mode**: the number of layers that occurred the most in the counting was taken as the one most likely to be correct.

They then had to guess the height of one brick and so calculate the total height of the wall. They could then make ladders long enough to reach the top of the wall.

The other average that you will use is the **median** (which finds the middle value), and there is no record of any use of this being used until the early 17th century.

These ancient examples demonstrate that you will not always work out the average in the same way – you must choose a method that is appropriate to the situation.

32.1 The mode

Average is a term often used when describing or comparing sets of data, for example, average rainfall over a year or the average mark in a test for a group of students.

In each of the above examples, you are representing the whole set of many values (rainfall on every day of the year or the marks of all the students) by just a single, 'typical' value, which is called the average.

The idea of an average is extremely useful, because it enables you to compare one set of data with another set by comparing just two values – their averages.

There are several ways of expressing an average, but the most commonly used averages are the **mode**, the **median** and the **mean**.

The **mode** is the value that occurs the most in a set of data. That is, it is the value with the highest **frequency**.

The mode is a useful average because it is very easy to find and it can be applied to non-numerical data (qualitative data). For example, you could find the modal style of skirts sold in a particular month.

Example 1

Suhail scored these numbers of goals in 12 football matches:

1 2 1 0 1 0 0 1 2 1 0 2

What is the mode of his scores?

The number that occurs most often in this list is 1. So, the mode is 1.

You can also say that the modal score or **modal value** is 1.

EXERCISE 32A

CORE

1 Find the mode for each set of data.

- a 3, 4, 7, 3, 2, 4, 5, 3, 4, 6, 8, 4, 2, 7
- b 47, 49, 45, 50, 47, 48, 51, 48, 51, 48, 52, 48
- c −1, 1, 0, −1, 2, −2, −2, −1, 0, 1, −1, 1, 0, −1, 2, −1, 2
- d $\frac{1}{2}, \frac{1}{4}, 1, \frac{1}{2}, \frac{3}{4}, \frac{1}{4}, 0, 1, \frac{3}{4}, \frac{1}{4}, 1, \frac{1}{4}, \frac{3}{4}, \frac{1}{4}, \frac{1}{2}$
- e 100, 10, 1000, 10, 100, 1000, 10, 1000, 100, 1000, 100, 10
- f 1.23, 3.21, 2.31, 3.21, 1.23, 3.12, 2.31, 1.32, 3.21, 2.31, 3.21

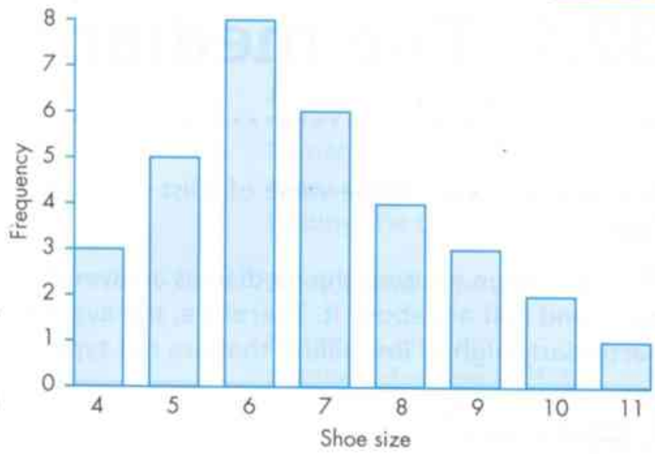
Advice and Tips

It helps to put the data in order or group all the same things together.

2 Find the mode for each set of data.

- a red, green, red, amber, green, red, amber, green, red, amber
- b rain, sun, cloud, sun, rain, fog, snow, rain, fog, sun, snow, sun
- c α, γ, α, β, γ, α, α, γ, β, α, β, γ, β, β, α, β, γ, β
- d ❋, ☆, ★, ★, ☆, ❋, ★, ✰, ★, ✰, ✭, ❋, ✪, ✰, ★, ✭, ✰

3 Halima did a survey to find the shoe sizes of students in her class. The bar chart illustrates her data.



a How many students are there in Halima's class?

b What is the modal shoe size?

c Can you tell which are the boys' shoes sizes and which are the girls' shoe sizes?

d Halima then decided to draw a bar chart to show the shoe sizes of the boys and the girls separately. Do you think that the mode for the boys and the mode for the girls will be the same as the mode for the whole class? Explain your answer.

4 The frequency table shows the marks that a class obtained in a spelling test.

Mark	3	4	5	6	7	8	9	10
Frequency	1	2	6	5	5	4	3	4

a Write down the mode for their marks.

b Do you think this is a typical mark for the class? Explain your answer.

5 Explain why the mode is often referred to as the 'shopkeeper's average'.

6 This table shows the colours of eyes of the students in a class.

	Blue	Brown	Green
Boys	4	8	1
Girls	8	5	2

a How many students are there in the class?

b What is the modal eye colour for:

i boys ii girls iii the whole class?

c After two students join the class the modal eye colour for the whole class is blue. Which of these statements is true?

- Both students had green eyes.
- Both students had brown eyes.
- Both students had blue eyes.
- You cannot tell what their eye colours were.

7 Here is a large set of raw data.

5 6 8 2 4 8 9 8 1 3 4 2 7 2 4 6 7 5 3 8

9 1 3 1 5 6 2 5 7 9 4 1 4 3 3 5 6 8 6 9

8 4 8 9 3 4 6 7 7 4 5 4 2 3 4 6 7 6 5 5

a What problems may occur if you attempted to find the mode by counting individual numbers?

b Explain a method that would make finding the mode more efficient and accurate.

c Use your method to find the mode of the data.

32.2 The median

The **median** is the **middle value** of a list of values when they are put in *order of size*, from lowest to highest.

The advantage of using the median as an average is that half the data values are below the median value and half are above it. Therefore, the average is only slightly affected by the presence of any particularly high or low values that are not typical of the data as a whole.

Example 2

Find the median for this list of numbers.

2, 3, 5, 6, 1, 2, 3, 4, 5, 4, 6

Putting the list in numerical order gives:

1, 2, 2, 3, 3, 4, 4, 5, 5, 6, 6

There are 11 numbers in the list, so the middle of the list is the 6th number.
Therefore, the median is 4.

Example 3

Find the median of the data shown in the frequency table.

Value	2	3	4	5	6	7
Frequency	2	4	6	7	8	3

First, add up the frequencies to find out how many pieces of data there are.

The total is 30 so the median value will be between the 15th and 16th values.

Now, add up the frequencies to give a running total, to find out where the 15th and 16th values are.

Value	2	3	4	5	6	7
Frequency	2	4	6	7	8	3
Running total	2	6	12	19	27	30

There are 12 data-values up to the value 4 and 19 up to the value 5.

Both the 15th and 16th values are 5, so the median is 5.

To find the median in a list of n values, written in order, use the rule:

$$\text{median} = \frac{n+1}{2}\text{th value}$$

CORE

EXERCISE 32B

1 Find the median for each set of data.

a 7, 6, 2, 3, 1, 9, 5, 4, 8

b 26, 34, 45, 28, 27, 38, 40, 24, 27, 33, 32, 41, 38

c 4, 12, 7, 6, 10, 5, 11, 8, 14, 3, 2, 9

d 12, 16, 12, 32, 28, 24, 20, 28, 24, 32, 36, 16

e 10, 6, 0, 5, 7, 13, 11, 14, 6, 13, 15, 1, 4, 15

f −1, −8, 5, −3, 0, 1, −2, 4, 0, 2, −4, −3, 2

g 5.5, 5.05, 5.15, 5.2, 5.3, 5.35, 5.08, 5.9, 5.25

> **Advice and Tips**
>
> Remember to put the data in order before finding the median.

> **Advice and Tips**
>
> If there is an even number of pieces of data, the median will be halfway between the two middle values.

2 A group of 15 students had lunch in the school's cafeteria. Given below are the amounts that they spent.

\$2.30, \$2.20, \$2, \$2.50, \$2.20, \$3.50, \$2.20, \$2.25,

\$2.20, \$2.30, \$2.40, \$2.20, \$2.30, \$2, \$2.35

a Find the mode for the data.

b Find the median for the data.

c Which is the better average to use? Explain your answer.

3 a Find the median of 7, 4, 3, 8, 2, 6, 5, 2, 9, 8, 3.

b Without putting them in numerical order, write down the median for each of these sets.

i 17, 14, 13, 18, 12, 16, 15, 12, 19, 18, 13

ii 217, 214, 213, 218, 212, 216, 215, 212, 219, 218, 213

iii 12, 9, 8, 13, 7, 11, 10, 7, 14, 13, 8

iv 14, 8, 6, 16, 4, 12, 10, 4, 18, 16, 6

> **Advice and Tips**
>
> Look for a connection between the original data and the new data. For example, in **i**, the numbers are each 10 more than those in part **a**.

4 Given below are the age, height and mass of each of the seven players in a netball team.

	Ella	Linda	Pat	Marion	Amina	Martha	Elisa
Age (yr)	13	15	12	11	11	15	14
Height (cm)	161	165	162	158	154	168	169
Mass (kg)	41	42	37	32	35	42	40

a Find the median age of the team. Which player has the median age?

b Find the median height of the team. Which player has the median height?

c Find the median mass of the team. Which player has the median mass?

d Who would you choose as the average player in the team? Give a reason for your answer.

CORE

5 The table shows the numbers of sandwiches sold in a shop over 25 days.

Sandwiches sold	10	11	12	13	14	15	16
Frequency	2	3	6	4	3	4	3

a What is the modal number of sandwiches sold?

b What is the median number of sandwiches sold?

6 **a** Write down a list of nine numbers that has a median of 12.

b Write down a list of 10 numbers that has a median of 12.

c Write down a list of nine numbers that has a median of 12 and a mode of 8.

d Write down a list of 10 numbers that has a median of 12 and a mode of 8.

7 A list contains seven even numbers. The largest number is 24. The smallest number is half the largest. The mode is 14 and the median is 16. Two of the numbers add up to 42. What are the seven numbers?

8 Look at this list of numbers.

4, 4, 5, 8, 10, 11, 12, 15, 15, 16, 20

a Add four numbers to make the median 12.

b Add six numbers to make the median 12.

c What is the least number of numbers to add that will make the median 4?

9 Explain why the median is not a good average to use in this set of payments.

Here are five payments.

\$3, \$5, \$8, \$100, \$3000

32.3 The mean

The **mean** of a set of data is the sum of all the values in the set divided by the total number of values in the set. That is:

$$\text{mean} = \frac{\text{sum of all values}}{\text{total number of values}}$$

This is what most people mean when they use the term **average**.

Another name for this average is the arithmetic **mean**.

The advantage of using the mean as an average is that it takes into account all the values in the set of data.

Example 4

The ages of 11 players in a football squad are:

21, 23, 20, 27, 25, 24, 25, 30, 21, 22, 28

What is the mean age of the squad?

Sum of all the ages = 266

Total number in squad = 11

Therefore, mean age = $\frac{266}{11}$ = 24.1818… = 24.2 (1 decimal place)

EXERCISE 32C

1 Find the mean for each set of data.

a 7, 8, 3, 6, 7, 3, 8, 5, 4, 9

b 47, 3, 23, 19, 30, 22

c 42, 53, 47, 41, 37, 55, 40, 39, 44, 52

d 1.53, 1.51, 1.64, 1.55, 1.48, 1.62, 1.58, 1.65

e 1, 2, 0, 2, 5, 3, 1, 0, 1, 2, 3, 4

2 Calculate the mean for each set of data, giving your answer correct to 1 decimal place.

a 34, 56, 89, 34, 37, 56, 72, 60, 35, 66, 67

b 235, 256, 345, 267, 398, 456, 376, 307, 282

c 50, 70, 60, 50, 40, 80, 70, 60, 80, 40, 50, 40, 70

d 43.2, 56.5, 40.5, 37.9, 44.8, 49.7, 38.1, 41.6, 51.4

e 2, 3, 1, 0, 2, 5, 4, 3, 2, 0, 1, 3, 4, 5, 0, 3, 1, 2

3 The table shows the marks that 10 students obtained in mathematics, English and science in their examinations.

Student	Ahmed	Badru	Camille	Dayar	Evrim	Fatima	George	Helga	Imran	Josie
Maths	45	56	47	77	82	39	78	32	92	62
English	54	55	59	69	66	49	60	56	88	44
Science	62	58	48	41	80	56	72	40	81	52

a Work out the mean mark for mathematics.

b Work out the mean mark for English.

c Work out the mean mark for science.

d Which student obtained marks closest to the mean in all three subjects?

e How many students were above the average mark in all three subjects?

CORE

4 Suni kept a record of the amount of time she spent on her homework over 10 days:

$\frac{1}{2}$ h, 20 min, 35 min, $\frac{1}{4}$ h, 1 h, $\frac{1}{2}$ h, $1\frac{1}{2}$ h, 40 min, $\frac{3}{4}$ h, 55 min

Calculate the mean time, in minutes, that Suni spent on her homework.

Advice and Tips

Convert all times to minutes, for example, $\frac{1}{4}$ h = 15 minutes.

5 These are the weekly wages of 10 people working in an office.

\$350 \$200 \$180 \$200 \$350 \$200 \$240 \$480 \$300 \$280

a Find the modal wage.

b Find the median wage.

c Calculate the mean wage.

d Which of the three averages best represents the office staff's wages? Give a reason for your answer.

Advice and Tips

Remember that the mean can be distorted by extreme values.

6 The ages of five people in a group of walkers are 38, 28, 30, 42 and 37.

a Calculate the mean age of the group.

b Steve, who is 41, joins the group. Calculate the new mean age of the group.

7 **a** Calculate the mean of 3, 7, 5, 8, 4, 6, 7, 8, 9 and 3.

b Calculate the mean of 13, 17, 15, 18, 14, 16, 17, 18, 19 and 13. What do you notice?

c Write down, without calculating, the mean for each of these sets of data.

i 53, 57, 55, 58, 54, 56, 57, 58, 59, 53

ii 103, 107, 105, 108, 104, 106, 107, 108, 109, 103

iii 4, 8, 6, 9, 5, 7, 8, 9, 10, 4

Advice and Tips

Look for a connection between the original data and the new data. For example in **i** the numbers are 50 more.

8 Two families were in a competition.

Speed family		Roberts family	
Brian	aged 59	Frank	aged 64
Kath	aged 54	Marylin	aged 62
James	aged 34	David	aged 34
Helen	aged 34	James	aged 32
John	aged 30	Tom	aged 30
Joseph	aged 24	Helen	aged 30
Joy	aged 19	Evie	aged 16

Each family had to choose four members with a mean age of between 35 and 36.

Choose two teams, one from each family, that have this mean age between 35 and 36.

CORE

9 Asif had an average batting score of 35 runs. He had scored 315 runs in nine games of cricket.

What is the least number of runs he needs to score in the next match if he is to get a higher average score?

10 The mean age of a group of eight walkers is 42. Joanne joins the group and the mean age changes to 40.

How old is Joanne?

32.4 The range

The **range** for a set of data is the highest value of the set minus the lowest value.

The range is *not* an average. It shows the **spread** of the data. You will use it when you are comparing two or more sets of similar data. You can also use it to comment on the **consistency** of two or more sets of data.

Example 5

Rachel's marks in 10 mental arithmetic tests were 4, 4, 7, 6, 6, 5, 7, 6, 9 and 6.

Adil's marks in the same tests were 6, 7, 6, 8, 5, 6, 5, 6, 5 and 6.

Compare their marks.

Rachel's mean mark is 60 ÷ 10 = 6 and her range is 9 – 4 = 5.

Adil's mean mark is 60 ÷ 10 = 6 and his range is 8 – 5 = 3.

Although the means are the same, Adil has a smaller range.

This shows that Adil's results are more consistent.

EXERCISE 32D

CORE

1 Find the range for each set of data.

a 3, 8, 7, 4, 5, 9, 10, 6, 7, 4

b 62, 59, 81, 56, 70, 66, 82, 78, 62, 75

c 1, 0, 4, 5, 3, 2, 5, 4, 2, 1, 0, 1, 4, 4

d 3.5, 4.2, 5.5, 3.7, 3.2, 4.8, 5.6, 3.9, 5.5, 3.8

e 2, –1, 0, 3, –1, –2, 1, –4, 2, 3, 0, 2, –2, 0, –3

CORE

2 The table shows the maximum and minimum temperatures at midday for five cities in England during a week in August.

	Birmingham	Leeds	London	Newcastle	Sheffield
Maximum temperature (°C)	28	25	26	27	24
Minimum temperature (°C)	23	22	24	20	21

a Write down the range of the temperatures for each city.

b What do the ranges tell you about the weather for England during the week?

3 Over a three-week period, a school sweet shop took these amounts.

	Monday	Tuesday	Wednesday	Thursday	Friday
Week 1	\$32	\$29	\$36	\$30	\$28
Week 2	\$34	\$33	\$25	\$28	\$20
Week 3	\$35	\$34	\$31	\$33	\$32

a Calculate the mean amount taken each week.

b Find the range for each week.

c What can you say about the total amounts taken for each of the three weeks?

4 In a golf tournament, the club coach had to choose either Maria or Fay to play in the first round. In the previous eight rounds, their scores were as follows.

Maria's scores: 75, 92, 80, 73, 72, 88, 86, 90

Fay's scores: 80, 87, 85, 76, 85, 79, 84, 88

a Calculate the mean score for each golfer.

b Find the range for each golfer.

c Which golfer would you choose to play in the tournament? Explain why.

Advice and Tips

The best person to choose may not be the one with the biggest mean but could be the most consistent player.

5 Dan has a choice of two buses to get to school: Number 50 or Number 63. Over a month, he kept a record of the numbers of minutes each bus was late when it set off from his home bus stop.

No. 50: 4, 2, 0, 6, 4, 8, 8, 6, 3, 9

No. 63: 3, 4, 0, 10, 3, 5, 13, 1, 0, 1

a For each bus, calculate the mean number of minutes late.

b Find the range for each bus.

c Which bus would you advise Dan to catch? Give a reason for your answer.

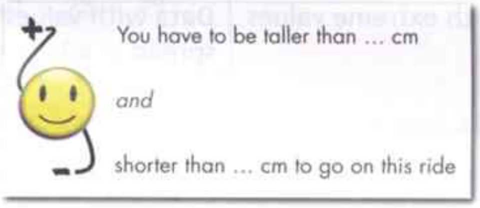
6 The table gives the ages and heights of 10 children.

Name	Age (years)	Height (cm)
Evrim	9	121
Isaac	4	73
Lilla	8	93
Lewis	10	118
Evie	3	66
Badru	6	82
Oliver	4	78
Halima	2	69
Isambard	9	87
Chloe	7	82

a Chloe is having a party. She wants to invite as many children as possible but does not want the range of ages to be more than 5.

Who will she invite?

b This is a sign at a theme park.



Isaac is the shortest person who can go on the ride and Isambard is the tallest.

What are the smallest and largest missing values on the sign?

7 **a** The age range of a school quiz team is 20 years and the mean age is 34.

Who would you expect to be in this team?

Explain your answer.

b Another team has an average age of $15\frac{1}{2}$ and a range of 1.

Who would you expect to be in this team?

Explain your answer.

32.5 Which average to use

An average must be truly **representative** of a set of data. So, when you have to find an average, it is crucial to choose the **appropriate** type of average for this particular set of data.

If you use the wrong average, your results will be distorted and give misleading information.

This table, which compares the advantages and disadvantages of each type of average, will help you to make the correct decision.

	Mode	**Median**	**Mean**
Advantages	Very easy to find Not affected by **extreme values** Can be used for non-numerical data	Easy to find for ungrouped data Not affected by extreme values	Easy to find Uses all the values The total for a given number of values can be calculated from it
Disadvantages	Does not use all the values May not exist	Does not use all the values Often not understood	Extreme values can distort it Has to be calculated
Use for	Non-numerical data Finding the most likely value	Data with extreme values	Data with values that are spread in a balanced way

EXERCISE 32E

CORE

1 These are the ages of the members of a hockey team.

29 26 21 24 26 28 35 23 29 28 29

a Give:

i the modal age

ii the median age

iii the mean age.

b What is the range of the ages?

2 a For each set of data, find the mode, the median and the mean.

i 6, 10, 3, 4, 3, 6, 2, 9, 3, 4

ii 6, 8, 6, 10, 6, 9, 6, 10, 6, 8

iii 7, 4, 5, 3, 28, 8, 2, 4, 10, 9

b For each set of data, decide which average is the best one to use and give a reason.

3 These are the numbers of copies of *The Evening Star* sold on 12 consecutive evenings by a shop during a promotion exercise organised by the newspaper's publisher.

65 73 75 86 90 112 92 87 77 73 68 62

a Find the mode, the median and the mean for the sales.

b The shopkeeper had to report the average sale to the publisher after the promotion. Which of the three averages would you advise the shopkeeper to use? Explain why.

4 The mean age of a group of 10 young people is 15.

a What do all their ages add up to?

b What will be their mean age in five years' time?

5 Decide which average you would use for each statistic. Give a reason for your answer.

a The average mark in an examination

b The average pocket money for a group of 16-year-old students

c The average shoe size for all the girls in one year at school

d The average height for all the artistes on tour with a circus

e The average hair colour for students in your school

f The average mass of all newborn babies in a hospital's maternity ward.

6 A pack of matches consisted of 12 boxes. The contents of each box were counted as:

34	31	29
35	33	30
31	28	29
35	32	31

On the box it stated 'Average contents 32 matches'. Is this correct?

7 Mr Brennan told each student their test mark and only gave the test statistics to the whole class. He gave the class the modal mark, the median mark and the mean mark.

a Which average would tell a student whether they were in the top half or the bottom half of the class?

b Which average tells the students nothing really?

c Which average allows a student to gauge how well they have done compared with everyone else?

8 Three players were hoping to be chosen for the basketball team.

The table shows their scores in the last few games they played.

Tom	16, 10, 12, 10, 13, 8, 10
David	16, 8, 15, 25, 8
Mohaned	15, 2, 15, 3, 5

The teacher said they would be chosen by their best average score.

Which average would each boy want to be chosen by?

CORE

9 a Find five numbers that have *both* the properties below:

- a range of 5
- a mean of 5.

b Find five numbers that have *all* the properties below:

- a range of 5
- a median of 5
- a mean of 5.

10 What is the average pay at a factory with 10 employees?

The boss said: '$43 295'

A worker said: '$18 210'

They were both correct.

Explain how this can be.

11 A list of nine numbers has a mean of 7.6. What number must be added to the list to give a new mean of 8?

12 A dance group of 17 people had a mean mass of 54.5 kg. To enter a competition there needed to be 18 people with an average mass of 54.4 kg or less. What is the maximum mass that the eighteenth person must have?

32.6 Stem-and-leaf diagrams

Here are the ages of 20 people

23, 13, 34, 44, 26, 12, 41, 31, 20, 18, 19, 31, 48, 32, 45, 14, 12, 27, 31, 19

Here are their ages in order.

12, 12, 13, 14, 18, 19, 19, 20, 23, 26, 27, 31, 31, 31, 32, 34, 41, 44, 45, 48

This is easier to read and analyse.

You can put the ages in a **stem-and-leaf** diagram. The number of values will be the 'stem' and the unit values will be the 'leaves'.

Key : 1 | 2 represents 12

1	2	2	3	4	8	9	9
2	0	3	6	7			
3	1	1	1	2	4		
4	1	4	5	8			

This is called a stem-and-leaf diagram. It gives a better idea of how the data is distributed.

A stem-and-leaf diagram should always have a key.

Example 6

Put the following data into a stem-and-leaf diagram.

45, 62, 58, 58, 61, 49, 61, 47, 52, 58, 48, 56, 65, 46, 54

a What is the modal value?

b What is the median value?

c What is the range of the values?

First, decide on the stem and leaf.

In this case, the tens digit will be the stem and the units digit will be the leaf.

Key: 4 | 5 represents 45

4	5	6	7	8	9	
5	2	4	6	8	8	8
6	1	1	2	5		

a The modal value is the most common, which is 58.

b There are 15 values, so the median will be the value that is, (15 + 1) ÷ 2, or the 8th value. Counting from either the top or the bottom, the median is 56.

c The range is the difference between the largest and the smallest value, which is 65 − 45 = 20

EXERCISE 32F

1 This stem-and-leaf diagram shows the marks some students scored in a test.

5	3	5	5	9	9							
6	0	0	4	5	5	6	8	8				
7	1	1	1	1	1	3	4	6	6	8	9	9
8	0	0	0	2	3	3	3	5	7			
9	0	1	4	4	5	6						

Key: 5 | 3 represents 53 marks

a How many students took the test?

b Find the median mark.

c Find the range.

d Find the modal mark.

e Explain why the median is more useful than the mode.

CORE

CORE

2 This stem-and-leaf diagram shows the times of athletes running 200 metres.

23.	8	9	9											
24.	1	3	3	5	7	8								
25.	0	0	1	1	3	5	8	8	9					
26.	1	4	6	6	7	8	9	9						
27.	0	0	0	2	2	2	2	2	2	3	5	8	8	8
28.	0	4	5	5	5	6	7							

Key: 23. | 8 represents 23.8 seconds

a How many runners took less than 26 seconds?

b Find the median time.

c Find the range of the times.

3 This stem-and-leaf diagram shows the times 50 people take to travel to work.

1	5	5	7	8	8	9	9						
2	0	0	0	2	3	5	5	5	5	6	9		
3	0	0	3	3	5	5	5	5	6	6	6	7	8
4	0	0	0	2	5								
5	3	5	5	5	5	6	6	9					
6	5	5											
7	0	6	6										
8	5												

Key: 5 | 3 represents 53 minutes

a How many people took more than an hour?

b Find the median time.

c Find the range of the times.

4 The heights of 15 tulips are measured.

43 cm, 39 cm, 41 cm, 29 cm, 36 cm,

34 cm, 43 cm, 48 cm, 38 cm, 35 cm,

41 cm, 38 cm, 43 cm, 28 cm, 48 cm

a Show the results in a stem-and-leaf diagram, using this key.

Key: 4 | 3 represents 43 cm

b What is the model height?

c What is the median height?

d What is the range of the heights?

5 A student records the number of text messages she receives each day for two weeks.

12, 18, 21, 9, 17, 23, 8, 2, 20, 13, 17, 22, 9, 9

a Show the results in a stem-and-leaf diagram, using this key.

Key: 1 | 2 represents 12 messages

b What was the modal number of text messages received in a day?

c What was the median number of text messages received in a day?

6 Zachia wanted to know how many people attended a daily youth club each day over a month. She recorded the data.

13, 19, 20, 9, 18, 24, 7, 8, 19, 14, 18, 23, 9, 10, 15, 31, 28, 26, 12, 24

a Show these results in a stem-and-leaf diagram.

b What is the median number of people at the youth club?

c What is the range of the numbers of people who attended the youth club?

7 This stem-and-leaf diagram shows the ages of the men and women working for a company.

Men		Women
9 9	1	8 9 9 9
8 5 4 2 2 1 0	2	0 0 0 1 1 3 3 5 8 9 9
9 9 7 6 6 4 3 1	3	0 2 2 3 3 5 6 6 7
8 7 6 6 6 5 3 3 3 2	4	0 3 3 4 5 6 6
7 7 6 5 4 1 0 0 0	5	1 2 8
4 3 1 1 1	6	

Key: 9 | 1 | 8 represents a man age 19 and a woman age 18

Copy and complete this table.

	Men	Women
Number of people	41	
Range of ages		
Median age		

8 This chart shows the heights of some girls and boys

Heights of girls		Heights of boys
8 7 4	12	8 9 9 9 9
7 7 5 5 0	13	0 4 5 5 6 7 7 7 7 8
9 8 8 8 6 4 3 2	14	1 1 1 3 5 5 5 5 6 6 9
7 5 3 1 0	15	0 3 3 4 5 5 7 8 8
4 2 2 1	16	1 2 2 2 4 6 8 8 8 9
	17	0 0 1 3

Key: 4 | 12 | 8 represents a girl 124 cm tall and a boy 128 cm tall

CORE

a How many children are less than 130 cm tall?

b Copy and complete this table.

	Girls	Boys
Number of children		49
Median height		
Range of heights		

c Complete these sentences

i The girls' median height is ... cm more/less than the boys' median height.

ii The girls' range is ... cm more/less than the boys' range.

Circle the correct word in more/less each time.

32.7 Using frequency tables

When you have gathered a lot of information, it is often convenient to put it together in a **frequency table**. From this table you can then find the values of the three averages and the range.

Example 7

A survey was done on the number of people in each car leaving a shopping centre. The results are summarised in the table.

Number of people in each car	1	2	3	4	5	6
Frequency	45	198	121	76	52	13

For the number of people in a car, calculate:

a the mode

b the median

c the mean.

a The modal number of people in a car is easy to spot. It is the number with the largest frequency, which is 198. Hence, the modal number of people in a car is 2.

b You can find the median number of people in a car by working out where the middle of the set of numbers is located. First, add up frequencies to get the total number of cars surveyed, which comes to 505. Next, calculate the middle position.

$(505 + 1) \div 2 = 253$

Now add the frequencies across the table to find which group contains the 253rd item. The 243rd item is the end of the group with 2 in a car. Therefore, the 253rd item must be in the group with 3 in a car. Hence, the median number of people in a car is 3.

c To calculate the mean number of people in a car, multiply the number of people in the car by the frequency. This is best done in an extra column. Add these products to find the total number of people and divide by the total frequency (the number of cars surveyed).

Number in car	Frequency	Number in these cars
1	45	1 × 45 = 45
2	198	2 × 198 = 396
3	121	3 × 121 = 363
4	76	4 × 76 = 304
5	52	5 × 52 = 260
6	13	6 × 13 = 78
Totals	505	1446

Hence, the mean number of people in a car is 1446 ÷ 505 = 2.9 (to 1 decimal place).

When you have gathered a lot of information, it is often convenient to put it together in a **frequency table**. From this table you can then find the values of the three averages and the range.

EXERCISE 32G

CORE

1 Find **i** the mode **ii** the median **iii** the mean from each frequency table below.

a A survey of the shoe sizes of all the boys in one year of a school gave these results.

Shoe size	4	5	6	7	8	9	10
Number of students	12	30	34	35	23	8	3

b A survey of the number of eggs laid by hens over a period of one week gave these results.

Number of eggs	0	1	2	3	4	5	6
Frequency	6	8	15	35	48	37	12

c This is a record of the number of babies born each week over one year in a small maternity unit.

Number of babies	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Frequency	1	1	1	2	2	2	3	5	9	8	6	4	5	2	1

d A school did a survey on how many times in a week students arrived late at school. These are the findings.

Number of times late	0	1	2	3	4	5
Frequency	481	34	23	15	3	4

CORE

2 A survey of the number of children in each family of a school's intake gave these results.

Number of children	1	2	3	4	5
Frequency	214	328	97	26	3

a Assuming each child at the school is shown in the data, how many children are at the school?

b Calculate the mean number of children in a family.

c How many families have this mean number of children?

d How many families would consider themselves average from this survey?

3 A dentist kept records of how many teeth he extracted from his patients.

In 1989 he extracted 598 teeth from 271 patients.

In 1999 he extracted 332 teeth from 196 patients.

In 2009 he extracted 374 teeth from 288 patients.

a Calculate the average number of teeth taken from each patient in each year.

b Explain why you think the average number of teeth extracted falls each year.

4 One hundred cases of apples delivered to a supermarket were inspected and the numbers of bad apples were recorded.

Bad apples	0	1	2	3	4	5	6	7	8	9
Frequency	52	29	9	3	2	1	3	0	0	1

Give:

a the modal number of bad apples per case

b the mean number of bad apples per case.

5 Two dice are thrown together 60 times. The sums of the scores are shown below.

Score	2	3	4	5	6	7	8	9	10	11	12
Frequency	1	2	6	9	12	15	6	5	2	1	1

Find:

a the modal score

b the median score

c the mean score.

6 During a one-month period, the number of days off taken by 100 workers in a factory were noted as follows.

Number of days off	0	1	2	3	4
Number of workers	35	42	16	4	3

Calculate:

a the modal number of days off

b the median number of days off

c the mean number of days off.

7 Two friends often played golf together. They recorded the numbers of shots they made to get their balls into each hole over the last five games to compare who was more consistent and who was the better player. Their results were summarised in the table.

Number of shots	1	2	3	4	5	6	7	8	9
Roger	0	0	0	14	37	27	12	0	0
Brian	5	12	15	18	14	8	8	8	2

a What is the modal score for each player?

b What is the range of scores for each player?

c What is the median score for each player?

d What is the mean score for each player?

e Which player is the more consistent and why?

f Who would you say is the better player and why?

8 A tea stain on a newspaper removed four numbers from this frequency table of goals scored in 40 football matches one weekend.

Goals	0	1	2			5
Frequency	4	6	9			3

The mean number of goals scored is 2.4.

What could the missing four numbers be?

9 Manju made day trips to Mumbai frequently during a year.

The table shows how many days in a week she travelled.

Days	0	1	2	3	4	5
Frequency	17	2	4	13	15	1

Explain how you would find the median number of days Manju travelled in a week to Mumbai.

32.8 Grouped data

E

Sometimes the information you are given is grouped in some way (called **grouped data**), as in Example 8, which shows the range of weekly pocket money given to Year 12 students in a particular class.

Normally, grouped tables use **continuous data**, which is data that can have any value within a range of values, for example, height, mass, time, area and capacity. In these situations, the **mean** can only be **estimated** as you do not have all the information.

Discrete data is data that consists of separate numbers, for example, goals scored, marks in a test, number of children and shoe sizes.

In both cases, when using a grouped table to estimate the mean, first find the midpoint of the interval by adding the two end-values and then dividing by two.

Example 8

Pocket money, p ($)	$0 < p \leqslant 1$	$1 < p \leqslant 2$	$2 < p \leqslant 3$	$3 < p \leqslant 4$	$4 < p \leqslant 5$
No. of students	2	5	5	9	15

a Write down the modal class.

b Calculate an estimate of the mean weekly pocket money.

a The modal class is easy to identify, since it is simply the one with the largest frequency. Here the modal class is $4 to $5.

b To estimate the mean, assume that each person in each class has the 'midpoint' amount, then build up the following table.

To find the midpoint value, add the two end-values together and then divide by two.

Pocket money, p ($$)	Frequency (f)	Midpoint (m)	$f \times m$
$0 < p \leqslant 1$	2	0.50	1.00
$1 < p \leqslant 2$	5	1.50	7.50
$2 < p \leqslant 3$	5	2.50	12.50
$3 < p \leqslant 4$	9	3.50	31.50
$4 < p \leqslant 5$	15	4.50	67.50
Totals	36		120

The estimated mean will be $120 ÷ 36 = $3.33 (rounded to the nearest cent).

If you had written 0.01–1.00, 1.01–2.00 and so on for the groups, then the midpoints would have been 0.505, 1.505 and so on. This would not have had a significant effect on the final answer as it is only an estimate.

Note that you *cannot* find the **median** or the range from a grouped table as you do not know the actual values.

EXERCISE 32H

1 For each table of values given below, find:

i the modal group

ii an estimate for the mean.

Advice and Tips

When you copy the tables, draw them vertically as in Example 7.

a

x	$0 < x \leqslant 10$	$10 < x \leqslant 20$	$20 < x \leqslant 30$	$30 < x \leqslant 40$	$40 < x \leqslant 50$
Frequency	4	6	11	17	9

b

y	$0 < y \leqslant 100$	$100 < y \leqslant 200$	$200 < y \leqslant 300$	$300 < y \leqslant 400$	$400 < y \leqslant 500$	$500 < x \leqslant 600$
Frequency	95	56	32	21	9	3

c

z	$0 < z \leqslant 5$	$5 < z \leqslant 10$	$10 < z \leqslant 15$	$15 < z \leqslant 20$
Frequency	16	27	19	13

d

Weeks	1–3	4–6	7–9	10–12	13–15
Frequency	5	8	14	10	7

2 Jason brought 100 pebbles back from the beach and found their masses, recording each mass to the nearest gram. His results are summarised in the table.

Mass, m (g)	$40 < m \leqslant 60$	$60 < m \leqslant 80$	$80 < m \leqslant 100$
Frequency	5	9	22

Mass, m (g)	$100 < m \leqslant 120$	$120 < m \leqslant 140$	$140 < m \leqslant 160$
Frequency	27	26	11

Find:

a the modal class of the pebbles

b an estimate of the total mass of all the pebbles

c an estimate of the mean mass of the pebbles.

3 A gardener measured the heights of all his daffodils to the nearest centimetre and summarised his results as follows.

Height (cm)	10–14	15–18	19–22	23–26	27–40
Frequency	21	57	65	52	12

a How many daffodils did the gardener have?

b What is the modal class of the daffodils?

c What is the estimated mean height of the daffodils?

EXTENDED

4 A survey was created to see how quickly an emergency service got to cars that had broken down. The table summarises the results.

Time (min)	1–15	16–30	31–45	46–60	61–75	76–90	91–105
Frequency	2	23	48	31	27	18	11

a How many calls were used in the survey?

b Estimate the mean time taken per call.

c Which average would the emergency service use for the average call-out time?

d What percentage of calls do the emergency service get to within the hour?

5 One hundred light bulbs were tested by their manufacturer to see whether the average life-span of the manufacturer's bulbs was over 200 hours. The table summarises the results.

Life span, h (hours)	$150 < h \leqslant 175$	$175 < h \leqslant 200$	$200 < h \leqslant 225$	$225 < h \leqslant 250$	$250 < h \leqslant 275$
Frequency	24	45	18	10	3

a What is the modal length of time a bulb lasts?

b What percentage of bulbs last longer than 200 hours?

c Estimate the mean life-span of the light bulbs.

d Do you think the test shows that the average life-span is over 200 hours? Explain your answer fully.

6 Three shops each claimed to have the lowest average price increase over the year. The table summarises their price increases.

Price increase (p)	1–5	6–10	11–15	16–20	21–25	26–30	31–35
Soundbuy	4	10	14	23	19	8	2
Springfields	5	11	12	19	25	9	6
Setco	3	8	15	31	21	7	3

Using their average price increases, make a comparison of the supermarkets and say which one has the lowest price increases over the year. Remember to justify your answers.

7 The table shows the distances run, over a month, by an athlete who is training for a half-marathon.

Distance, d (km)	$0 < d \leqslant 5$	$5 < d \leqslant 10$	$10 < d \leqslant 15$	$15 < d \leqslant 20$	$20 < d \leqslant 25$
Frequency	3	8	13	5	2

A half-marathon is 21 kilometres. It is recommended that an athlete's daily average distance should be at least a third of the distance of the race for which they are training. Is this athlete doing enough training?

8 The table shows the points scored in a general-knowledge competition by all the players.

Points	0–9	10–19	20–29	30–39	40–49
Frequency	8	5	10	5	2

Balvir noticed that two frequencies were the wrong way round and that this made a difference of 1.7 to the arithmetic mean.

Which two frequencies were the wrong way round?

9 The profit made each week by a charity shop is shown in the table below.

Profit	\$0–\$500	\$501–\$1000	\$1001–\$1500	\$1501–\$2000
Frequency	15	26	8	3

Estimate the mean profit made each week.

10 The table shows the number of members of 100 football clubs.

Members	20–29	30–39	40–49	50–59	60–69
Frequency	16	34	27	18	5

Roger claims that the median number of members is 39.5.

Is he correct? Explain your answer.

32.9 Cumulative frequency diagrams

E

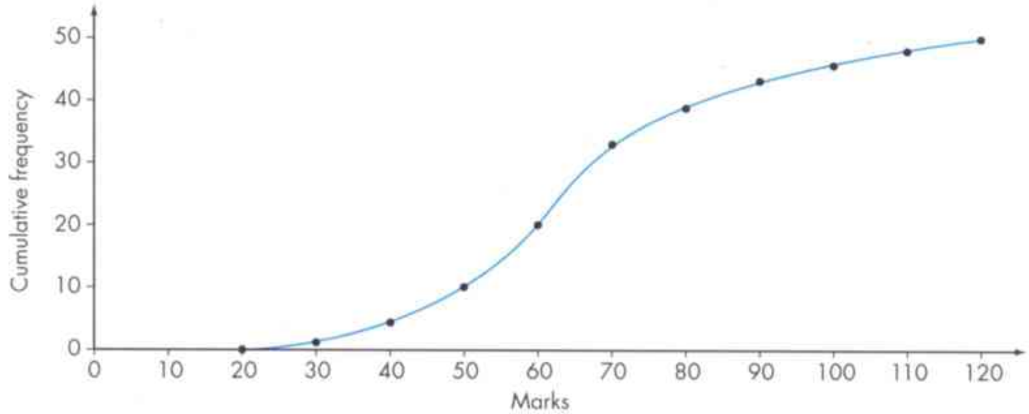
The **inter-quartile range** is a measure of the **dispersion** of a set of data. The advantage of the interquartile range is that it eliminates extreme values, and bases the measure of spread on the middle 50% of the data. This section will show you how to find the interquartile range and the median of a set of data by drawing a **cumulative frequency diagram.**

Look at the marks of 50 students in a mathematics test, which have been put into a grouped table. Note that it includes a column for the **cumulative frequency**, which you can find by adding each frequency to the sum of all preceding frequencies.

Mark	No. of students	Cumulative frequency
21 to 30	1	1
31 to 40	3	4
41 to 50	6	10
51 to 60	10	20

Mark	No. of students	Cumulative frequency
61 to 70	13	33
71 to 80	6	39
81 to 90	4	43
91 to 100	3	46
101 to 110	2	48
111 to 120	2	50

This data can then be used to plot a graph of the top value of each group against its cumulative frequency. The points to be plotted are (30, 1), (40, 4), (50, 10), (60, 20), etc., which will give the graph shown below. Note that the cumulative frequency is *always* the vertical axis.



Also note that the scales on both axes are labelled at each graduation mark, in the usual way. *Do not* label the scales as shown below. It is *wrong*.

The plotted points can be joined by a freehand curve, to give a cumulative frequency diagram.

The median

The **median** is the middle item of data, once all the items have been put in order of size, from lowest to highest. So, if you have n items of data plotted as a cumulative frequency diagram, you can find the median from the middle value of the cumulative frequency, that is the $\frac{1}{2}n$th value.

But remember, if you want to find the median from a simple list of discrete data, you **must** use the $\frac{1}{2}(n + 1)$th value. The reason for the difference is that the cumulative frequency diagram treats the data as continuous, even for data such as examination marks, which are discrete. You can use the $\frac{1}{2}n$th value when working with cumulative frequency diagrams because you are only looking for an *estimate* of the median.

There are 50 values in the table on this and the previous page. To find the median:

- The middle value will be the 25th value.
- Draw a horizontal line from the 25th value to meet the graph.
- Now look down to the horizontal axis.

This will give an estimate of the median. In this example, the median is about 64 marks.

The inter-quartile range

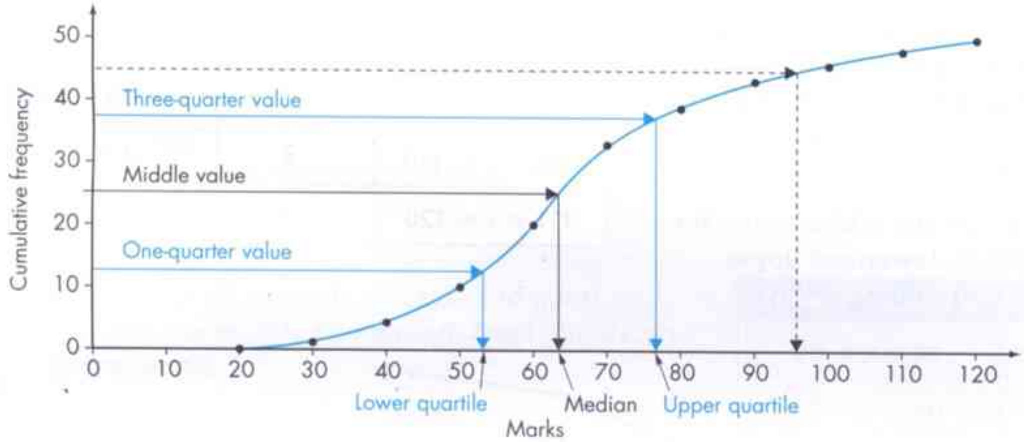
By dividing the cumulative frequency into four parts, you can obtain **quartiles** and the inter-quartile range.

The **lower quartile** is the value one-quarter of the way up the cumulative frequency axis and is given by the $\frac{1}{4}n$th value.

The **upper quartile** is the value three-quarters of the way up the cumulative frequency axis and is given by the $\frac{3}{4}n$th value.

The inter-quartile range is the difference between the lower and upper quartiles.

These are illustrated on the graph below.



The quarter and three-quarter values out of 50 values are the 12.5th value and the 37.5th value. Draw lines across to the cumulative frequency curve from these values and down to the horizontal axis. These give the lower and upper quartiles. In this example, the lower quartile is 54 marks, the upper quartile is 77 marks and the inter-quartile range is 77 – 54 = 23 marks.

As well as the median and the quartiles we can find percentiles. For example to find the 90th percentile:

90% of 50 is 45.

Draw a line across from a cumulative frequency of 45 and then down to the horizontal axis.

This shows that a mark of 95 is the 90th percentile.

So 90% of the students scored $\leqslant$ 95 marks.

Other percentiles:

- The median is the 50th percentile.
- The upper quartile is the 75th percentile.
- The lower quartile is the 25th percentile.

Example 9

This table shows the marks of 100 students in a mathematics test.

a Draw a cumulative frequency curve.

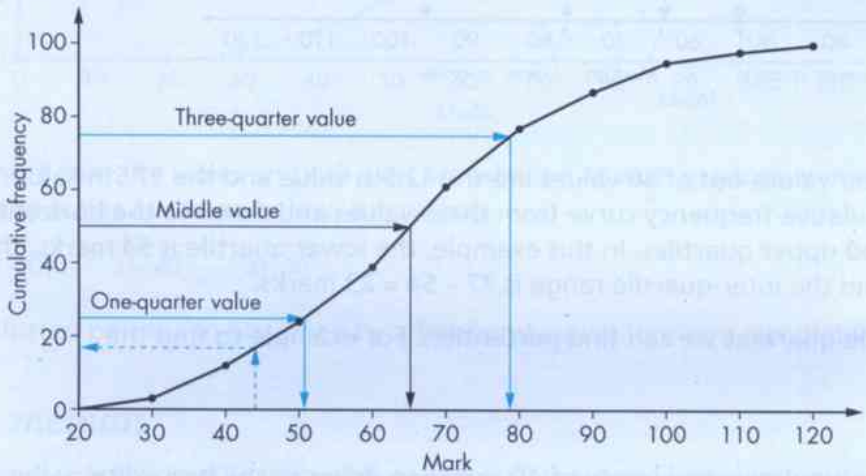
b Use your graph to find the median and the inter-quartile range.

c Students who score less than 44 have to have extra teaching. How many students will have to have extra teaching?

Mark, x	No. of students	Cumulative frequency
$21 \leqslant x \leqslant 30$	3	3
$31 \leqslant x \leqslant 40$	9	12
$41 \leqslant x \leqslant 50$	12	24
$51 \leqslant x \leqslant 60$	15	39
$61 \leqslant x \leqslant 70$	22	61
$71 \leqslant x \leqslant 80$	16	77
$81 \leqslant x \leqslant 90$	10	87
$91 \leqslant x \leqslant 100$	8	95
$101 \leqslant x \leqslant 110$	3	98
$111 \leqslant x \leqslant 120$	2	100

You will meet several ways of giving groups (for example, 21–30, 20, $x \leqslant 30$, 21, x, 30) but the important thing to remember is to plot the top point of each group against the corresponding cumulative frequency.

a and **b** Draw the graph and add the lines for the median (50th value), lower and upper quartiles (25th and 75th values).



The required answers are read from the graph.

Median = 65 marks

Lower quartile = 51 marks

Upper quartile = 79 marks

Interquartile range = 79 − 51 = 28 marks

c At 44 on the mark axis, draw a perpendicular line to intersect the graph, and at the point of intersection draw a horizontal line across to the cumulative frequency axis, as shown. Number of students needing extra teaching is 18.

Note: An alternative way in which the table in Example 9 could have been set out is shown below. This arrangement has the advantage that the points to be plotted are taken straight from the last two columns. Decide which method you prefer.

Mark, x	No. of students	Less than	Cumulative frequency
$21 \leqslant x \leqslant 30$	3	30	3
$31 \leqslant x \leqslant 40$	9	40	12
$41 \leqslant x \leqslant 50$	12	50	24
$51 \leqslant x \leqslant 60$	15	60	39
$61 \leqslant x \leqslant 70$	22	70	61
$71 \leqslant x \leqslant 80$	16	80	77
$81 \leqslant x \leqslant 90$	10	90	87
$91 \leqslant x \leqslant 100$	8	100	95
$101 \leqslant x \leqslant 110$	3	110	98
$111 \leqslant x \leqslant 120$	2	120	100

EXERCISE 32I

EXTENDED

1 A class of 30 students was asked to guess when one minute had passed. The table shows the results.

Time, x (seconds)	No. of students
$20 < x \leqslant 30$	1
$30 < x \leqslant 40$	3
$40 < x \leqslant 50$	6
$50 < x \leqslant 60$	12
$60 < x \leqslant 70$	3
$70 < x \leqslant 80$	3
$80 < x \leqslant 90$	2

- **a** Copy the table and complete a cumulative frequency column.
- **b** Draw a cumulative frequency diagram.
- **c** Use your diagram to estimate the median time and the inter-quartile range.

2 A group of 50 pensioners was given the task in question **1**. The results are shown in the table.

Time, x (seconds)	No. of pensioners
$10 < x \leqslant 20$	1
$20 < x \leqslant 30$	2
$30 < x \leqslant 40$	2
$40 < x \leqslant 50$	9
$50 < x \leqslant 60$	17
$60 < x \leqslant 70$	13
$70 < x \leqslant 80$	3
$80 < x \leqslant 90$	2
$90 < x \leqslant 100$	1

- **a** Copy the table and complete a cumulative frequency column.
- **b** Draw a cumulative frequency diagram.
- **c** Use your diagram to estimate the median time and the inter-quartile range.
- **d** Which group, the students or the pensioners, was better at estimating time? Give a reason for your answer.

EXTENDED

3 The sizes of 360 senior schools are recorded in the table.

a Copy the table and complete a cumulative frequency column.

b Draw a cumulative frequency diagram.

c Use your diagram to estimate the median size of the schools and the inter-quartile range.

d Schools with fewer than 350 students are threatened with closure. About how many schools are threatened with closure?

e Use your graph to estimate the 90th percentile.

f Use your graph to estimate the 40th percentile.

No. of students	No. of schools
100–199	12
200–299	18
300–399	33
400–499	50
500–599	63
600–699	74
700–799	64
800–899	35
900–999	11

4 The temperature at a seaside town was recorded for 50 days. It was recorded to the nearest degree. The table shows the results.

a Copy the table and complete a cumulative frequency column.

b Draw a cumulative frequency diagram. Note that as the temperature is to the nearest degree the top values of the groups are 7.5 °C, 10.5 °C, 13.5 °C, 16.5 °C, etc.

c Use your diagram to estimate the median temperature and the inter-quartile range.

d Use your diagram to estimate the 10th percentile.

Temperature (°C)	No. of days
5–7	2
8–10	3
11–13	5
14–16	6
17–19	6
20–22	9
23–25	8
26–28	6
29–31	5

5 A game consists of throwing three darts and recording the total score. The results of the first 80 people to throw are recorded in the table.

a Draw a cumulative frequency diagram to show the data.

b Use your diagram to estimate the median score and the inter-quartile range.

c People who score over 90 get a prize. About what percentage of the people get a prize?

Total score, x	No. of players
$1 \leqslant x \leqslant 20$	9
$21 \leqslant x \leqslant 40$	13
$41 \leqslant x \leqslant 60$	23
$61 \leqslant x \leqslant 80$	15
$81 \leqslant x \leqslant 100$	11
$101 \leqslant x \leqslant 120$	7
$121 \leqslant x \leqslant 140$	2

6 One hundred children were asked to say how much pocket money they got in a week. The results are in the table.

a Copy the table and complete a cumulative frequency column.

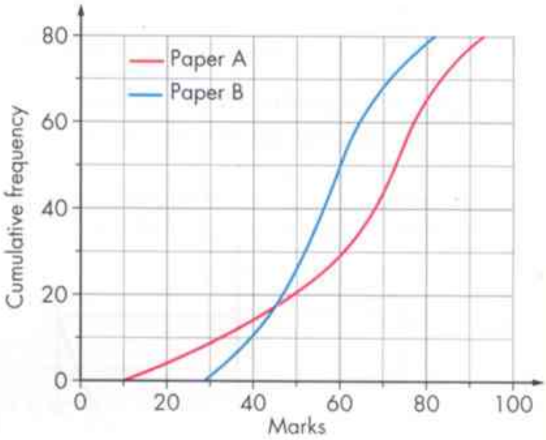
b Draw a cumulative frequency diagram.

c Use your diagram to estimate the median amount of pocket money and the inter-quartile range.

d Estimate the 10th percentile and the 90th percentile.

Amount of pocket money (cents)	No. of children
51–100	6
101–150	10
151–200	20
201–250	28
251–300	18
301–350	11
351–400	5
401–450	2

7 Johan set his class an end-of-course test with two papers, A and B. He produced the cumulative frequency graphs, as shown.



a What is the median score for each paper?

b What is the inter-quartile range for each paper?

c Which is the harder paper? Explain how you know.

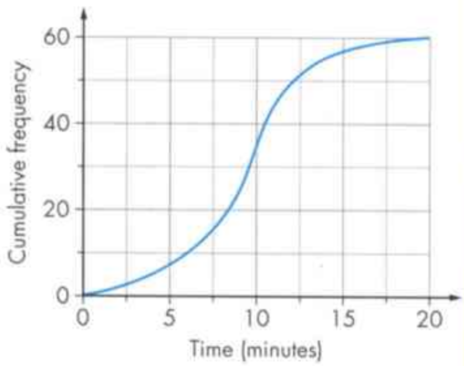
Johan wanted 80% of the students to pass each paper and 20% of the students to get a top grade in each paper.

d What marks for each paper give:

i a pass **ii** the top grade?

8 The lengths of time taken by 60 helpline telephone calls were recorded.

A cumulative frequency diagram of this data is shown here.



a Estimate the percentage of calls lasting more than 10 minutes.

b What is the 15th percentile for call lengths?

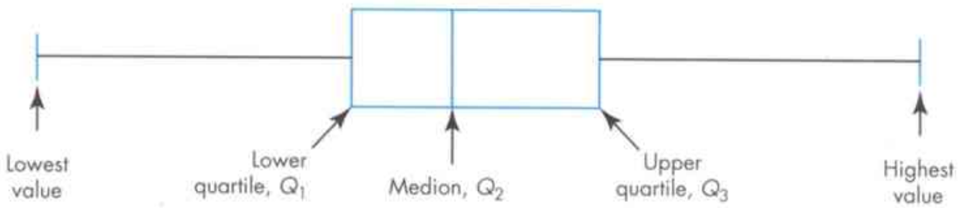
9 Byron was given a cumulative frequency diagram showing the marks obtained by students in a mental maths test.

He was told the top 10% were given the top grade.

How would he find the marks needed to gain this top award?

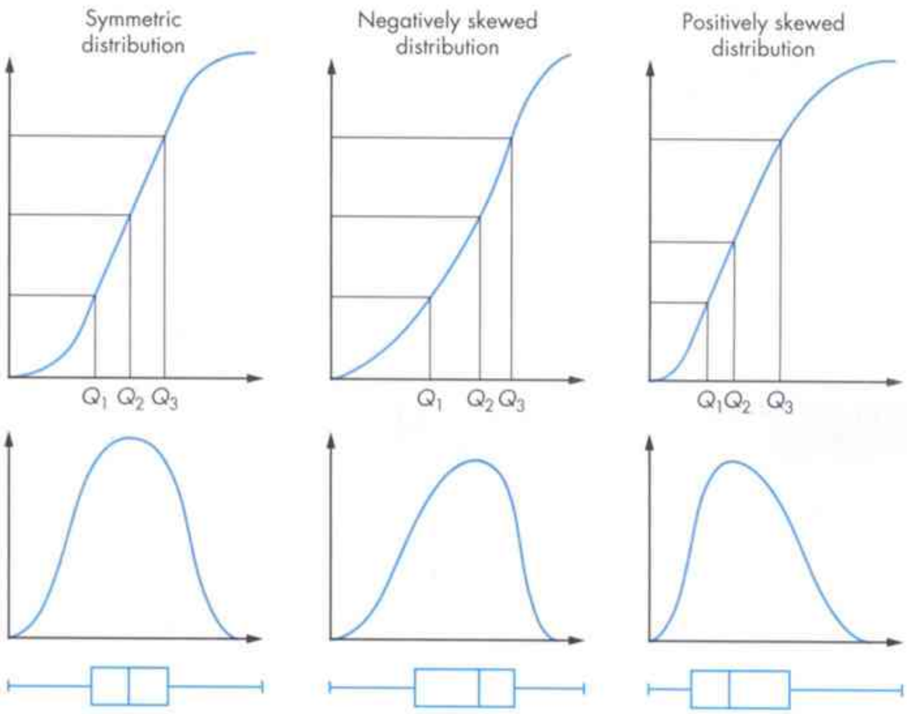
32.10 Box-and-whisker plots

Another way of displaying data for comparison is by means of a **box-and-whisker plot**. This requires five pieces of data. These are the **lowest value**, the **lower quartile** (Q_1), the **median** (Q_2), the **upper quartile** (Q_3) and the **highest value**. They are drawn in the following way.



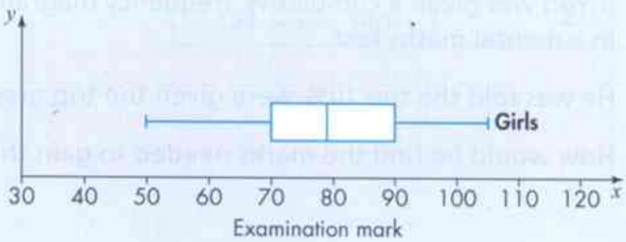
These data values are always placed against a scale so that they are accurately plotted.

The following diagrams show how the cumulative frequency curve, the frequency curve and the box-and-whisker plot are connected for three common types of distribution.



Example 10

The box-and-whisker plot for the girls' marks in last year's examination is shown here.

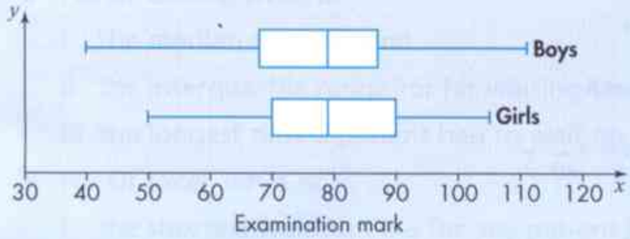


The boys' results for the same examination are: lowest mark 39, lower quartile 65, median 78, upper quartile 87, highest mark 112.

a On the same grid, draw the box-and-whisker plot for the boys' marks.

b Comment on the differences between the two distributions of marks.

a The data for boys and girls is plotted on the grid below.

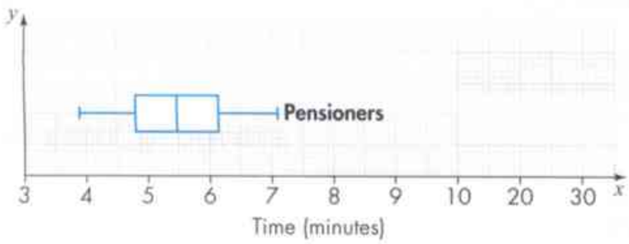


b The girls and boys have the same median mark but both the lower and upper quartiles for the girls are higher than those for the boys, and the girls' range is slightly smaller than the boys'.

This suggests that the girls did better than the boys overall, even though a boy got the highest mark.

EXERCISE 32J

1 The box-and-whisker plot shows the times taken for a group of pensioners to do a set of 10 long-division calculations.

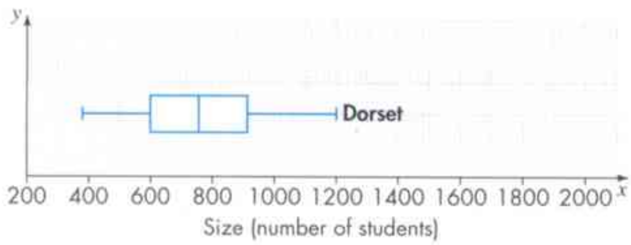


The same set of calculations was given to some students in Year 11. Their results are: shortest time 3 minutes 20 seconds, lower quartile 6 minutes 10 seconds, median 7 minutes, upper quartile 7 minutes 50 seconds and longest time 9 minutes 40 seconds.

a Copy the diagram and draw a box-and-whisker plot for the students' times.

b Comment on the differences between the two distributions.

2 The box-and-whisker plot shows the sizes of secondary schools in Dorset.



EXTENDED

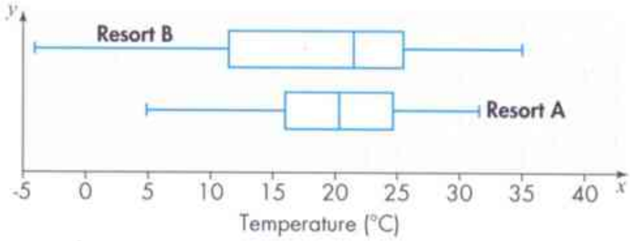
EXTENDED

The data for schools in Rotherham is: smallest 280 students, lower quartile 1100 students, median 1400 students, upper quartile 1600 students, largest 1820 students.

a Copy the diagram and draw a box-and-whisker plot for the sizes of schools in Rotherham.

b Comment on the differences between the two distributions.

3 The box-and-whisker plots for the noon temperature at two resorts, recorded over a year, are shown on the grid below.



a Comment on the differences in the two distributions.

b Mary wants to go on holiday in July. Which resort would you recommend? Why?

4 The following table shows some data on the annual salaries for 100 men and 100 women.

	Lowest Salary	Lower quartile	Median salary	Upper quartile	Highest salary
Men	\$6500	\$16 000	\$20 000	\$22 000	\$44 500
Women	\$7000	\$14 000	\$16 000	\$21 500	\$33 500

a Draw box-and-whisker plots to compare both sets of data.

b Comment on the differences between the distributions.

5 The table shows the monthly salaries of 100 families.

Monthly salary (\$)	No. of families
1451–1500	8
1501–1550	14
1551–1600	25
1601–1650	35
1651–1700	14
1701–1750	4

a Draw a cumulative frequency diagram to show the data.

b Estimate the median monthly salary and the interquartile range.

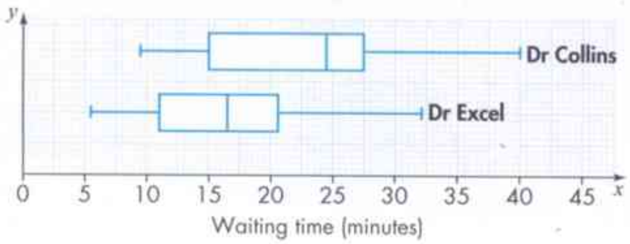
c The lowest monthly salary was \$1480 and the highest was \$1740.

i Draw a box-and-whisker plot to show the distribution of salaries.

ii Is the distribution symmetric, negatively skewed or positively skewed?

6 A health practice had two doctors, Dr Excel and Dr Collins.

The following box-and-whisker plots were created to illustrate the waiting times for their patients during October.



a For Dr Collins, what is:

i the medián waiting time

ii the interquartile range for his waiting time

iii the longest time a patient had to wait on October?

b For Dr Excel, what is:

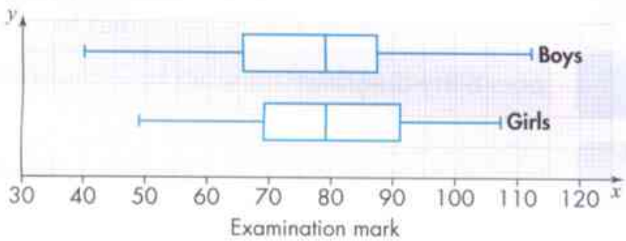
i the shortest waiting time for any patient in October

ii the median waiting time

iii the interquartile range for his waiting time?

c Anwar was deciding which doctor to try to see. Which one would you advise he sees? Why?

7 The box-and-whisker plot for a school's end-of-year mathematics tests are shown below.



What is the difference between the means of the boys' and the girls' test results?

Check your progress

Core

- I can calculate the mean, median, mode and range for individual and discrete data
- I can distinguish between the uses for the mean, median, mode and range
- I can construct and interpret stem-and-leaf tables
- I can calculate an estimate of the mean for grouped and continuous data
- I can identify the modal class from a grouped frequency distribution
- I can construct and use cumulative frequency diagrams
- I can estimate and interpret the median, percentiles, quartiles and inter-quartile range

Extended

- I can construct and interpret box-and-whisker plots

Chapter 33

Probability

Topics	Level	Key words
1 The probability scale	CORE	chance, outcome, event, probability, impossible, certain
2 Calculating probabilities	CORE	outcome, equally likely, probability fraction, random
3 Probability that an event will not happen	CORE	outcome
4 Probability in practice	CORE	experimental probability, trials, relative frequency
5 Using Venn diagrams	CORE	
6 Possibility diagrams	CORE	possibility diagram
7 Tree diagrams	CORE	tree diagram
8 Conditional probability	EXTENDED	

In this chapter you will learn how to:

CORE	EXTENDED
• Calculate the probability of a single event as either a fraction, decimal or percentage. (C8.1 and E8.1) • Understand and use the probability scale from 0 to 1. (C8.2 and E8.2) • Understand that: *the probability of an event occurring* = 1 – *the probability of the event not occurring*. (C8.3 and E8.3) • Understand relative frequency as an estimate of probability. Expected frequency of occurrences. (C8.4 and E8.4) • Calculate the probability of simple combined events, using possibility diagrams and tree diagrams and Venn diagrams. (C8.5 and E8.5)	• Calculate conditional probability using Venn diagrams, tree diagrams and tables. (E8.6)

Why this chapter matters

Chance is a part of everyday life. Judgements are frequently made based on the probability of something happening.

For example:

- there is an 80% chance that my team will win the game tomorrow
- there is a 40% chance of rain tomorrow
- she has a 50–50 chance of having a baby girl
- there is a 10% chance of the bus being on time tonight.

In everyday life we talk about the probability of something happening. Two people might give different probabilities to the same events because of their different views. For example, some people might not agree that there is an 80% chance of your team winning the game. They might say that there is only a 70% chance of them winning tomorrow. A lot depends on what people believe or have experienced.

When people first started to predict the weather scientifically over 150 years ago, they used probabilities to do it. For example, meteorologists looked for three important indicators of rain:

- the number of nimbus clouds in the sky
- falling pressure on a barometer
- the direction of the wind and whether it was blowing from a part of the country with high rainfall.

If all three of these things occurred together rain would almost certainly follow soon.

Now, in the 21st century, probability theory is used to control the flow of traffic through road systems (below left) or the running of telephone exchanges (below right), and to look at patterns of the spread of infections.

33.1 The probability scale

Almost daily, you hear somebody talking about the probability of whether something will happen. They usually use words such as **'chance'**, 'likelihood' or 'risk' rather than 'probability'. For example:

'What is the likelihood of rain tomorrow?'

'What chance does she have of winning the 100 metre sprint?'

'Is there a risk that his company will go bankrupt?'

You can give a value to the chance of any of these **outcomes** or **events** happening – and millions of others, as well. This value is called the **probability**.

It is true that some things are certain to happen and that some things cannot happen; that is, the chance of something happening can be anywhere between **impossible** and **certain**. This situation is represented on a sliding scale called the **probability scale**, as shown here.

Note: All probabilities lie somewhere in the range of **0** to **1**.

An outcome or an event that cannot happen (is impossible) has a probability of 0. For example, the probability that donkeys will fly is 0.

An outcome or an event that is certain to happen has a probability of 1. For example, the probability that the sun will rise tomorrow is 1.

Example 1

Put arrows on the probability scale to show the probability of each of the outcomes of these events.

a You will get a head when throwing a coin.

b You will get a six when throwing a dice.

c You will have maths homework this week.

a This outcome is an even chance. (Commonly described as a fifty-fifty chance.)

b This outcome is fairly unlikely.

c This outcome is likely.

The arrows show the approximate probabilities on the probability scale.

EXERCISE 33A

1 State whether each of these events is impossible, very unlikely, unlikely, even chance, likely, very likely or certain.

- **a** Someone in your class is left-handed.
- **b** You will live to be 100.
- **c** You get a score of seven when you throw a dice.
- **d** You will watch some TV this evening.
- **e** A new-born baby will be a girl.

2 Draw a probability scale and put an arrow to show the approximate probability of each of these events happening.

- **a** The next car you see will have been made in Japan.
- **b** A person in your class will have been born in the 20th century.
- **c** It will rain tomorrow.
- **d** In the next Olympic Games, someone will run the 1500 m race in 3 minutes.
- **e** During this week, you will have noodles with a meal.

3 **a** Draw a probability scale and mark an arrow to show the approximate probability of each of these events.

- A The next person to come into the room will be male.
- B The person sitting next to you in mathematics is over 16 years old.
- C Someone in the class will have a mobile phone.

b What number on your scale corresponds to each arrow?

4 **a** Give two events of your own for which you think the probability of an outcome is:

- A impossible
- B very unlikely
- C evens
- D likely
- E certain.

b Draw a probability scale numbered from 0 to 1 and put an arrow for each of your events.

c What number on your scale corresponds to each arrow?

5 'The train was late yesterday so it is very likely that it will be late today.'

Is this true?

33.2 Calculating probabilities

In Exercise 33A, you may have had difficulty in knowing exactly where to put some of the arrows on the probability scale. It would have been easier for you if each result of the event could have been given a value, from 0 to 1, to represent the probability for that result.

For some events, this can be done by first finding all the possible results, or **outcomes**, for a particular event. For example, when you throw a coin there are two **equally likely** outcomes: it lands heads up or tails up. (The 'head' of a coin is the side which usually shows a head, the 'tail' is the side which shows the value of the coin.)

If you want to calculate the probability of getting a head, there is only one outcome that is possible. So, you can say that there is a 1 in 2, or 1 out of 2, chance of getting a head. This is usually given as a **probability fraction**, namely $\frac{1}{2}$. So, you would write the event as:

$$\text{P(head)} = \frac{1}{2}$$

Probabilities can also be written as decimals or percentages, so that:

$$\text{P(head)} = \frac{1}{2} \text{ or } 0.5 \text{ or } 50\%$$

The probability of an outcome is defined as:

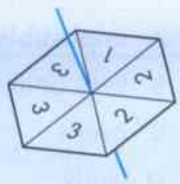
$$\text{P(outcome)} = \frac{\text{number of ways the outcome can happen}}{\text{total number of possible outcomes}}$$

This definition always leads to a fraction, which should be cancelled to its simplest form.

Another probability term you will meet is at **random**. This means that the outcome cannot be predicted or affected by anyone.

Example 2

The spinner shown here is spun and the score on the side on which it lands is recorded.



What is the probability that the score is:

a 2

b odd

c less than 5?

a There are two 2s out of six sides, so $\text{P}(2) = \frac{2}{6} = \frac{1}{3}$.

b There are four odd numbers, so $\text{P(odd)} = \frac{4}{6} = \frac{2}{3}$.

c All of the numbers are less than 5, so this is a certain event.

P(less than 5) = 1

Example 3

Bernice is always early, just on time or late for work.

The probability that she is early is 0.1, the probability she is just on time is 0.5.

What is the probability that she is late?

As all the possibilities are covered – that is 'early', 'on time' and 'late' – the total probability is 1. So:

P(early) + P(on time) = 0.1 + 0.5 = 0.6

So, the probability of Bernice being late is 1 – 0.6 = 0.4.

EXERCISE 33B

1. There are ten balls in a bag. One is red, two are blue, three are yellow and four are green. A ball is taken out without looking.

   What is the probability that it is:

   a red
   b green
   c green or yellow
   d red or green
   e white?

Advice and Tips

If an event is impossible, just write the probability as 0, not as a fraction such as $\frac{0}{6}$. If it is certain, write the probability as 1, not as a fraction such as $\frac{6}{6}$.

2. An 8-sided spinner has the numbers 1, 2, 3, 4, 5, 6, 7 and 8 on it. It is spun once.

   What is the probability that the score is:

   a 3
   b more than 3
   c an even number?

Advice and Tips

Remember to cancel the fractions if possible.

3. A bag contains only blue balls. If I take one out at random, what is the probability of each of these outcomes?

   a I get a black ball.
   b I get a blue ball.

4. Ten number cards with the numbers 1 to 10 inclusive are placed in a hat. Amir takes a number card out of the bag without looking. What is the probability that he draws:

   a the number 7
   b an even number
   c a number greater than 6
   d a number less than 3
   e a number between 3 and 8?

CORE

CORE

5 A pencil case contains six red pens and five blue pens. Paulo takes out a pen without looking at it. What is the probability that he takes out:

a a red pen

b a blue pen

c a pen that is not blue?

6 A bag contains 50 balls. 10 are green, 15 are red and the rest are white. Galenia takes a ball from the bag at random. What is the probability that she takes:

a a green ball

b a white ball

c a ball that is not white

d a ball that is green or white?

7 There are 500 students in a school and 20 students in Ali's class. One person is chosen at random to welcome a special visitor.

What is the probability the person is in Ali's class?

8 Anton, Bianca, Charlie, Debbie and Elisabeth are in the same class. Their teacher wants two students to do a special job.

a Write down all the possible combinations of two people, for example, Anton and Bianca, Anton and Charlie. (There are 10 combinations altogether.)

b How many pairs give two boys?

c What is the probability of choosing two boys?

d How many pairs give a boy and a girl?

e What is the probability of choosing a boy and a girl?

f What is the probability of choosing two girls?

Advice and Tips

Try to be systematic when writing out all the pairs.

9 A bag contains 25 coloured balls. 12 are red, 7 are blue and the rest are green. Ravi takes a ball at random from the bag.

a Find:

i P(he takes a red)

ii P(he takes a blue)

iii P(he takes a green).

b Add together the three probabilities. What do you notice?

c Explain your answer to part **b**.

10 The weather tomorrow will be sunny, cloudy or raining.

If P(sunny) = 40%, P(cloudy) = 25%, what is P(raining)?

11 At morning break, Priya has a choice of coffee, tea or hot chocolate.

If P(she chooses coffee) = 0.3 and P(she chooses hot chocolate) = 0.2, what is P(she chooses tea)?

12 The following information is known about the classes at Bradway School.

Year	Y1		Y2		Y3		Y4		Y5		Y6	
Class	P	Q	R	S	T	U	W	X	Y	Z	K	L
Girls	7	8	8	10	10	10	9	11	8	12	14	15
Boys	9	10	9	10	12	13	11	12	10	8	16	17

A class representative is chosen at random from each class.

Which class has the best chance of choosing a boy as the representative?

13 The teacher chooses, at random, a student to ring the school bell.

Tom says: 'It's even chances that the teacher chooses a boy or a girl.'

Explain why Tom might not be correct.

33.3 Probability that an event will not happen

In some questions in Exercise 33B, you were asked for the probability of something not happening. For example, in question 5 you were asked for the probability of picking a pen that is not blue. You could answer this because you knew how many pens there were in the case. However, sometimes you do not have this type of information.

The probability of throwing a six on a fair, six-sided dice is $P(6) = \frac{1}{6}$.

There are five **outcomes** that are not sixes: 1, 2, 3, 4, 5.

So, the probability of *not* throwing a six on a dice is:

$$P(\text{not a } 6) = \frac{5}{6}$$

Notice that:

$$P(6) = \frac{1}{6} \text{ and } P(\text{not a } 6) = \frac{5}{6}$$

So,

$$P(6) + P(\text{not a } 6) = 1$$

If you know that $P(6) = \frac{1}{6}$, then P(not a 6) is:

$$1 - \frac{1}{6} = \frac{5}{6}$$

So, if you know P(outcome happening), then:

P(outcome not happening) = 1 – P(outcome happening)

Example 4

A box of coloured pencils has 20 different pencils. There are four red pencils, five blue, one green, three yellow, two brown, one black, and four other colours.

A pencil is chosen at random. What is the probability that it is not red?

There are 4 red pencils out of 20.

The probability that a red is chosen is $= \frac{4}{20} = \frac{1}{5}$.

The probability that a red is **not** chosen is $1 - \frac{1}{5} = \frac{4}{5}$.

EXERCISE 33C

CORE

1 a The probability that a football team will win their next match is $\frac{1}{4}$. What is the probability that the team will not win?

b The probability that snow will fall during the winter holidays is 0.45. What is the probability that it will not snow?

c The probability that Paddy wins a game of chess is 0.7 and the probability that he draws the game is 0.1. What is the probability that he loses the game?

2 Look at Example 4.

What is the probability that the pencil is:

a not blue

b not yellow

c not black?

3 These letter cards are put into a bag.

M	A	T	H	E	M
A	T	I	C	A	L

a Lee takes a letter card at random.

i What is the probability he takes a letter A?

ii What is the probability he does not take a letter A?

b Ziad picks an M and keeps it. Tasnim now takes a letter from those remaining.

i What is the probability she takes a letter A?

ii What is the probability she does not take a letter A?

4 Hamzah is told: 'The chance of your winning this game is 0.3.'

Hamzah says: 'So I have a chance of 0.7 of losing.'

Explain why Hamzah might be wrong.

33.4 Probability in practice

Suppose you toss a coin many times and count how many times it lands with the 'heads' side showing.

The value of 'number of heads ÷ number of tosses' is called an **experimental probability.** As the number of **trials**, or experiments, increases, the value of the experimental probability gets closer to the theoretical probability, which in this case is $\frac{1}{2}$.

Experimental probability is also known as the **relative frequency** of an event. The relative frequency of an event is an estimate for the theoretical probability. It is given by:

$$\text{relative frequency of an outcome or event} = \frac{\text{frequency of the outcome or event}}{\text{total number of trials}}$$

Example 5

The frequency table shows the speeds of 160 vehicles that pass a radar speed check on a fast road.

Speed (km/h)	20–29	30–39	40–49	50–59	60–69	70+
Frequency	14	23	28	35	52	8

a What is the experimental probability that a car is travelling faster than 70 km/h?

b If 500 vehicles pass the speed check, estimate how many will be travelling faster than 70 km/h.

a The experimental probability is the relative frequency, which is $\frac{8}{160} = \frac{1}{20}$.

b The number of vehicles travelling faster than 70 km/h will be $\frac{1}{20}$ of 500.

That is:

500 ÷ 20 = 25

EXERCISE 33D

1 Naseer throws a fair, six-sided dice and records the number of sixes that he gets after various numbers of throws. The table shows his results.

Number of throws	10	50	100	200	500	1000	2000
Number of sixes	2	4	10	21	74	163	329

a Calculate the experimental probability of scoring a 6 at each stage that Naseer recorded his results.

b How many ways can a dice land?

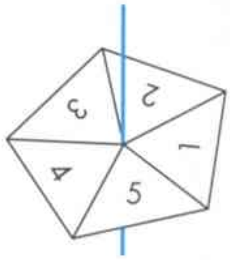
c How many of these ways give a 6?

d What is the theoretical probability of throwing a 6 with a dice?

e If Naseer threw the dice a total of 6000 times, how many sixes would you expect him to get?

CORE

2 Marie made a five-sided spinner, like the one shown in the diagram. She used it to play a board game with her friend Sarah.



The girls thought that the spinner was not very fair as it seemed to land on some numbers more than others. They threw the spinner 200 times and recorded the results. The results are shown in the table.

Side spinner lands on	1	2	3	4	5
Number of times	19	27	32	53	69

a Work out the experimental probability of each number.

b How many times would you expect each number to occur if the spinner is fair?

c Do you think that the spinner is fair? Give a reason for your answer.

3 A bottle contains 20 balls. The balls are either black or white.

Kenny conducts an experiment to see how many black balls there are in the bottle. He tips one ball at a time into a clear sealed tube at the end of the bottle. He records the number of black balls and tips them back into the bottle.

The results are shown in the table.

Number of samples	Number of black balls	Experimental probability
10	2	
100	25	
200	76	
500	210	
1000	385	
5000	1987	

a Copy the table and complete it by calculating the experimental probability of getting a black ball at each stage.

b Using this information, how many black balls do you think there are in the bottle?

4 Use a set of number cards from 1 to 10 and work with a partner. Take turns to choose a card and keep a record each time of what card you get, before returning it to the pack. Shuffle the cards each time and repeat the experiment 60 times. Put your results in a copy of this table.

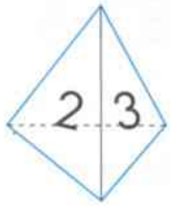
Score	1	2	3	4	5	6	7	8	9	10
Total										

a How many times would you expect to get each number?

b Do you think you and your partner conducted this experiment fairly?

c Explain your answer to part **b**.

5 A four-sided dice has faces numbered 1, 2, 3 and 4. The score is the face on which it lands. Five students throw the dice to see if it is biased. They each throw it a different number of times. Their results are shown in the table.



Student	Total number of throws	Score			
		1	2	3	4
Ali	20	7	6	3	4
Balvir	50	19	16	8	7
Caryl	250	102	76	42	30
Deema	80	25	25	12	18
Emma	150	61	46	26	17

a Which student will have the best set of results for finding the probability of each score? Why?

b Add up all the score columns and work out the relative frequency of each score. Give your answers to 2 decimal places.

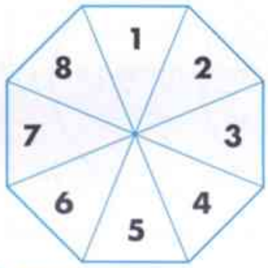
c Is the dice biased? Explain your answer.

6 Andrew made an eight-sided spinner.

He tested it to see if it was fair.

He spun the spinner and recorded the results.

Unfortunately his little sister spilt something over his results table, so he could not see the middle part.



Number spinner lands on	1	2	3			6	7	8
Frequency	18	19	22			19	20	22

Assuming the spinner was a fair one, try to complete the missing parts of the table for Andrew.

7 At a computer factory, tests were carried out to see how many faulty computer chips were produced in one week.

	Monday	Tuesday	Wednesday	Thursday	Friday
Sample	850	630	1055	896	450
Number faulty	10	7	12	11	4

On which day was it most likely that the highest number of faulty computer chips were produced?

8 Steve tossed a coin 1000 times to see how many heads he got.

He said: 'If this is a fair coin, I should get 500 heads.'

Explain why he is wrong.

Chapter 33 . Topic 5

33.5 Using Venn diagrams

Sometimes you want to find the probability of two events happening at the same time.

Venn diagrams are one way of answering these questions.

Example 6

There is a set of 15 cards. Each card has an integer from 1 to 15.

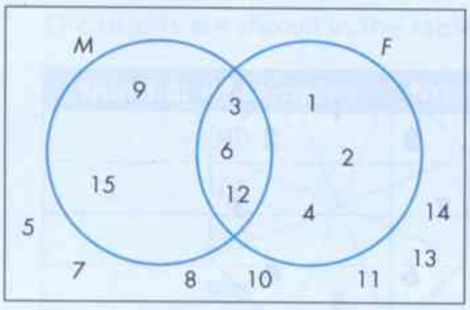
You take a card at random. Find the probability that it is

a A multiple of 3 and a factor of 12

b A multiple of 3 and not a factor of 12

c Neither a multiple of 3 nor a factor of 12.

Show the sets M = {multiples of 3} and F = {factors of 12} in a Venn diagram.



a The numbers 3, 6 and 12 are in both M and F so $n(M \cap F) = 3$. The probability is $\frac{3}{15} = \frac{1}{5}$.

b The numbers 9 and 15 are in M but not in F. The probability is $\frac{2}{15}$.

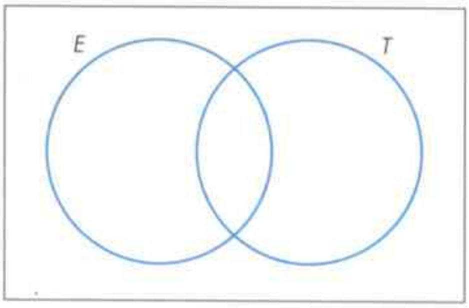
c There are 7 numbers outside both sets. The probability is $\frac{7}{15}$.

EXERCISE 33E

CORE

1 ξ = [integers from 1 to 21} E = {even numbers} T = {multiples of 3}

a Show these sets on this Venn diagram.



b A set of cards have the integers from 1 to 21.

A card is chosen at random.

Find the probability that it is

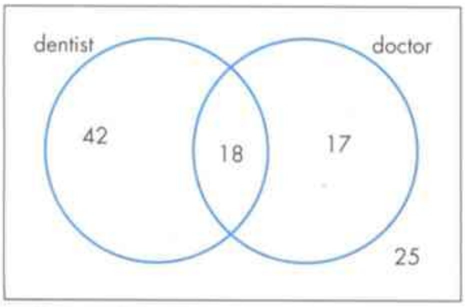
i an even number

ii a multiple of 3

iii both an even number and a multiple of 3.

2 There are 100 people in a survey.

This Venn diagram shows how many went last year to the doctor or the dentist or both.



For example, 42 people went to the dentist but not the doctor.

One person is chosen at random. Find the probability that the person went to

a the dentist

b the doctor

c both the dentist and the doctor

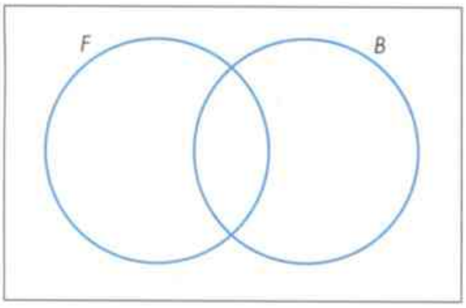
d neither.

3 60 people are asked if they play football (F) or basketball (B)

Here are the results

	Football and basketball	Only football	Only basketball	Neither
Number of people	12	30	8	10

a Show the numbers in this Venn diagram.



b A person is chosen at random.

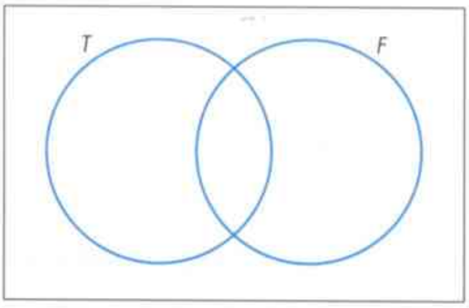
Find the probability the person plays

i football and basketball **ii** football **iii** basketball

CORE

4 ξ = {integers from 1 to 100} T = {multiples of 10} F = {multiples of 15}

a Put the elements of T and F in this Venn diagram



b A number between 1 and 100 is chosen at random.

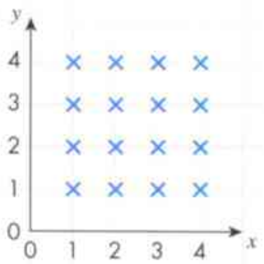
Find the probability that the number is

i a multiple of 15

ii a multiple of both 10 and 15

iii not a multiple of 10 or 15.

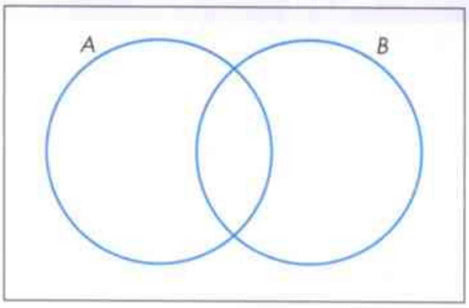
5



ξ = {points marked with a cross}

A = {points on the line $y = x$} B = {point on the line $y = 4 - x$}

a Put the coordinates of the points in this Venn diagram.



b One of the points is chosen at random.

Find the probability that it is

i on line A

ii on line B

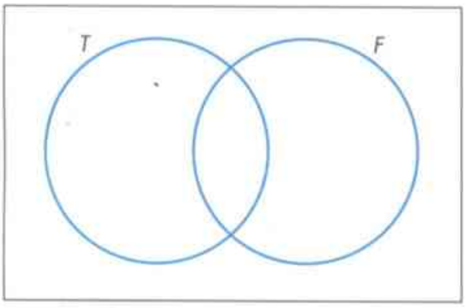
iii on both lines

6 ξ = {integers from 1 to 100}

T = {multiples of 10} F = {multiples of 4}

a How many elements are in set T?

b On this Venn diagram show the number of elements in each region.



c A computer generates an integer between 1 and 100 at random.

Find the probability that it is

i a multiple of 10

ii a multiple of 4

iii a multiple of both 10 and 4

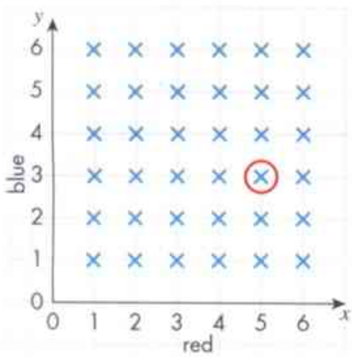
iv not a multiple of 10 or 4

33.6 Possibility diagrams

Venn diagrams are not the only way to show two events happening at the same time.

Suppose that you throw two dice. One is red and one is blue.

You can show all the possible outcomes in a **possibility diagram**.



There are 36 different outcomes.

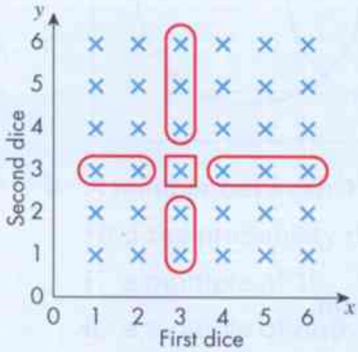
The cross with a circle round it is 5 on the red dice and 3 0n the blue dice.

Example 7

Two dice are thrown. Find the probability of getting

a two 3s **b** exactly one 3 **c** at least one 3 **d** no 3s

Here is a possibility diagram.



a There are 36 outcomes. There is only one way to get two 3s. It is shown with a square box.

The probability is $\frac{1}{36}$

b There are 10 ways to get one 3. They are in the four loops.

The probability is $\frac{10}{36} = \frac{5}{18}$

c At least one 3 means getting one or two. There are 10 + 1 = 11 ways.

The probability = $\frac{11}{36}$

d No 3s means any of the 25 crosses that are not in a loop.

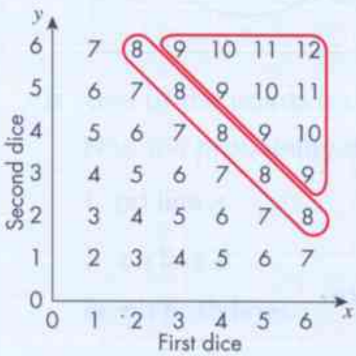
Probability = $\frac{25}{36}$

Example 8

Two dice are thrown and the numbers are added. Find the probability of a total of

a 8 **b** 8 or more **c** less than 8

Here is a possibility diagram. The totals have been written for each outcome.



a The diagonal loop shows 5 ways to get a total of 9.

Probability = $\frac{5}{36}$

b The triangular loop shows 10 ways to get a total more than 8.

To get a total of 8 or more there are 5 + 10 = 15 ways.

Probability = $\frac{15}{36} = \frac{5}{12}$

c The crosses that are not in a loop are the ways to get a total of less than 8.

There are 21 of these.

Probability = $\frac{21}{36} = \frac{7}{12}$

EXERCISE 33F

1 Two dice are thrown. Use a possibility diagram like the one in Example 7 to answer these questions.

- **a** What is the most likely score?
- **b** Which two scores are least likely?
- **c** Write down the probabilities of throwing all the scores from 2 to 12.
- **d** What is the probability of a score that is:
  - **i** bigger than 10
  - **ii** between 3 and 7
  - **iii** even
  - **iv** a square number
  - **v** a prime number
  - **vi** a triangular number?

2 Two dice are thrown. Use a possibility diagram like the one in Example 8 to answer these questions.

- **a** the score is an even 'double'
- **b** at least one of the dice shows 2
- **c** the score on one dice is twice the score on the other dice
- **d** at least one of the dice shows a multiple of 3?

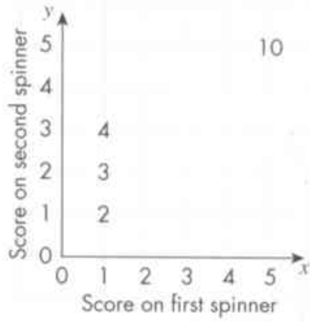
3 Two dice are thrown. Use a possibility diagram to find these probabilities.

- **a** both dice show a 6
- **b** at least one of the dice will show a 6
- **c** exactly one dice shows a 6

CORE

CORE

4 The diagram shows the scores for the event 'the difference between the scores when two fair, six-sided dice are thrown'. Copy and complete the diagram.



For the event described above, what is the probability of a difference of:

a 1

b 0

c 4

d 6

e an odd number?

5 When two fair coins are thrown together, what is the probability of scoring:

a two heads

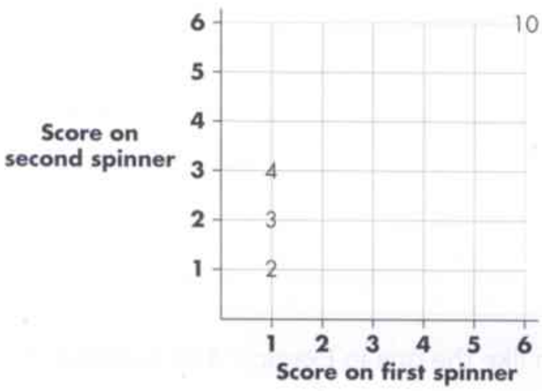
b a head and a tail

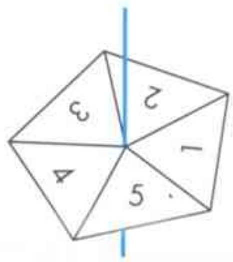
c at least one tail

d no tails?

Use a diagram of the outcomes when two coins are thrown together.

6 Two five-sided spinners are spun together and the total score of the faces that they land on is worked out. Copy and complete the possibility diagram shown.





a What is the most likely score?

b When two five-sided spinners are spun together, what is the probability that:

i the total score is 5

ii the total score is an even number

iii the score is a 'double'

iv the total score is less than 7?

7 Two eight-sided spinners showing the numbers 1 to 8 were thrown at the same time.

a Draw a possibility diagram to show the product of the two scores.

b What is the probability that the product of the two spinners is an even square number?

8 Isaac throws two dice. He multiplies the numbers together. Find the probability that the product is between 19 and 35.

33.7 Tree diagrams

Here is the question in Example 7 again.

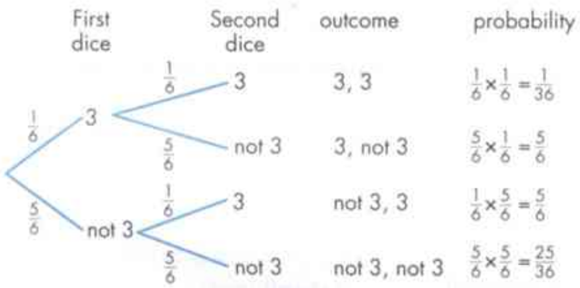
Two dice are thrown.

Find the probability of getting

a two 3s **b** exactly one 3 **c** at least one 3 **d** no 3s

You answered this question with a possibility diagram.

You can also use a **tree diagram**.



The probability that a dice shows 3 is $\frac{1}{6}$.

The probability that a dice does not show 3 (a 'not 3') is $\frac{5}{6}$.

Write these probabilities on each branch.

The branches show four outcomes, with each dice either '3' or 'not 3'.

To find the probability of each outcome, *multiply* the probabilities on the branches.

a The probability of 3, 3 is $\frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$. This is the answer to part **a**.

b The probability of 3, not 3 is $\frac{1}{6} \times \frac{5}{6} = \frac{5}{36}$. This is 3 on the first but not the second.

The probability of not 3, 3 is $\frac{5}{6} \times \frac{1}{6} = \frac{5}{36}$. This is 3 on the second but not the first.

Add these two: $\frac{5}{36} + \frac{5}{36} = \frac{10}{36} = \frac{5}{18}$. This is the answer to part **b**.

c For at least one 3, add the three probabilities:

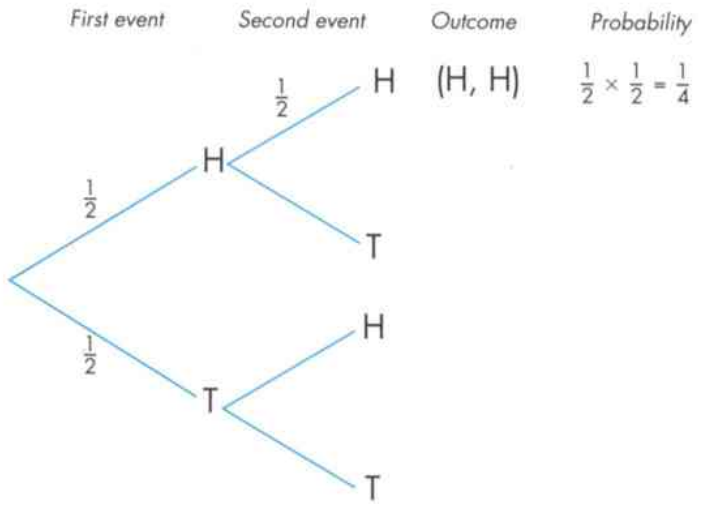
$\frac{1}{36} + \frac{5}{36} + \frac{5}{36} = \frac{11}{36}$ This is the answer to part **c**.

d The probability of not 3, not 3 is $\frac{5}{6} \times \frac{5}{6} = \frac{25}{36}$. This is the answer to part **d**.

EXERCISE 33G

CORE

1 A coin is tossed twice. Copy and complete this tree diagram to show all the possible outcomes.



Use your tree diagram to work out the probability of getting:

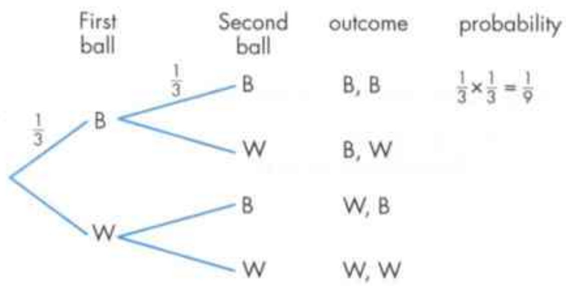
a two heads **b** a head and a tail **c** at least one tail.

2 There are two black balls and one white one in a bag.

Luis takes one at random.

He replaces it and then takes another one at random.

a Copy and complete this tree diagram.



$\frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$

b Find the probability that he takes

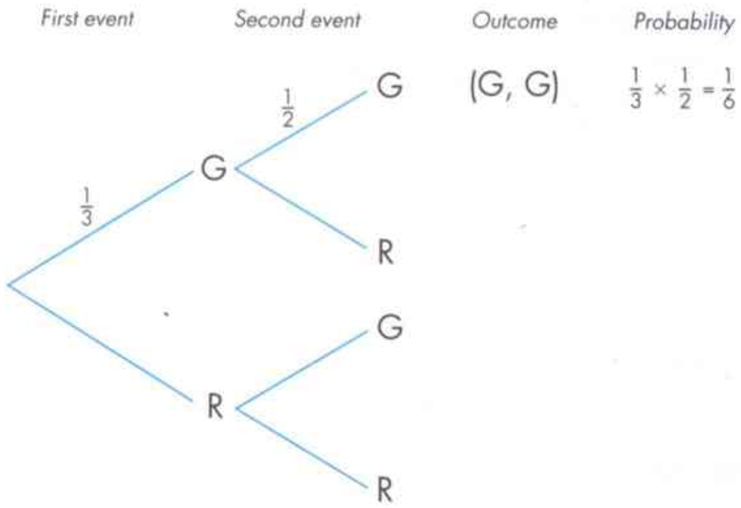
i two white balls ii one ball of each colour iii at least one black ball

3 On my way to work, I drive through two sets of road works with traffic lights that only show green or red. I know that the probability of the first set being green is $\frac{1}{3}$ and the probability of the second set being green is $\frac{1}{2}$.

a What is the probability that the first set of lights will be red?

b What is the probability that the second set of lights will be red?

c Copy and complete this tree diagram to show the possible outcomes when passing through both sets of lights.



d Using the tree diagram, what is the probability that:

i I do not get held up at either set of lights

ii I get held up at exactly one set of lights

iii I get held up at least once.

e Over a school term I make 90 journeys to work. On how many days can I expect to get two green lights?

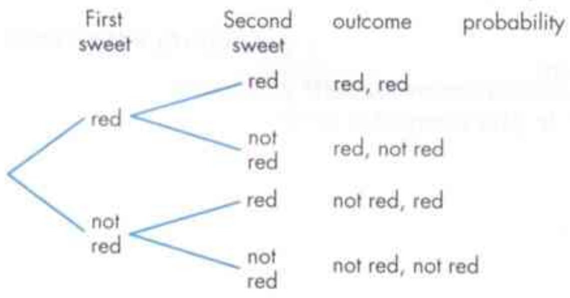
4 There are two bags of sweets.

The first bag has 5 sweets and 4 of them are red.

The second bag has 5 sweets and 2 of them are red.

Delia takes one sweet from each bag.

a Copy and complete this tree diagram.



b Find the probability that she takes

i two red sweets

ii at least one red sweet

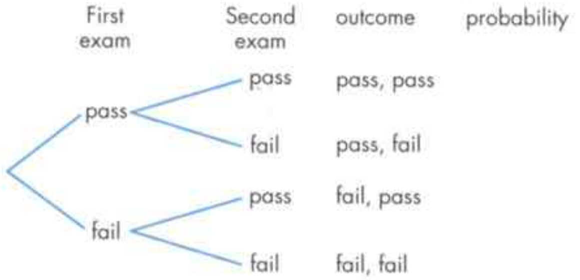
iii no red sweets

5 An English exam has two parts, an oral test and a written test.

The probability that Chen passes the oral test is 0.9

The probability that Chen passes the written test is 0.6

a Copy and complete this tree diagram



b Find the probability that Chen

i passes both parts

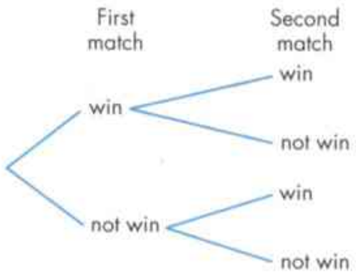
ii passes only one of the two.

6 A football team is playing two matches.

The probability that the team will win the first match is 0.6

The probability that the team will win the second match is 0.7

a Copy and complete this tree diagram to show probabilities.



b Find the probability that the team will win at least one of the two matches.

7 Ahmed is playing a computer game.

The probability that he wins each time is 0.3.

He plays 2 games. Find the probability that:

a Ahmed wins both games

b the computer wins both games

c Ahmed and the computer both win one game.

8 The probability of rain on Monday is 0.25.

The probability of rain on Tuesday is 0.6.

Find the probability of:

a rain on Monday but not on Tuesday

b no rain on either day

c rain on at least one day.

9 Abebe is running in an 800 m race and a 1500 m race.

The probability that he wins the 800 m is 0.65

The probability that he wins the 1500 m is 0.4

Find the probability that he wins one race and loses the other.

CORE

10 A fruit bowl contains six oranges and eight lemons. Kevin takes two pieces of fruit at random.

a If the first piece is an orange, what is the probability that the second is:

i an orange

ii a lemon?

b What is the probability that:

i both are oranges

ii both are lemons?

11 A bag contains three black balls and seven red balls. A ball is taken out and not replaced. This is repeated twice. What is the probability that:

a all three are black

b exactly two are black

c exactly one is black

d none are black?

12 On my way to work, I pass two sets of traffic lights. The probability that the first is green is $\frac{1}{3}$. If the first is green, the probability that the second is green is $\frac{1}{3}$. If the first is red, the probability that the second is green is $\frac{2}{3}$. What is the probability that:

a both are green

b none are green

c exactly one is green

d at least one is green?

13 An engineering test is in two parts, a written test and a practical test. 90% of those who take the written test pass. When a person passes the written test, the probability that he or she will also pass the practical test is 60%. When a person fails the written test, the probability that he or she will pass the practical test is 20%.

a What is the probability that someone passes both tests?

b What is the probability that someone passes one test?

c What is the probability that someone fails both tests?

d What is the combined probability of the answers to parts **a**, **b** and **c**?

33.8 Conditional probability

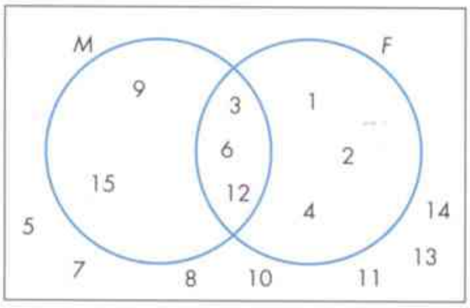
E

Here is the Venn diagram from Example 6.

There is a set of 15 cards. Each card has an integer from 1 to 15.

You take a card at random.

M = {multiples of 3} and F = {factors of 12}



There are 5 numbers in set M.

The probability of picking a multiple of 3 is $\frac{5}{15}$.

There are 6 numbers in set F.

The probability of picking a factor of 12 is $\frac{6}{15} = \frac{2}{5}$.

Suppose you know the card is a factor of 12.

Now the probability that is a multiple of three is different.

You want the probability that it is in M, given that it is in F.

This is $\frac{3}{6} = \frac{1}{2}$.

This is called a **conditional probability**.

Suppose you know the card is a multiple of 3.

What is the conditional probability that the number is a factor of 12 given that it is a multiple of 3?

You want the probability that it is in F, given that it is in M.

This is $\frac{3}{5}$.

You sometimes need conditional probabilities on a tree diagram.

Example 9

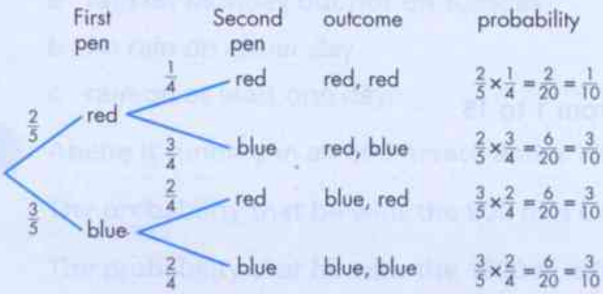
There are 3 red pens and 2 blue pens in a box.

Two pens are taken out at random.

Find the probability that they are

a both red **b** both blue **c** one of each colour.

Here is the tree diagram.



The probabilities on the branches are not the same each time.

When the first pen is taken, the probabilities are red = $\frac{2}{5}$ and blue = $\frac{3}{5}$.

These are on the first pair of branches.

If the first pen is red, there are 1 red and 3 blue pens left.

The conditional probabilities are red = $\frac{1}{4}$ and blue = $\frac{3}{4}$.

These go on the top pair of branches for the second pen.

If the first pen is blue, there are 2 red and 2 blue pens left.

The conditional probabilities are red = $\frac{2}{4}\left(\text{or } \frac{1}{2}\right)$ and blue = $\frac{2}{4}\left(\text{or } \frac{1}{2}\right)$.

These go on the bottom pair of branches for the second pen.

a The probability they are both red is $\frac{2}{5} \times \frac{1}{4} = \frac{2}{20} = \frac{1}{10}$.

b The probability they are both blue is $\frac{3}{5} \times \frac{2}{4} = \frac{6}{20} = \frac{3}{10}$.

c One of each colour is either the second or the third outcome.

Add the two probabilities.

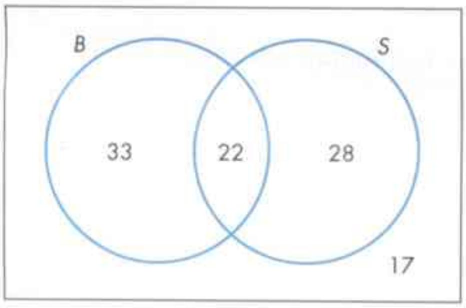
$$\left(\frac{2}{5} \times \frac{3}{4}\right) + \left(\frac{3}{5} \times \frac{2}{4}\right) = \frac{6}{20} + \frac{6}{20} = \frac{12}{20} = \frac{3}{5}$$

EXERCISE 33H

EXTENDED

1 100 people are asked if they have a brother or a sister.

The numbers are in this Venn diagram.



B = {people with a brother} S = {people with a sister}

One person is chosen at random.

a Find the probability that the person has a brother.

b The person has a brother. Find the probability that the person has a sister.

c Another person is chosen at random. This person has a sister. Find the probability that this person has a brother.

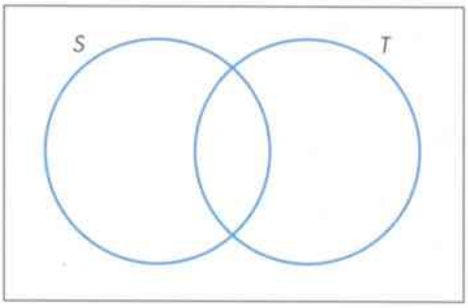
EXTENDED

2 ξ = {integers from 1 to 25}

S = {factors of 16}

T = {factors of 24}

a Put the elements of ξ in this Venn diagram.



An integer between 1 and 25 is chosen at random.

b Find the probability that it is a factor of both 16 and 24.

c If the number is a factor if 24, find the probability that it is a factor of 16.

d If the number is a factor if 16, find the probability that it is a factor of 24.

e If the number is a not a factor if 16, find the probability that it is a not a factor of 24.

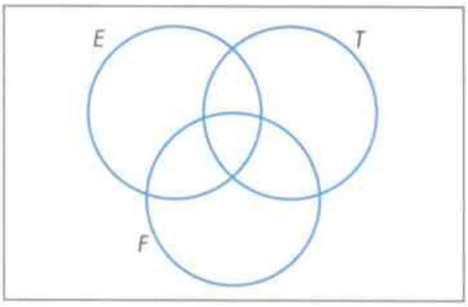
3 ξ = {integers from 1 to 30}

E = {even numbers}

T = {multiples of 3}

F = {multiples of 5}

a Put the elements of ξ in this Venn diagram.



An integer between 1 and 30 is chosen at random.

b Find the probability that it is a multiple of 3.

c Find the probability that it is a multiple of 3 and 5.

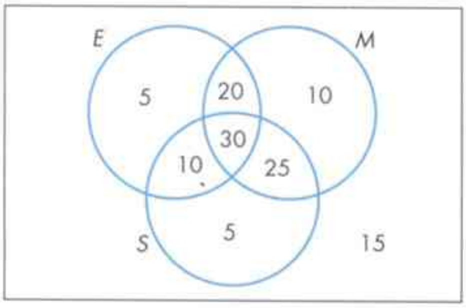
d If the number is a multiple of 5, find the probability that it is also a multiple of 3.

e If the number is a multiple of 5, find the probability that it is an even multiple of 3.

f If the number is a multiple of 3, find the probability that it is also a multiple of 5.

4 120 students take exams in English (E), maths (M) and science (S).

This Venn diagram shows how many students pass each exam.



A student is chosen at random.

a Find the probability that the student passed maths.

b Find the probability that the student passed all three subjects.

A student is chosen at random. This student passed maths.

c Find the probability that the student passed science.

d Find the probability that the student passed English.

A student is chosen at random. This student failed maths.

e Find the probability that the student also failed science.

5 Two dice are thrown. The numbers are added together.

a Show the possible totals on a possibility diagram.

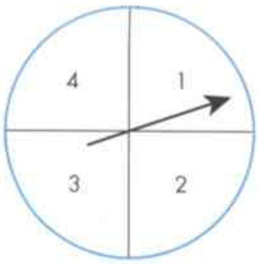
b Find the probability of a total of 6.

c If the total is 6, find the probability that one of the numbers is 2.

d Find the probability that at least one of the numbers is 2.

e If at least one of the numbers is 2, find the probability that the total is 6.

6



A spinner can show the numbers 1, 2, 3 and 4 with equal probabilities.

The spinner is spun twice and the numbers are added.

a Find the probability that the total is less than 5.

b If the total is 5, find the probability that the numbers on each spin are the same.

c If both numbers are the same, find the probability that the total is less than 5.

EXTENDED

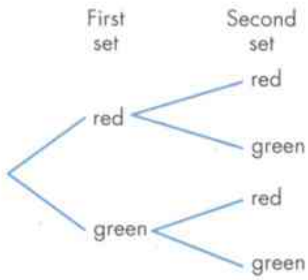
7 Driving to work, Karim passes two sets of traffic lights.

The probability that the first set is red is 0.3.

If the first set is red, the probability that the second set is red is 0.8.

If the first set is green, the probability that the second set is red is 0.4.

a Put probabilities on the branches of this tree diagram.



b Find the probability that both sets of lights are red.

c Find the probability that both sets of lights are green.

d Find the probability that one set is red and the other is green.

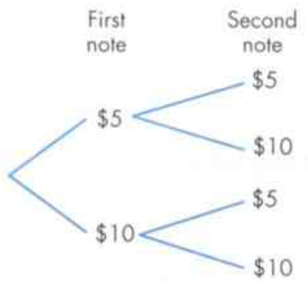
8 Laura has 4 \$5 notes and 6 \$10 notes in her purse.

She takes out two notes at random.

a Find the probability that the first note is \$5.

b If the first note is \$5, what is the probability that the second note is also \$5?

c Put probabilities on the branches of this tree diagram.



d Find the probability that the total value of the two notes is:

i \$10 ii \$15 iii \$20

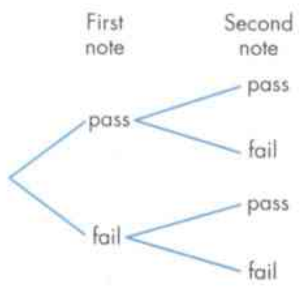
9 An engineering exam has two parts, a written test and a practical test.

The probability that a student passes the written test is 0.9.

If a student passes the written test, the probability of passing the practical test is 0.8.

If a student fails the written test, the probability of failing the practical test is 0.4.

a Put probabilities on the branches of this tree diagram.



b Find the probability that a student passes both parts.

c Find the probability that a student passes one part and fails the other.

10 A box has 3 red counters and 5 white counters.

Sara takes out counters at random, one at a time.

a Find the probability that the first two counters are red.

b Find the probability that the first two counters are white.

c Sara takes out 3 counters all together. Find the probability that all are the same colour.

11 Aaron and Barak play a game.

There are 5 white balls and one red ball in a bag.

Aaron takes out 3 balls, one at a time, without looking.

If one of the balls is red, Aaron wins the game. If they are all white Barak wins.

What is the probability that Aaron wins? Give a reason for your answer.

Check your progress

Core

- I can calculate the probability of a single event as a fraction, a decimal or a percentage
- I can understand and use the probability scale from 0 to 1
- I understand that the probability of an event not occurring = 1 – the probability of the event occurring
- I understand relative frequency as an estimate of probability
- I can calculate the probability of simple combined events using possibility diagrams, tree diagrams or Venn diagrams with two sets
- I can calculate the probability of simple combined events using Venn diagrams with three sets

Extended

- I can calculate conditional probability using tables, tree diagrams or Venn diagrams

Examination questions: Statistics and probability

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PAPER 1

CORE

1 Jim scores the following marks in 8 tests.

7 8 8 y 6 9 10 5

His mean mark is 7.5.

Calculate the value of y. [2]

Cambridge International IGCSE Mathematics 0580 *Paper 11 Q9 Oct/Nov 2015*

2 Chico has a bag of sweets.

He takes a sweet from the bag at random.

The table shows the probabilities of taking each flavour of sweet.

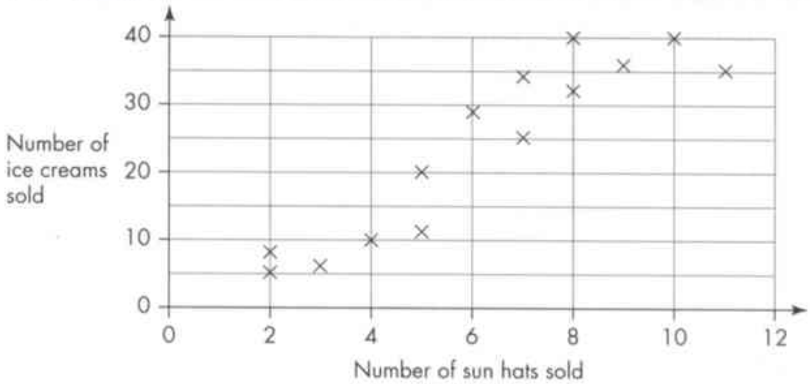
Flavour	Lemon	Lime	Strawberry	Blackcurrant	Orange
Probability	0.15	0.22		0.18	0.24

a Complete the table. [2]

b Find the probability that the sweet is lemon or lime. [1]

Cambridge International IGCSE Mathematics 0580 *Paper 11 Q15 Oct/Nov 2015*

3 The scatter diagram shows the number of sun hats and ice creams sold by a shop each day for two weeks.



a Write down the type of correlation shown by the diagram. [1]

b Describe the relationship between the number of sun hats sold and the number of ice creams sold. [1]

Cambridge International IGCSE Mathematics 0580 *Paper 11 Q9 May/June 2015*

4 These are the heights, correct to the nearest centimetre, of 12 children.

132 114 151 130 132 145 163 142 153 170 132 125

Find the median height. [2]

Cambridge International IGCSE Mathematics 0580 *Paper 11 Q5 Oct/Nov 2014*

5 Cheryl recorded the midday temperatures in Seoul for one week in January.

Day	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Temperature (°C)	−4	−5	−3	−11	−8	−3	−1

a Write down the mode. [1]

b On how many days was the temperature lower than the mode? [1]

Cambridge International IGCSE Mathematics 0580 *Paper 11 Q10 Oct/Nov 2014*

6 The table shows the average monthly temperature (°C) for Fairbanks, Alaska.

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Temperature (°C)	−23.4	−19.8	−11.7	−0.8	9.2	15.4	16.9	13.8	7.5	−5.8	−21.4	−21.8

a Find

i the difference between the highest and the lowest temperatures,

ii the median. [1]

b A month is chosen at random from the table. [2]

Find the probability that its average temperature is below zero. [1]

Cambridge International IGCSE Mathematics 0580 *Paper 11 Q19 May/June 2014*

7 S P A C E S

One of the 6 letters is taken at random.

a Write down the probability that the letter is S. [1]

b The letter is replaced and again a letter is taken at random.

This is repeated 600 times.

How many times would you expect the letter to be S? [1]

Cambridge International IGCSE Mathematics 0580 *Paper 11 Q14 Oct/Nov 2013*

8 The probability of Sachin's team winning any match is 0.45.

a Write down the probability of Sachin's team not winning any match. [1]

b In a season there are 40 matches.

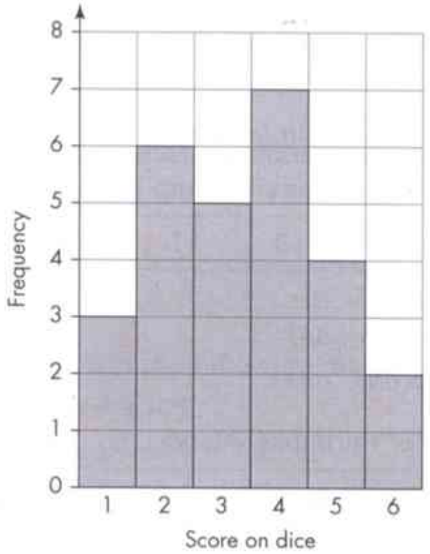
How many matches should Sachin's team expect to win in a season? [2]

Cambridge International IGCSE Mathematics 0580 *Paper 11 Q12 May/June 2013*

CORE

9 Marco throws a six-sided dice 27 times.

The bar chart shows his results.



a Write down the mode. [1]

b Work out the probability that Marco throws a number less than 5. [2]

c Calculate the mean. [3]

Cambridge International IGCSE Mathematics 0580 *Paper 11 Q20 May/June 2013*

10 8 15 7 8 7 15 4 13 4 3 10 2 9 4 5

a Write down the mode. [1]

b Work out the median. [2]

Cambridge International IGCSE Mathematics 0580 *Paper 11 Q16 Oct/Nov 2013*

PAPER 3

1 **a** 120 children take part in an athletics competition.

i Complete the table to show the number of children in each group.

	Girls	Boys	Total
Age 15			65
Age 16	44		
Total	70		120

[2]

ii One child is selected at random.
Find the probability that it is a girl aged 16.
Give your answer as a fraction in its lowest terms. [2]

iii Write down the ratio number of girls aged 15 : number of boys aged 15.
Give your answer in its simplest form. [2]

b Here are the distances, in metres, recorded in the boys' shot putt.
9.23 6.21 9.86 8.64 7.15 7.72 9.01 7.34 6.53 6.89

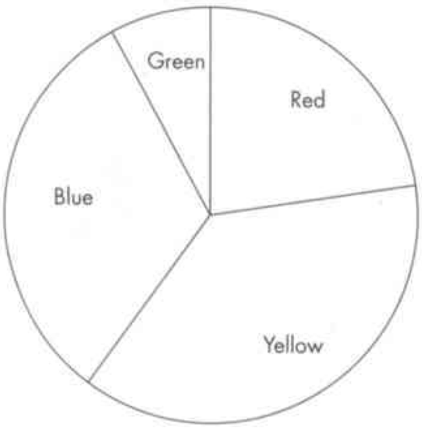
i Find the median. [2]

ii Find the range. [1]

iii Another boy was a late entry to the competition.
After his attempt, the range increased by 20 cm.
Work out the two possible distances of his attempt. [2]

Cambridge International IGCSE Mathematics 0580 *Paper 31 Q1 Oct/Nov 2015*

2 All the children in a school are asked to choose their favourite colour.
The pie chart shows the results.



a Write down the least favourite colour chosen. [1]

b 27 children choose yellow as their favourite colour.
Work out the total number of children in the school. [3]

c Work out the percentage of the children in the school who choose red. [2]

Cambridge International IGCSE Mathematics 0580 *Paper 31 Q5 May/June 2015*

CORE

3 12 athletes took part in the 100 metres race.

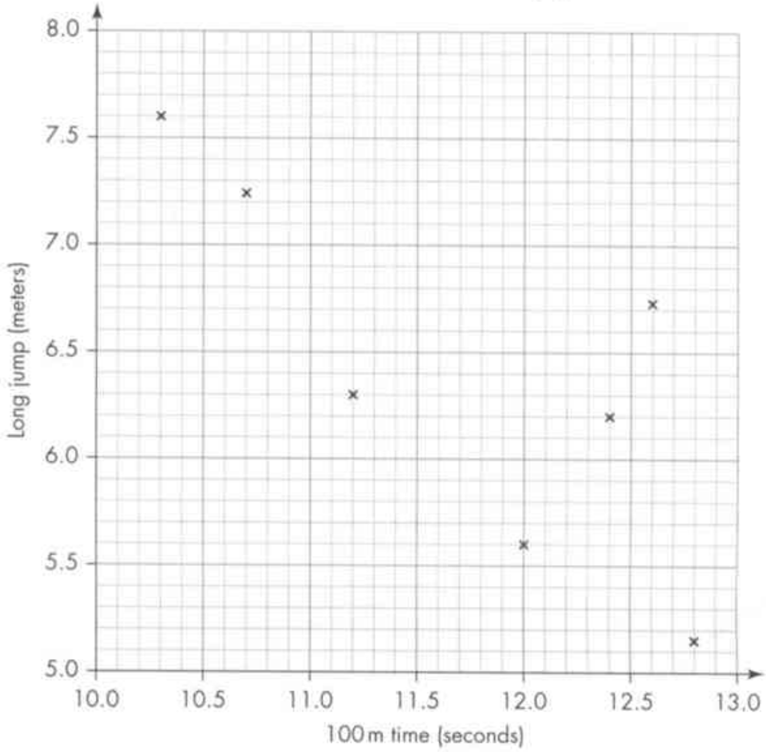
11 of these athletes also took part in the long jump.

The times and distances, each measured correct to 3 significant figures, for these athletes are shown in the table.

Athlete	A	B	C	D	E	F	G	H	I	J	K	L
100 m time (seconds)	12.1	10.3	12.8	10.7	12.6	11.2	12.0	12.4	10.6	12.7	11.8	11.1
Long jump (metres)	X	7.60	5.15	7.25	6.72	6.30	5.60	6.20	6.90	5.70	6.85	6.70

a The scatter diagram shows the times and distances for athletes B to H.

i Plot the times and distances for athletes I, J, K and L. [2]



ii On the scatter diagram, draw a line of best fit. [1]

iii Athlete A did not take part in the long jump.

Use your line of best fit to estimate a long jump distance for athlete A. [1]

iv What type of correlation is shown on the scatter diagram? [1]

v Describe in words the relationship between the time for 100 metres and the distance in the long jump. [1]

b Use the table of times and distances to work out

i the mean of the 100 metres times, [2]

ii the percentage of athletes who ran 100 metres in less than 11.5 seconds, [2]

iii the range of the distances jumped by the 11 athletes, B to L. [1]

Cambridge International IGCSE Mathematics 0580 *Paper 31 Q3 Oct/Nov 2014*

4 **a** Amir asked 15 friends how many hours they spent playing sport last weekend.
His results are shown in the table below.

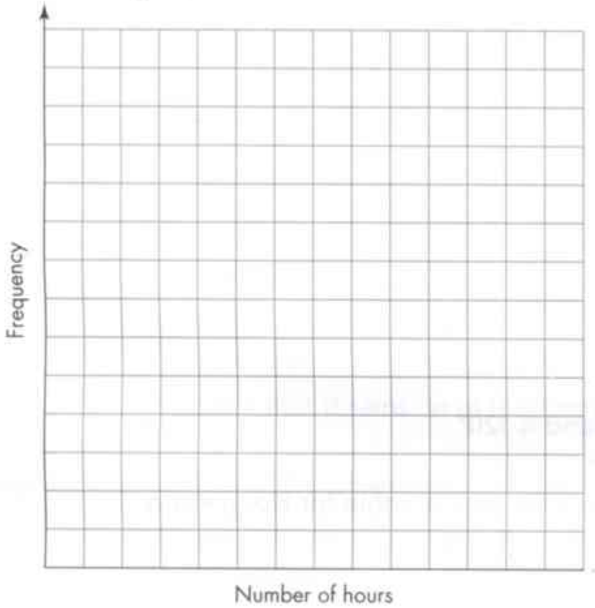
Number of hours	0	1	2	3	4	5
Frequency	6	2	3	1	2	1

i Write down the mode. [1]

ii Find the median. [1]

iii Calculate the mean. [3]

iv On the grid, draw a bar chart to show the information given in the table. [4]



b Amir also asked these 15 friends which was their favourite sport.
His results are shown in the table below.

Football	4
Cricket	5
Basketball	2
Badminton	4

Amir picks one of these friends at random.
Write down the probability that his friend's favourite sport is

i cricket, [1]

ii **not** football, [1]

iii basketball or badminton. [1]

Cambridge International IGCSE Mathematics 0580 *Paper 31 Q3 Oct/Nov 2012*

PAPER 2

EXTENDED

1 Paul and Sammy take part in a race.

The probability that Paul wins the race is $\frac{9}{35}$.

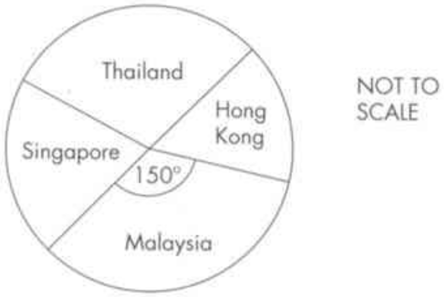
The probability that Sammy wins the race is 26%.

Who is more likely to win the race?

Give a reason for your answer. [2]

Cambridge International IGCSE Mathematics 0580 *Paper 21 Q5 May/June 2015*

2



A travel brochure has 72 holidays in four different countries.

The pie chart shows this information

a There are 24 holidays in Thailand.
Show that the sector angle for Thailand is 120°. [2]

b The sector angle for Malaysia is 150°.
The sector angle for Singapore is twice the sector angle for Hong Kong.
Calculate the number of holidays in Hong Kong. [3]

Cambridge International IGCSE Mathematics 0580 *Paper 21 Q17 May/June 2014*

3 Leon scores the following marks in 5 tests.

8 4 8 y 9

His mean mark is 7.2.

Calculate the value of y. [2]

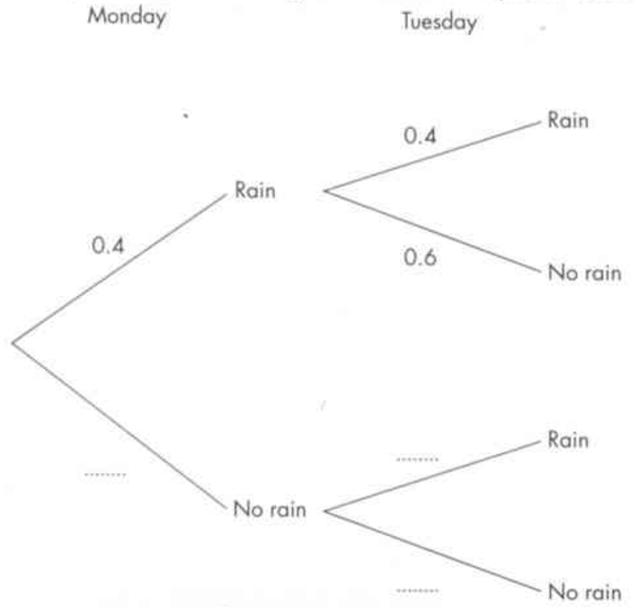
Cambridge International IGCSE Mathematics 0580 *Paper 21 Q6 May/June 2012*

4 If it rains today the probability that it will rain tomorrow is 0.4.

If it does not rain today the probability that it will rain tomorrow is 0.2.

On Sunday it rained.

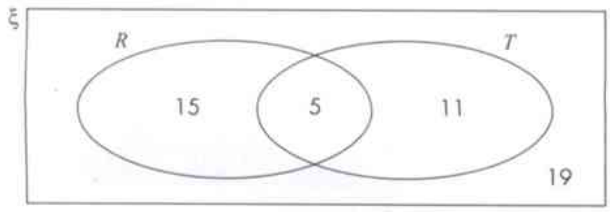
a Complete the tree diagram for Monday and Tuesday. [2]



b Find the probability that it rains on at least one of the two days shown in the tree diagram. [3]

Cambridge International IGCSE Mathematics 0580 *Paper 21 Q18 Oct/Nov 2014*

5



The Venn diagram shows the number of red cars and the number of two-door cars in a car park.

There is a total of 50 cars in the car park.

R = {red cars} and T = {two-door cars}.

a A car is chosen at random.

Write down the probability that

i it is red and it is a two-door car, [1]

ii it is not red and it is a two-door car. [1]

b A two-door car is chosen at random.

Write down the probability that it is not red. [1]

c Two cars are chosen at random.

Find the probability that they are both red. [2]

Cambridge International IGCSE Mathematics 0580 *Paper 21 Q22 Oct/Nov 2013*

EXTENDED

6 The table shows the probability that a person has blue, brown or green eyes.

Eye colour	Blue	Brown	Green
Probability	0.4	0.5	0.1

Use the table to work out the probability that two people, chosen at random,

a have blue eyes, [2]

b have different coloured eyes. [4]

Cambridge International IGCSE Mathematics 0580 *Paper 21 Q20 Oct/Nov 2015*

7 In this question, give all your answers as fractions.

A box contains 3 red pencils, 2 blue pencils and 4 green pencils.

Raj chooses 2 pencils at random, without replacement.

Calculate the probability that

a they are both red, [2]

b they are both the same colour, [3]

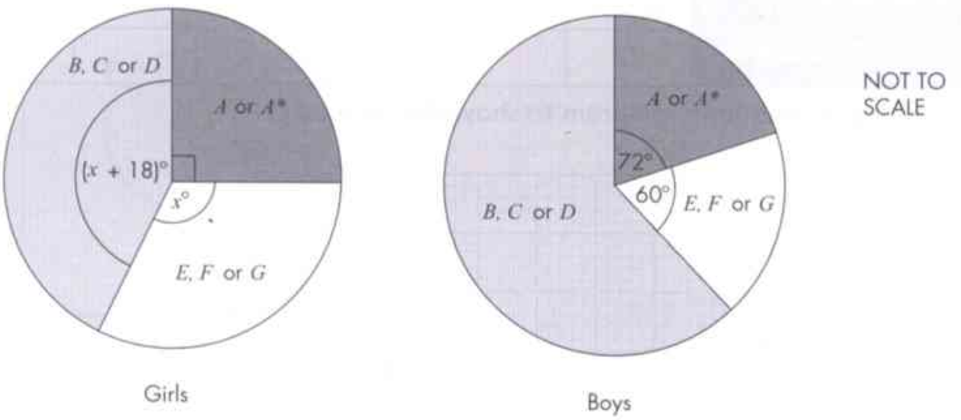
c exactly one of the two pencils is green. [3]

Cambridge International IGCSE Mathematics 0580 *Paper 21 Q21 May/June 2012*

PAPER 4

EXTENDED

1



The pie charts show information on the grades achieved in mathematics by the girls and boys at a school.

a For the **Girls'** pie chart, calculate

i x, [2]

ii the angle for grades B, C or D. [1]

b Calculate the percentage of the **Boys** who achieved grades E, F or G. [2]

c There were 140 girls and 180 boys.

i Calculate the percentage of students (girls and boys) who achieved grades A or A^*. [3]

ii How many more boys than girls achieved grades B, C or D? [2]

d The table shows information about the times, t minutes, taken by 80 of the girls to complete their mathematics examination.

Time (t minutes)	$40 < t \leqslant 60$	$60 < t \leqslant 80$	$80 < t \leqslant 120$	$120 < t \leqslant 150$
Frequency	5	14	29	32

i Calculate an estimate of the mean time taken by these 80 girls to complete the examination. [4]

ii On a histogram, the height of the column for the interval $60 < t \leqslant 80$ is 2.8 cm.

Calculate the heights of the other three columns.

Do not draw the histogram. [4]

iii $40 < t \leqslant 60$ column height = ...cm

$80 < t \leqslant 120$ column height = ...cm

$120 < t \leqslant 150$ column height = ...cm [4]

Cambridge International IGCSE Mathematics 0580 *Paper 41 Q1 Oct/Nov 2012*

2 120 students take a mathematics examination.

a The time taken, m minutes, for each student to answer question 1 is shown in this table.

Time (m minutes)	$0 < m \leqslant 1$	$1 < m \leqslant 2$	$2 < m \leqslant 3$	$3 < m \leqslant 4$	$4 < m \leqslant 5$	$5 < m \leqslant 6$
Frequency	72	21	9	11	5	2

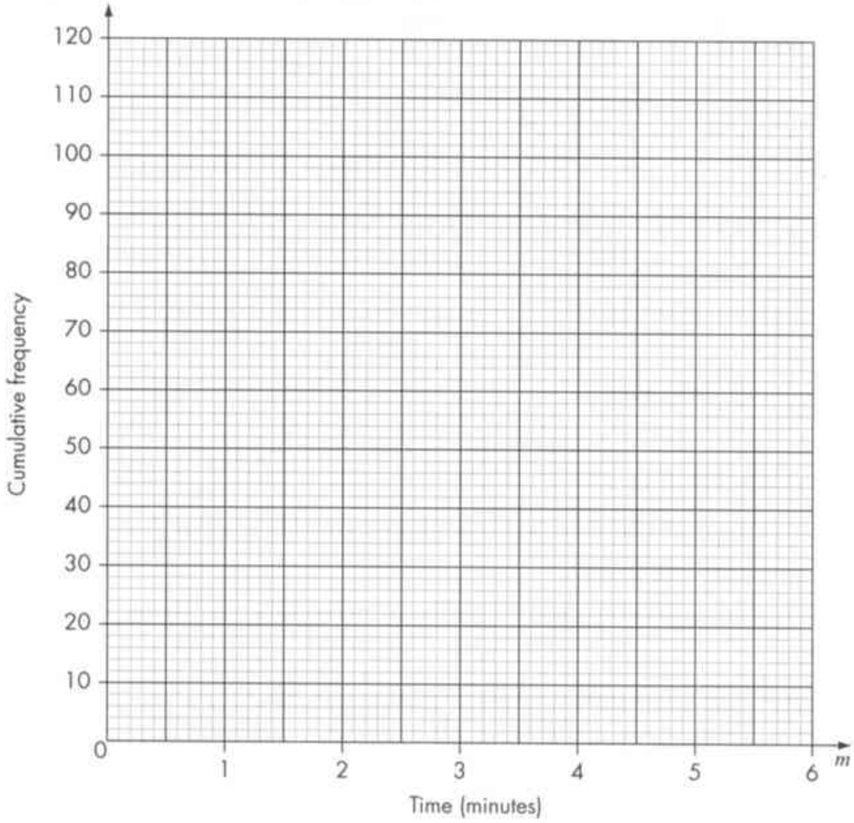
Calculate an estimate of the mean time taken. [4]

EXTENDED

b i Using the table in part (a), complete this cumulative frequency table. [2]

Time (m minutes)	$m \leqslant 1$	$m \leqslant 2$	$m \leqslant 3$	$m \leqslant 4$	$m \leqslant 5$	$m \leqslant 6$
Cumulative frequency	72					120

ii Draw a cumulative frequency diagram to show the time taken. [3]



iii Use your cumulative frequency diagram to find

a the median, [1]

b the inter-quartile range, [2]

c the 35th percentile. [2]

c A new frequency table is made from the table shown in **part (a)**.

Time (m minutes)	$0 < m \leqslant 1$	$1 < m \leqslant 3$	$3 < m \leqslant 6$
Frequency	72		

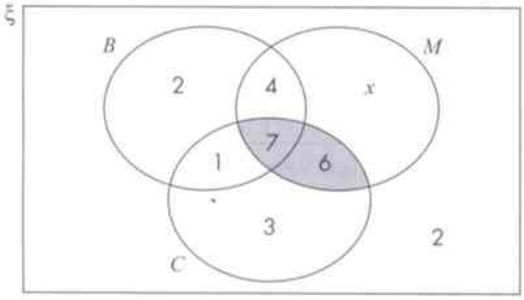
i Complete the table above. [2]

ii A histogram was drawn and the height of the first block representing the time $0 < m \leqslant 1$ was 3.6 cm.

Calculate the heights of the other two blocks. [3]

Cambridge International IGCSE Mathematics 0580 *Paper 41 Q6 Oct/Nov 2015*

3 30 students were asked if they had a bicycle (B), a mobile phone (M) and a computer (C). The results are shown in the Venn diagram.



a Work out the value of x. [1]

b Use set notation to describe the shaded region in the Venn diagram. [1]

c Find $n(C \cap (M \cup B)')$. [1]

d A student is chosen at random.

i Write down the probability that the student is a member of the set M'. [1]

ii Write down the probability that the student has a bicycle. [1]

e Two students are chosen at random from the students who have computers.

Find the probability that each of these students has a mobile phone but no bicycle. [3]

Cambridge International IGCSE Mathematics 0580 *Paper 41 Q4 M/J 2015*

4 **a** A square spinner is biased.

The probabilities of obtaining the scores 1, 2, 3 and 4 when it is spun are given in the table.

Score	1	2	3	4
Probability	0.1	0.2	0.4	0.3

i Work out the probability that on one spin the score is 2 or 3. [2]

ii In 5000 spins, how many times would you expect to score 4 with this spinner? [1]

iii Work out the probability of scoring 1 on the first spin and 4 on the second spin. [2]

b In a bag there are 7 red discs and 5 blue discs.

From the bag a disc is chosen at random and not replaced.

A second disc is then chosen at random.

Work out the probability that at least one of the discs is red.

Give your answer as a fraction. [3]

Cambridge International IGCSE Mathematics 0580 *Paper 41 Q6 May/June 2014*

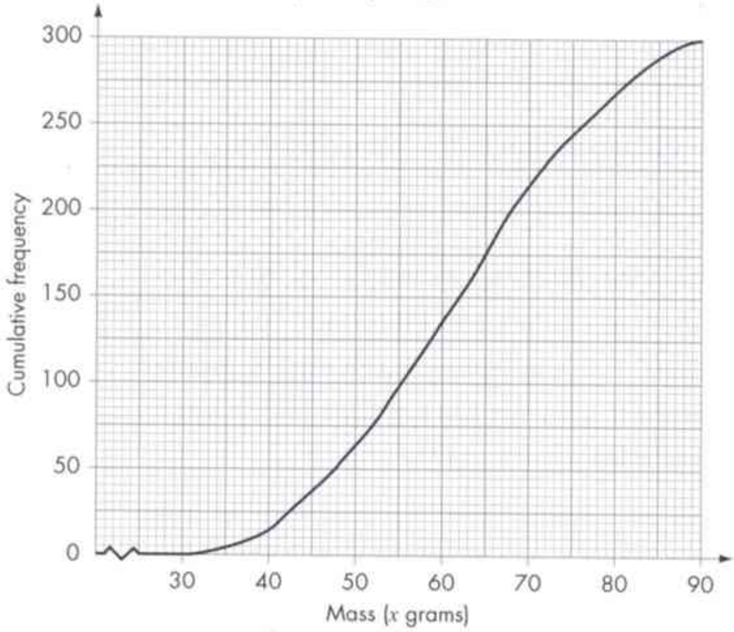
EXTENDED

5 Lauris records the mass and grade of 300 eggs. The table shows the results.

Mass (x grams)	$30 < x \leqslant 40$	$40 < x \leqslant 50$	$50 < x \leqslant 60$	$60 < x \leqslant 70$	$70 < x \leqslant 80$	$80 < x \leqslant 90$
Frequency	15	48	72	81	54	30
Grade	small		medium	large	very large	

a Find the probability that an egg chosen at random is graded very large. [1]

b The cumulative frequency diagram shows the results from the table.



Use the cumulative frequency diagram to find

i the median, [1]

ii the lower quartile, [1]

iii the inter-quartile range, [1]

iv the number of eggs with a mass greater than 65 grams. [2]

Cambridge International IGCSE Mathematics 0580 *Paper 21 Q18 Oct/Nov 2012*

Examination questions: Mixed type

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PAPER 3

CORE

1 **a** Luis buys a season ticket to watch his local football team.

The season ticket costs $595.

i Luis buys the season ticket online and gets a 5% discount on the $595.

Work out how much Luis pays for the season ticket online. [2]

ii A ticket to watch one match costs $3 8.

Luis watches 16 matches.

How much did Luis save by buying a season ticket online instead of 16 tickets at $38 each? [2]

b The football stadium has 26 272 seats.

The number of people who attend one match is 23 854,

Calculate the percentage of the 26 272 seats that are **empty**. [2]

c The total number of people attending matches at the stadium last season was 506 762.

Write 506 762 in standard form, correct to 3 significant figures. [2]

Cambridge International IGCSE Mathematics 0580 *Paper 31 Q3 Oct/Nov 2015*

2 Three friends are going on holiday.

They travel by plane.

a Ahmed's suitcase has mass m kilograms.

i The mass of Sonia's suitcase is 5 kg more than the mass of Ahmed's suitcase.

Write down an expression, in terms of m, for the mass of Sonia's suitcase. [1]

ii The mass of Hala's suitcase is twice the mass of Ahmed's suitcase.

Write down an expression, in terms of m, for the mass of Hala's suitcase. [1]

iii The total mass of the three suitcases is 47 kg.

Write down an equation in terms of m. [1]

iv Solve your equation and find the mass of each suitcase. [3]

Ahmed's suitcase .... kg

Sonia's suitcase .... kg

Hala's suitcase .... kg

Cambridge International IGCSE Mathematics 0580 *Paper 31 Q4 Oct/Nov 2015*

3 Denzil grows tomatoes. He selects a random sample of 25 tomatoes.

The mass of each tomato, to the nearest 5 grams, is shown below.

55	65	50	75	65
80	70	70	55	60
70	60	65	50	75
65	70	75	80	70
55	65	70	80	55

a **i** Complete the frequency table.

You may use the tally column to help you. [2]

Mass (grams)	Tally	Frequency
50		
55		
60		
65		
70		
75		
80		

ii Write down the mode. [1]

iii Find the range. [1]

iv Show that the mean mass is 66 g. [2]

b Denzil picks 800 tomatoes.

4% of the 800 tomatoes are damaged.

How many of these tomatoes are not damaged? [2]

e Denzil sells 750 of his tomatoes.

i The mean mass of a tomato is 66 g.

Calculate the mass of the 750 tomatoes in kilograms. [3]

ii Denzil sells his tomatoes at $1.40 per kilogram.

Calculate the total amount he receives from selling all the 750 tomatoes. [1]

iii The cost of growing these tomatoes was $33.

Calculate his percentage profit. [3]

Cambridge International IGCSE Mathematics 0580 *Paper 31 Q4 May/June 2014*

CORE

4 Sonia works in a toy shop.

a **i** One week she works for 30 hours and is paid $180.

Calculate the amount she is paid per hour. [1]

ii The next week Sonia works for 38 hours and is paid $220.

Find the difference in her pay per hour for these two weeks. [2]

b The shop sells bags of 40 marbles.

One bag has marbles in the ratio red : blue : green = 1 : 3 : 4.

i Calculate the number of marbles of each colour. [2]

ii A second bag of 40 marbles contains 11 red marbles, 9 blue marbles and 20 green marbles.

All the marbles from the two bags are mixed together.

Write down the ratio of marbles red : blue : green.

Give your answer in its simplest form. [2]

Cambridge International IGCSE Mathematics 0580 *Paper 31 Q3 May/June 2015*

PAPER 4

1 **a** Luc is painting the doors in his house.

He uses $\frac{3}{4}$ of a tin of paint for each door.

Work out the least number of tins of paint Luc needs to paint 7 doors. [3]

b Jan buys tins of paint for $17.16 each.

He sells the paint at a profit of 25%.

For how much does Jan sell each tin of paint? [2]

c The cost of $17.16 for each tin of paint is 4% more than the cost in the previous year.

Work out the cost of each tin of paint in the previous year. [3]

d In America a tin of paint costs $17.16.

In Italy the same tin of paint costs €13.32.

The exchange rate is $1 = €0.72.

Calculate, in dollars, the difference in the cost of the tin of paint. [2]

Cambridge International IGCSE Mathematics 0580 *Paper 41 Q1 Oct/Nov 2015*

2 **a** A company makes compost by mixing loam, sand and coir in the following ratio.

loam : sand : coir = 7 : 2 : 3

i How much loam is there in a 72 litre bag of the compost? [2]

ii In a small bag of the compost there are 13.5 litres of coir.

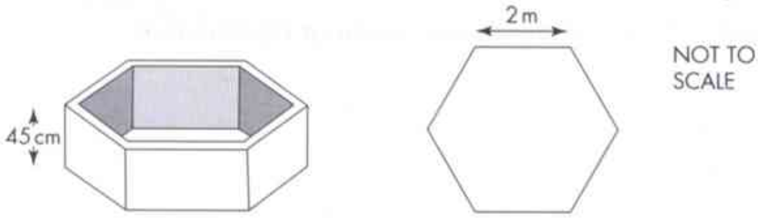
How much compost is in a small bag? [2]

iii The price of a large bag of compost is $8.40.

This is an increase of 12% on the price last year.

Calculate the price last year. [3]

b Teresa builds a raised garden bed in the shape of a hexagonal prism.



The garden bed has a height of 45 cm.

The cross section of the inside of the garden bed is a regular hexagon of side 2 m.

i Show that the area of the cross section of the inside of the garden bed is 10.4 m^2, correct to 3 significant figures. [3]

ii Calculate the volume of soil needed to fill the garden bed. [2]

iii Teresa wants to fill the garden bed with organic top soil.

She sees this advertisement in the local garden centre.

ORGANIC TOP SOIL	Number of tonnes purchased		
	1 to 5	6 to 10	Over 10
Cost per tonne	\$47.00	\$45.50	\$44.00

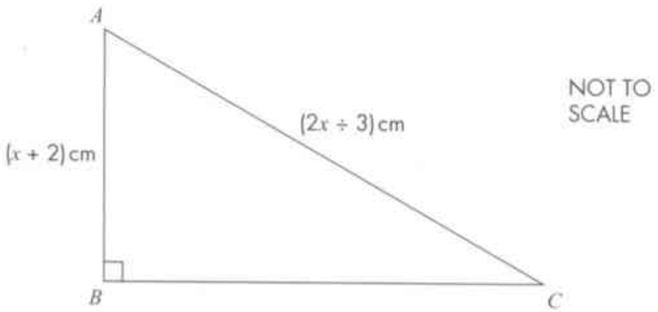
Organic top soil is sold in one tonne bags.

1 m^3 of organic top soil has a mass of 1250 kg.

Calculate the cost of the organic top soil needed to fill the garden bed completely. [1 tonne = 1000 kg] [4]

Cambridge International IGCSE Mathematics 0580 *Paper 41 Q1 Oct/Nov 2014*

3 **a**



In triangle ABC, $AB = (x + 2)$ cm and $AC = (2x + 3)$ cm.

$\sin ACB = \frac{9}{16}$

Find the length of BC. [6]

b A bag contains 7 white beads and 5 red beads.

i The mass of a red bead is 2.5 grams more than the mass of a white bead.

The total mass of all the 12 beads is 114.5 grams.

Find the mass of a white bead and the mass of a red bead. [5]

ii Two beads are taken out of the bag at random, without replacement.

Find the probability that

a they are both white, [2]

b one is white and one is red. [3]

Cambridge International IGCSE Mathematics 0580 *Paper 41 Q7 May/June 2013*

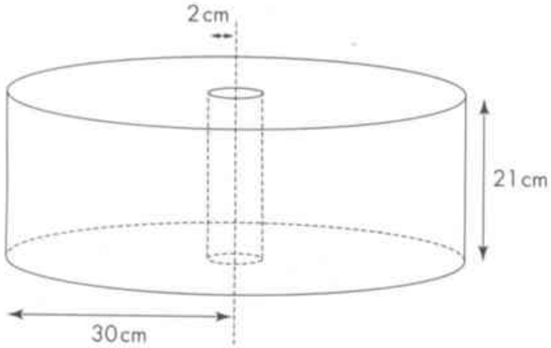
EXTENDED

4 **a** The running costs for a papermill are \$75 246.
This amount is divided in the ratio labour costs : materials = 5 : 1.
Calculate the labour costs. [2]

b In 2012 the company made a profit of \$135 890.
In 2013 the profit was \$150 675.
Calculate the percentage increase in the profit from 2012 to 2013. [3]

c The profit of \$135 890 in 2012 was an increase of 7% on the profit in 2011.
Calculate the profit in 2011. [3]

d



Paper is sold in cylindrical rolls.
There is a wooden cylinder of radius 2 cm and height 21 cm in the centre of each roll.
The outer radius of a roll of paper is 30 cm.

i Calculate the volume of paper in a roll. [3]

ii The paper is cut into sheets which measure 21 cm by 29.7 cm.
The thickness of each sheet is 0.125 mm.

a Change 0.125 millimetres into centimetres. [1]

b Work out how many whole sheets of paper can be cut from a roll. [4]

Cambridge International IGCSE Mathematics 0580 *Paper 41 Q3 May/June 2014*

Glossary

π **pi** the value of the circumference of any circle divided by its diameter. Approximately 3.142

12-hour clock used to give a time of day using times up to 12 and am for morning times and pm for afternoon times

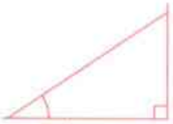
24-hour-clock used to give a time of day using times up to 24 and does not need am or pm.

Absolute value the positive value of the difference between a number and zero

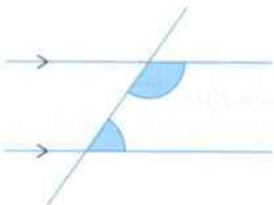
Acceleration the rate at which the velocity of a moving object changes

Acute angles are angles which are smaller than 90°

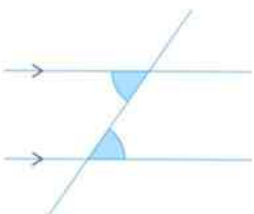
Adjacent side the side adjacent (next) to the known or required angle in a right-angled triangle



Allied angles are made when a line crosses a pair of parallel lines. Allied angles are sometimes called interior angles



Alternate angles are made when a line crosses a pair of parallel lines. The alternate angles are on alternate sides of the line

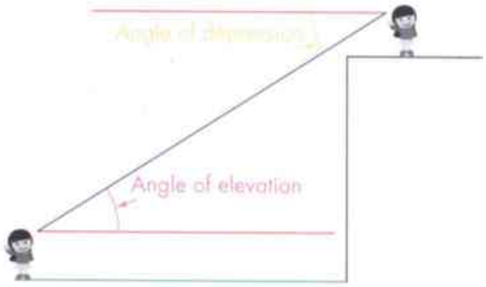


Amplitude the maximum displacement of a cyclical function from its central position

Angle an angle measures the amount of turn or the change in direction between two lines

Angle bisector a line which divides an angle into two equal parts

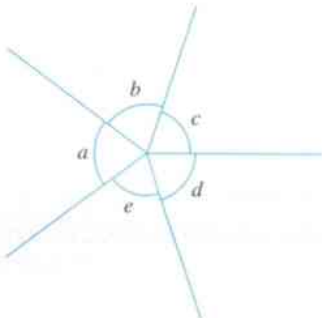
Angle of depression looking down this is the angle between the horizontal and the line of sight



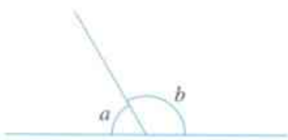
Angle of elevation looking up this is the angle between the horizontal and the line of sight

Angle of rotation the angle through which a shape is turned when it is rotated about a point

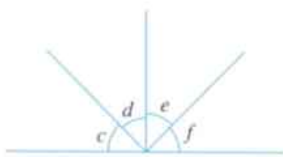
Angles at a point the sum of the angles at a point is 360° For example: $a + b + c + d + e = 360°$



Angles on straight lines the angles at a point on a straight line add up to 180°, for example:



$a + b = 180°$



$c + d + e + f = 180°$

Annual rate the amount something changes in a year, often referring to an interest rate which is given as a percentage

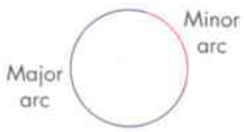
Anticlockwise the opposite direction to which the hands of a clock turn

Appropriate sensible for the context being considered

Approximation a value that is close but not exactly Appropriate sensible for the context being considered equal to another value which can be used to give an idea of the size of the value. For example, a journey taking 58 minutes may be described taking about an hour. The sign ≈indicates 'is approximately equal to'

(Arbitrary) constant of integration the constant introduced due to integrating a function can take any value so is an arbitrary constant

Arc part of the circumference of a circle



Area the amount of space in a 2-D shape

Area scale factor the factor by which an area is multiplied

Area sine rule a formula used to find the area of a triangle

Arithmetic progression a sequence of numbers are in arithmetic progression if they difference between consecutive numbers are constant

Average a value which is chosen to represent a set of data. Mode, mean, and median are common examples of an average

Average speed the ratio of distance ÷ time for a journey

Axis (plural **axes**) a fixed reference line for the measurement of coordinates

Axis of symmetry a line through a shape so that one side is a reflection of the other.

Bar chart a type of frequency diagram drawn using bars or rectangles of equal widths to display discrete data

Base

1) base number is the number which is being raised to a power
2) the base of a 2-D shape is the horizontal line drawn at the bottom of the shape
3) the base of a 3-D object is the flat part of the object upon which it stands

Bearing an angle measured clockwise from North to describe a direction

Bisect cut in half

Boundary

1) edge of an area
2) class boundary is the largest or smallest value in a class

Box plot a graph that shows the distribution of data along a number line

Brackets used to group terms together in algebra

Cancel dividing both the numerator and denominator in a fraction by the same number

Capacity the amount a container holds when it is full

Cartesian coordinates are an ordered pair (x, y) of numbers used to specify the position of a point where the x-value, or x-coordinate, gives the distance parallel to the x-axis and the y-value, or y-coordinate, gives the distance parallel to the y-axis

Cartesian plane the plane in which the x-axis and y-axis lie

Centilitre one hundredth of a litre

Centimetre one hundredth of a metre

Centre

1) of a circle is where the compass point is placed when drawing a circle using a pair of compasses
2) of a transformation is a fixed point from which a transformation is described

Centre of enlargement the lines joining corresponding points in an object and its image all meet at the centre of enlargement

Centre of rotation the fixed point about which an object is rotated

Certain having a probability of 1

Chance likelihood

Chord a line across a circle

Circumference the distance around the outside (perimeter) of a circle

Class frequency the number of values in a class

Class interval the width of a group in a grouped frequency distribution

Classes are groups

Clockwise the hands of a clock turn in this direction

Coefficient a constant term which is multiplied by a variable. For example, $2x$, or $5x^2$, the '2' and the '5' are coefficients of x and x^2 respectively

Collinear lying on the same straight line

Column vector an ordered set of 2 or 3 numbers used to give the position of a point or to describe the change in position of a point

Common factors factors which are common to more than one term or number

Compasses drawing instruments used to draw arcs and circles

Complement the complement of set A is everything outside of set A

Completing the square a way of simplifying or solving a quadratic equation by adding an expression to both sides to make one part of the equation a perfect square

Composite function a function that is made from two or more separate functions

Compound interest the overall interest earned on investment when the total interest earned in each period is added back to the original capital

Cone a 3-D shape with a circular base and a curved sloping face

Congruent two shapes are congruent if they have exactly the same shape and size

Consecutive next to each other

Consistency how varied a set of values are

Constant a value that does not change

Constant of proportionality the constant value of the ratio between two proportional quantities

Constant speed a particle has constant speed if its speed does not change

Construct use only pencil, straight edge and compasses

Continuous data data that can have any value in a range

Conversion graph a graph used to convert from one unit to another

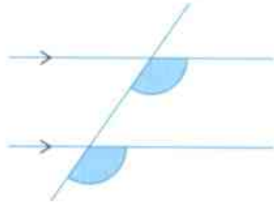
Coordinate method the way in which an object or an image can be found in 'enlargements' by counting squares

Coordinate grid a grid used to show the positions of points

Coordinates used to specify the location of a point. *See* Cartesian coordinates

Correlation a connection between two sets of data

Corresponding angles angles which are in the same position and are equal



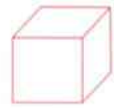
Corresponding sides sides which are in the same position

Cosine the ratio of the adjacent side to the hypotenuse

Cosine rule a rule connecting sides and an angle of any triangle usually used when the triangle is not right-angled. $a^2 = b^2 + c^2 - 2bc \text{ Cos } A$

Cross-section a face formed by cutting through a 3-D object

Cube a 3-D solid consisting of six square faces



Cube number the number you get when you multiply a number by itself and then again. For example 8 is a cube number as $2 \times 2 \times 2 = 8$. 8 is called the cube of 2 and can be written as 2^3, 2 cubed

Cube root the opposite of cubing a number, so the cube root of 8 is 2

Cubic function is a function in which the highest power of x is x^3

Cubic sequence a sequence in which the values are obtained using n^3 in some way.

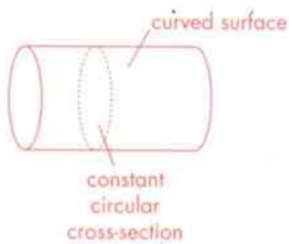
Cuboid a 3-D solid consisting of 6 rectangular faces

Cumulative frequency obtained by adding frequencies together to accumulate them

Curved surface a part of a cylinder

Cyclic quadrilateral a quadrilateral whose vertices lie on a circle

Cylinder a prism whose constant cross-section is a circle.



Dashed line a line which has regular gaps, sometimes called broken

Decagon a polygon with 10 sides

Decay a reduction which follows a predictable pattern

Deceleration the rate at which speed or velocity decreases with time

Decimal a number written using only digits and a decimal point, for example 12.34

Decimal equivalent a number written as a decimal which has the same value as a number written in a different form

Decimal place the position of a digit after the decimal point in a number

Decrease become smaller or reduce

Denominator the number on the 'bottom' of a fraction

Derivative the result of differentiation, the gradient function

Diameter the distance from one point on a circle to another passing through the centre of the circle

Difference

1) the result when one number is subtracted from another
2) the gap between consecutive numbers in a sequence

Difference of two squares a results used to factorise algebraic expressions $x^2 - y^2 = (x + y)(x - y)$

Differentiate find the gradient function

Differentiation the process used to find the gradient function

Digit a single number, for example in the number 234 the digit in the 'tens column' is '3'

Direct proportion two quantities are in direct proportion if one increases as the other increases

Direct variation same as direct proportion

Directed number a positive or negative number or zero

Direction in vectors the direction is indicated by an arrow on the vector

Discrete data data that takes only particular values in a range

Dispersion spread or variation

Displacement when an object moves from a position A to a position B the displacement is the magnitude of the vector AB

Distance a measured length between two points

Distance travelled the measured length between two points of a journey

Distance–time graph a graph plotting the distance travelled against the time taken

Dividend a number being divided by another. For example 12 ÷ 4 = 3, here 12 is the dividend

Divisor a number being divided into another. For example 12 ÷ 4 = 3, here 4 is the divisor.

Edge the line where two faces of a solid meet

Element a member of a set

Eliminate remove. For example when two simultaneous equations are solved you eliminate one of the letters, that is you combine the two equations by removing one of the letters

Empty set a set with no elements. Also called the null set

Enlargement a transformation in which the shape of an object remains the same but the size usually changes

Equally likely have the same probability

Equals has the same value as

Equation a statement which involves two expressions which have the same value so have an '=' sign between them

Equation of line the relationship between the x-coordinate and the y-coordinate for each of a set of points

Equidistant at the same distance

Equilateral triangle an equilateral triangle is a triangle with all its sides equal. All three angles in an equilateral triangle are 60°

Equivalent fractions two or more fractions which have the same value as each other. For example $\frac{2}{4}$ and $\frac{3}{6}$ are equivalent fractions, both are equivalent to $\frac{1}{2}$

Estimate find an approximate value for

Estimated a value which has been found approximately

Glossary

Event a set of outcomes in probability

Exact form written without using decimals or any approximations

Exchange rate the equivalence between two different currencies

Expand multiply all the terms inside the brackets by those outside the brackets. (opposite of factorise)

Expansion when brackets are expanded the result is called the expansion of the brackets

Experimental probability the ratio of the number of times the event occurs to the total number of trials

Exponential having a constant base raised to a variable power

Exponential decay a reduction which follows a pattern predictable using an exponential function

Exponential form written in the form of a^x

Exponential functions functions which involve a constant base raised to a variable power

Exponential growth an increase which follows a pattern predictable using an exponential function

Exponential sequence a set of numbers which follow an exponential pattern

Expression a series of terms connected by plus and minus signs

External angles angles turned through when going round the perimeter of a polygon

Extreme values values which stand out as being particularly large or small relative to the other values in a set.

Face one flat surface of a solid which is enclosed by edges

Factor a whole number which divides exactly into another whole number

Factor pair a pair of numbers which multiply to give another number. For example, 3 and 4 are a factor pair of 12.

Factorise take all common factors outside brackets. (opposite of expand)

Formula (plural **formulae** or **formulas**) a rule expressed in words or letters

Fraction a number which is written using two parts called the numerator and denominator

Frequency the number of times a value occurs in a set of data

Frequency density the ratio of the frequency to the class-width in a frequency distribution. Frequency density is used to draw histograms

Frequency table a table showing frequencies

Function a rule which takes one number and changes it into another.

Gradient a measure of how steep a line is

Gram (g) a basic unit of mass in the metric system

Greater than 'greater than' means the same as 'more than'

Grouped data data which has been sorted into groups or classes

Grouped frequency table a table which shows the number of values in each of a set of groups or classess.

Heptagon a polygon with 7 sides

Hexagon a six-sided polygon

Highest common factor (HCF) the largest factor which is common to two or more other numbers

Histogram a chart drawn using rectangles that uses the area to represent frequencies

Hypotenuse the longest side of a right-angled triangle. It is always opposite the right angle.

Image the new shape after a transformation

Impossible an event with probability 0

Improper fraction a fraction in which the numerator is greater than the denominator

Included in inequalities lines on the boundary of a region which are included in that region are drawn using a solid line

Included angle the angle between two adjacent sides

Increase go up in value

Index (plural **indices**)

1) the power of a number
2) a quantity which allows comparisons to be made over time, for example a cost of living index allows costs over time to be compared

Inequality a statement about the relative size of two values or expressions using the symbols < (less than), ⩽ (less than or equal to), > (greater than), ⩾ (greater than or equal to)

Infinite going on for ever

Integer a whole number

Intercept the place a line crosses an axis, for example the y-intercept is the place a line crosses the y-axis

Interior angles

1) Interior angles are the angles inside a polygon. The sum, S, of the interior angles of a polygon with n sides is given by the formula $S = 180(n - 2)°$
2) Allied angles are sometimes called interior angles

Inter-quartile range the distance between the lower and upper quartiles

Intersection

1) a set of elements that belong to both of two other sets
2) the point where two lines cross

Inverse something that has an opposite or reverse effect

Inverse functions functions which have the reverse effect to each other

Inverse operations operations that have the reverse or opposite effect to each other. For example, addition and subtraction are inverse operations, multiplication and division are inverse operations

Inverse proportion the relationship between two variables where one decreases as the other increases

Irrational number a number that cannot be written as a fraction

Irregular polygon any polygon which is not regular

Isosceles triangle a triangle with two equal sides. An isosceles triangle has two equal angles (at the foot of the equal sides).

Key an explanation of what a diagram shows. For example, in a pictogram the key will show a symbol and how many items it represents and it states what the items are

Kilogram (kg) a measure of mass in the metric system. 1 kg = 1000 g

Kilometre (km) a measure of distance in the metric system. 1 km = 1000 m

Kite a kite is a quadrilateral with two pairs of equal adjacent sides.

Length how long an object is

Less than the sign < is used to mean 'less than'. For example, 2 < 3 means 2 is smaller than 3, or 2 is less than 3

Like terms terms containing the same variable raised to the same power. These terms can then be added or subtracted to be combined

Limits of accuracy the upper and lower bounds when approximating

Line a one-dimensional object extending infinitely in both directions

Line bisector a line which cuts another line in two equal parts at right angles

Line of best fit a single straight line which passes through a set of points and is as close as possible to as many of them as possible

Line of symmetry a line drawn so that one side of the line is a reflection of the other side

Line segment the part of a line that joins two points

Linear of a straight line

Linear equations equations that do not contain any powers or roots such as $\sqrt{x}$, or y^2

Linear programming problem solving using graphs of straight lines

Linear scale factor a multiplier which is linear

Linear sequence a sequence of numbers so that the difference between consecutive numbers is constant

Litre a basic unit of capacity in the SI system

Locus (plural **loci**) the path of a moving point

Loss the difference between the amount taken when it is sold and the amount paid for it initially when the amount taken is smaller than the amount paid initially

Lower bound the smallest possible value of a rounded quantity

Lower quartile the value which 25% of the data are below or equal to

Lowest common multiple (LCM) the smallest number which is a multiple of two or more other numbers. For example the LCM of 6 and 10 is 30

Lowest terms a fraction which has been cancelled as much as possible so that it is not possible to cancel it further is said to be in its lowest terms.

Magnitude the size or length of a vector

Map scale the scale on a map indicates how many centimetres on the ground are represented by one centimetre on the map

Mapping the process which changes one number to another

Mapping diagram a diagram showing two sets of numbers and how each number in one set is mapped to each number in the other

Mass the amount of matter in an object

Maximum the largest possible value

Mean a measure of average, found by adding all the values and dividing by how many there are

Median a measure of average, found by listing all the values in order and taking the value in the middle

Metric system the system of weights and measures most commonly in use

Middle value the value in the middle

Midpoint the point which is exactly half way between two others

Millilitre (ml) a unit used for measuring capacity. 1000 millilitres = 1 litre

Millimetre (mm) a unit used for measuring distance. 1000 millimetres = 1 metre

Minimum the smallest possible value

Mirror line a line of symmetry

Mixed number a number containing a whole number part and a fraction

Modal class the class which has the highest frequency

Modal value the value with the highest frequency

Mode the value with the highest frequency

More than the sign > is used to mean 'more than'. For example, 5 > 4 means 5 is larger than 4, or 5 is more than 4

Multiple a number which is obtained by multiplying two other numbers

Multiplier a number which is used to multiply another number is called a multiplier

Mutually exclusive two events are mutually exclusive if one prevents the other from happening. For example, when you follow a maze and get to a junction where there are exactly two possible choices: you can turn left or right, these are mutually exclusive as you cannot go both left and right at the same time.

Natural number the counting numbers 1, 2, 3, ...

Negative less than 0

Negative correlation a correlation where one variable decreases as the other increases

Negative enlargement an enlargement involving a negative scale factor. A negative enlargement produces an image shape on the opposite side of the centre of enlargement to the original shape

Negative index an index which is below 0

Net a 2-D pattern that can be cut out and folded into a 3-D shape

Nonagon a polygon with 9 sides

Non-linear not following the pattern of a straight line

***n*th term** counting from the first term in a sequence it is the term in position n. It is used to give a general formula that can be used to find every term in the sequence

Null set a set with no elements

Number line a line labelled with numbers. Sometimes used to help working with negative numbers

Numerator the number which is on the 'top' of a fraction.

Object the original shape before a transformation

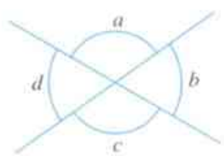
Obtuse angles greater than 90° but less than 180°

Obtuse-angled triangle a triangle with one angle greater than 90°

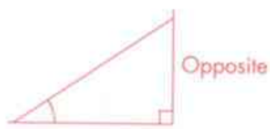
Octagon a regular eight-sided polygon

Operation +, −, ×, ÷ are all operations

Opposite angles are made by two straight lines crossing each other. The diagram shows a and c are opposite angles, and b and d are opposite angles. Opposite angles are equal so $a = c$ and $b = d$. Opposite angles are sometimes called vertically opposite angles



Opposite side the side opposite the known or required angle in a right-angled triangle



Order when numbers are organised into a sequence they are said to be in order

Order of rotational symmetry the number of times a shape will fit onto itself exactly in one full rotation when rotated about a point

Origin (0, 0) the point where the x-axis and y-axis cross

Outcome result.

Parabola the shape of the graph of a quadratic function

Parallel lines which have exactly the same direction, are always the same distance apart and never meet

Parallelogram a quadrilateral in which opposite sides are parallel

Patterns predictable arrangements or sequences of numbers

Pentagon a five-sided polygon

Percentage out of each hundred

Percentage change the difference between the current and the original value as a fraction of the original value multiplied by 100

Percentage decrease a negative percentage change where the current value is lower than the original value

Percentage increase a positive percentage change where the current value is higher than the original value

Percentage loss the difference between the current and the original value as a fraction of the original value multiplied by 100 when the values are money and the current value is lower than the initial value

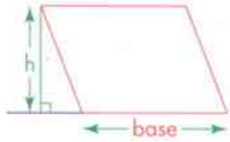
Percentage profit the difference between the current and the original value as a fraction of the original value multiplied by 100 when the values are money and the current value is higher than the initial value

Perimeter the distance around the outside of a 2-D shape

Perpendicular two lines are perpendicular if they are at right angles to each other

Perpendicular bisector a line that cuts a line in half at right angles

Perpendicular height the height of a 2-D shape measured from the base of the shape and at right angles to it. The perpendicular height of a shape does not have to be inside the shape, as seen in this parallelogram, the perpendicular height is labelled h in the diagram



Pictogram a chart which uses identical pictures to show frequencies

Pie chart a circular chart showing how a whole set of data is divided into parts. The angles, or areas, are used to represent frequencies

Plane area the area of a flat shape

Plane of symmetry a plane in a solid which divides the solid into two parts each of which is a reflection of the other

Point of contact the position where a line touches a curve

Point of inflexion a point on a curve where the gradient changes from decreasing to increasing or from increasing to decreasing

Polygon a 2-dimensional shape made using only straight lines

Position vector a vector which gives the position of a point

Positive greater than 0

Positive correlation a correlation where one variable increases as the other increases

Possibility diagram a diagram which shows all the possible outcomes of an event

Power an index

Power 0 any value or expression raised to the power 0 has a value of 1. For example $3^0 = 1$, and $(3x)^0 = 1$

Power 1 raising any value or expression to the power 1 does not change its value. For example $3^1 = 3$, and $(3x)^1 = 3x$

Prime factorisation writing a number as a product of all its factors which are prime

Prime number a number which has exactly 2 factors

Principal the amount of money invested

Prism a 3-D solid which has the same cross-section throughout its shape (a uniform cross-section). Triangular prisms (with a triangular cross-section) and cylinders (with a circular cross-section) are examples of prisms

Probability how likely an individual outcome of an event is to occur. Probability is measured on a scale from 0 to 1

Probability fraction a probability which is written as a fraction

Product the product of two or more numbers is obtained by multiplying the numbers together

Product of prime factors *see* prime factorisation

Profit when you sell something for more than you paid for it the difference is the profit

Proof a set of statements which together form a step by step mathematical argument

Proper fraction a fraction in which the numerator is less than the denominator

Proper subset a set of elements which is contained within another set, but not the same as that set. For example, Set $A = \{1, 2, 3, 4, 5\}$ then set $D = \{1, 2, 3\}$ is a proper subset of A

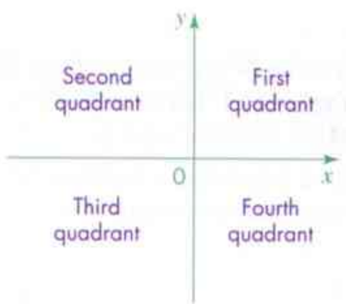
Protractor used to measure angles

Prove make a proof

Pyramid a solid shape with triangular faces. The base of a pyramid does not have to be triangular

Pythagoras' theorem a relationship between the sides of a right angled triangle. $a^2 + b^2 = c^2$ where c is the hypotenuse of the triangle.

Quadrant the axes divide a page into four quadrants



Quadratic equation an equation in which the highest power of x is x^2

Quadratic expression an expression in which the highest power of x is x^2

Quadratic formula the formula used to solve a quadratic equation $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, where a, b, and c are the coefficients of x^2, x, and the constant term, respectively

Quadratic graph a graph of a quadratic equation

Quadratic sequence a sequence of numbers form a quadratic sequence if they differences between consecutive numbers are in an arithmetic progression. The formula for the nth term of a quadratic sequence is a formula in which the highest power of n is n^2

Quadrilateral a polygon with four sides

Qualitative descriptive rather than numerical

Quantity an amount of something

Quartile a value which is one quarter way through a set of data when it is listed in order. Lower quartile is the value which is one quarter way from the lowest value, upper quartile is the value which is one quarter way from the highest value.

Radius (plural **radii**) the distance from one point on a circle to the centre

Random a random number is a number which is not predictable

Range

1) the distance between the largest and smallest value in a set of data
2) the set of y values for a function

Rate the rate of interest when borrowing or saving money

Rate of increase how quickly a variable or value increases relative to another variable

Ratio a way to compare two quantities with each other

Rational number a number which can be written as a fraction $\left(\frac{a}{b}\right)$ where a and b are integers

Ray methods the way in which an object or an image can be found in 'enlargements' by drawing lines or 'rays' from the centre of enlargement

Real number any rational or irrational number

Rearrange rewrite in a different order

Reciprocal the reciprocal of a number is 1 divided by the number. For example, the reciprocal of 3 is $\frac{1}{3}$, and the reciprocal of $\frac{3}{5}$ is $\frac{5}{3}$

Rectangle a quadrilateral with two pairs of parallel sides and four right angles

Recurring decimal a decimal with an infinite number of decimal places the last digit or group of digits repeats

Reflection a transformation which results in a copy of the object looking as though it is the object viewed in a mirror

Reflex angles are greater than 180°

Regular polygon A polygon is regular if all its interior angles are equal and all of its sides are the same length

Relative frequency the ratio of the number of the successful outcome's to the total number of trials, which can then be used as an estimate of probability

$$relative\ frequency = \frac{number\ of\ successful\ outcomes}{number\ of\ trials}$$

Representative in place of, for example when a single value is used to represent a set of values it is used in place of all the values

Required to be found. The required angle is an angle whose size is to be found

Rhombus a parallelogram with all its sides equal

Right angle an angle of 90°

Right-angled triangle a triangle which has an angle of 90°

Rotation turning an object or shape about a point by a given angle in either a clockwise or anti-clockwise direction

Rotational symmetry a shape has rotational symmetry if when it is rotated about a point it fits exactly onto itself at least once before being turned 360°

Round

1) circular in shape
2) replace a number with one which is approximately equal

Rounded down replace a number with one which is approximately equal but smaller than the original number

Rounded up replace a number with one which is approximately equal but smaller than the original number

Rule a general statement

Ruler used to draw straight lines and measure lengths.

Sample a selection from a larger population

Scalar a single number

Scale the ratio between the lengths on a scale drawing and the actual length represented

Scale drawing an accurate drawing in which the lengths are in proportion to the original

Scale factor a number which tells you how many times larger an image is of an object

Scatter diagram points plotted on Cartesian axes which are used to see any correlation or to make predictions

Sector part of a circle enclosed between part of the circumference and two radii

Segment part of a circle enclosed between part of the circumference and a chord

Semi-circle half a circle

Sequence a set of numbers which follow a rule

Set a collection of items

Set square used to find right angles

Significant figures the digits of a number

Similar two shapes are similar if they have the same shape but not the same size

Simple interest - a charge for borrowing or lending money. $I = \frac{PRT}{100}$

Simplest form the way in which a fraction or ratio is written so that the smallest possible whole numbers are used

Simplify

1) in an expression or equation this means to collect any like terms so there are as few terms as possible
2) in a fraction or ratio this means write a fraction so that the smallest possible whole numbers are used

Simultaneous linear equations two equations with two unknowns

Sine the ratio of the opposite side to the hypotenuse

Sine rule a rule connecting the sides and angles of a triangle which is not right-angled

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \text{ or } \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Single fraction a fraction which consists of exactly one numerator and one denominator only

Single unit value in unitary method it is the cost of one unit or the time it takes for one item

Slant height the sloping distance from the circular base of a cone to the apex at the top

Solid line a line which has no breaks or gaps

Solid shapes 3-D objects

Soluble can be solved or worked out

Solution answer to a problem

Solve

1) a triangle work out any missing sides or angles
2) an equation work out the value of the letter

Speed the rate of change of distance with time

Speed–time graph a graph which shows how speed varies with time

Sphere a solid shape which appears round no matter what position it is viewed from, for example a football

Spread the distance(s) between the values

Square

1) a square is a regular four-sided polygon
2) to square a number is to multiply it by itself, for example the square of 5 is 25

Square number the result of squaring an integer

Square root the opposite of squaring a number. For example the square root of 36 is 6

Standard form a way to write very large and very small numbers using a number between 1 and 10 multiplied by a power of 10

Stem-and-leaf diagram shows data arranged by place value, for the purpose of comparing frequencies

Straight-line graph the graph of a linear function

Subject the subject of a formula is the letter in front of the equals sign. For example, in $t = 2s + 3$, t is the subject of the formula

Subset if all the elements of a set C are in a set A then C is a subset of A

Substitute replace a letter with either an expression or a value

Subtended made. The angle subtended at the circumference of a circle is the angle made there

Supplementary supplementary angles add up to 180°

Surface area the total area of the faces of a solid

Symbol each of +, −, ×, ÷ is a symbol.

Tally chart a chart used to collect data with a tally used to record each value. Tallies are marked as |s and grouped in 5s as a 5-bar gate. For example this tally represents 7 values being recorded Note the 5-bar gate is 4 vertical lines with one diagonal line

Tangent

1) a straight line that touches a circle at one point only
2) the ratio of the opposite side to the adjacent side in a right-angled triangle

Term

1) an expression forming part of a larger equation or part of an equation
2) a single item in a sequence

Term to term rule a rule which links one term in a sequence to the next term

Terminating decimal a decimal that has a finite number of digits

Three-figure bearing a bearing which has 3 digits exactly

Time in speed distance and time, time is how long a journey takes, measured in hours minutes or seconds

Timetable a table giving arrival and/or departure times

Tonne a metric unit for measuring mass.
1 tonne = 1000 kg

Top heavy a fraction in which the numerator is greater than the denominator. *See* improper fraction

Transformation a change in the position or size of a shape. Reflections, rotations, translations or enlargements are all examples of transformations

Translation a transformation which moves a shape from one position to another without changing the orientation or size of the shape

Transversal a line crossing a pair of parallel lines

Trapezium a quadrilateral with two parallel sides

Tree diagram a diagram with branches which shows all the possible outcomes of an event and their probabilities

Trial an experiment which is repeated a number of times

Turning point a point on a graph where the gradient changes from being either positive to negative or from negative to positive.

Union the union of sets A and B is the set of all elements which are in either set A or in set B or in both

Unitary method a way to solve proportion problems by finding the value of a single item

Universal set the set containing all the things being considered

Upper bound the largest possible value of a rounded quantity

Upper quartile the value below which three-quarters of the data lie.

Variable a quantity that can take different values

Variation

1) *see* proportion
2) how the values in a data set are spread out

Vector a way of writing the position of a point or describing a movement from one position to another

Venn diagram a way to show the elements of different sets

Vertex a corner of a shape, where two edges meet

Vertical height distance of one point above another point or line

Vertically opposite angles vertically opposite angles are the same as opposite angles

Volume the amount of space a solid shape occupies

Volume scale factor a multiplier for enlargements using volumes.

Weak correlation when there is little connection between two sets of data

Width distance from one side to the other.

Zero correlation when there is no connection between two sets of data.

Answers to Chapter 1

1.1 Square numbers and cube numbers

Exercise 1A

1 36, 49, 64, 81, 100, 121, 144, 169, 196, 225, 256, 289, 324, 361, 400

2 4, 9, 16, 25, 36, 49

3 **a** 50, 65, 82 **b** 98, 128, 162 **c** 51, 66, 83 **d** 48, 63, 80

4 **a** 25, 169, 625, 1681
b Answers in each row are the same

5 **a** 13 **b** 5 **c** 9 **d** 11 **e** 20

6 36 and 49

Exercise 1B

1 **a** 12, 24, 36 **b** 20, 40, 60 **c** 15, 30, 45 **d** 18, 36, 54 **e** 35, 70, 105

2

	Square number	Factor of 70
Even number	16	10
Multiple of 7	49	35

3 4761 (69^2)

4 24 seconds

5 30 seconds

6 **a** 12 **b** 9 **c** 6 **d** 13 **e** 15 **f** 16 **g** 10 **h** 17

Exercise 1C

1 1, 8, 27, 64, 125, 216, 343, 512, 729, 1000, 1331, 1728

2 9, 36, 100: square numbers
$1 + 8 + 27 + 64 + 125 = 225$
$1 + 8 + 27 + 64 + 125 + 216 = 441$
$1 + 8 + 27 + 64 + 125 + 216 + 343 = 784$

3 **a** 126 217 344
b 124 215 342
c 250 432 686
d 216 125 64

4 **a** 153, 370, 371
b Each answer is the sum of the cubes of its digits

5 1729

6 692 = 4761 and 693 = 328509 The answers use all the digits from 0 to 9 exactly once.

1.2 Multiples of whole numbers

Exercise 1D

1 **a** 3, 6, 9, 12, 15 **b** 7, 14, 21, 28, 35 **c** 9, 18, 27, 36, 45 **d** 11, 22, 33, 44, 55 **e** 16, 32, 48, 64, 80

2 **a** 72, 132, 216 **b** 161, 91 **c** 72, 102, 132, 78, 216

3 **a** 98 **b** 99 **c** 96 **d** 95 **e** 98 **f** 96

4 4 or 5 (as 2, 10 and 20 are not realistic answers)

5 **a** 18 **b** 28 **c** 15

6 5 numbers: 18, 36, 54, 72, 90

1.3 Factors of whole numbers

Exercise 1E

1 **a** 1, 2, 5, 10 **b** 1, 2, 4, 7, 14, 28
c 1, 2, 3, 6, 9, 18 **d** 1, 17
e 1, 5, 25 **f** 1, 2, 4, 5, 8, 10, 20, 40
g 1, 2, 3, 5, 6, 10, 15, 30 **h** 1, 3, 5, 9, 15, 45
i 1, 2, 3, 4, 6, 8, 12, 24 **j** 1, 2, 4, 8, 16

2 **a** 55 **b** 67 **c** 29 **d** 39 **e** 65 **f** 80

3 **a** 2 **b** 2 **c** 3 **d** 5 **e** 3 **f** 3 **g** 7 **h** 5 **i** 10 **j** 11

4 5

Exercise 1F

1 **a** 168 **b** 105 **c** 84 **d** 84 **e** 96 **f** 54 **g** 75 **h** 144

2 **a** 8 **b** 7 **c** 4 **d** 14 **e** 4 **f** 9 **g** 5 **h** 4 **i** 3 **j** 16 **k** 5 **l** 18

3 18 and 24

1.4 Prime numbers

Exercise 1G

1 23 and 29

2 97

3 All these numbers are not prime.

4 3, 5, 7

5 Only if all 31 bars are in a single row, as 31 is a prime number and its only factors are 1 and 31.

1.5 Prime factorisation

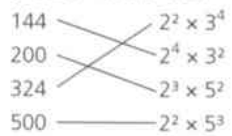
Exercise 1H

1 **a** 36 **b** 105 **c** 250 **d** 816 **e** 714 **f** 1715 **g** 1089 **h** 1352

2 **a** $2 \times 3^2 \times 5$ **b** $2^3 \times 19$ **c** 2^6
d $2 \times 3 \times 5 \times 11$ **e** 17^2 **f** $2^5 \times 5^2$
g It is a prime number and cannot be factorised.
h $7 \times 11 \times 13$

3 $77 = 7 \times 11$; $129 = 3 \times 43$; $221 = 13 \times 17$

4 **a** $900 = 2^2 \times 3^2 \times 5^2$ **b** $1800 = 2^3 \times 3^2 \times 5^2$
c $1350 = 2 \times 3^3 \times 5^2$

5


6 a $2 \times 3 \times 5 \times 7 = 210$
b The answer to **a** $\times 11 = 2 \times 3 \times 5 \times 7 \times 11 = 2310$
7 a $2^2 \times 3^2 \times 17$ b $2 \times 3^2 \times 17$ c $2^3 \times 3 \times 17$
8 71, 73 and 79 because they are prime numbers
9 $456\,533 = 7^3 \times 11^3$

1.6 More about HCF and LCM

Exercise 1I

1 a $2 \times 3^2 = 18$ b $2^3 \times 3^4 = 648$
2 a 35 b 735
3 a $2^4 \times 3 \times 5$ b $2 \times 3^2 \times 7$ c 6 d 5040
4 a $2^3 \times 3^2$ and $2^2 \times 3^3$ b 36 c 216
5 a 16 b 576
6 a 33 b 2772
7 a $2^2 \times 3 = 12$ b $2^3 \times 3^2 \times 5 = 360$
8 a 5 b 1575
9 a 15 b 23 625
10 a 72 b $2^5 \times 3^4$
11 a 1 b 12 600
12 a $72 \times 162 = 11664$ b $18 \times 648 = 11664$
c You could do a similar calculation for the numbers in questions 2 to 6. You should find that the two products are equal each time.
d There is not a similar result in this case.

1.7 Real numbers

Exercise 1J

1 a yes b no c yes d yes e no
f yes g no h yes i yes
2 a rational b rational c irrational
d rational e irrational f rational
g rational h irrational i irrational
3 a $\frac{1}{300}$ b $\frac{10}{3}$ or $3\frac{1}{3}$ c $\frac{4}{17}$ d 0.4
4 $\frac{1}{2}$ and 0.5
5 $\frac{5}{6}$
6 a 2.5 and 3.5 is one possible answer
b 0.4×2.5 is one possible answer
c Not possible
7 $\sqrt{2} \times \sqrt{8}$ is a possible answer
8 π and $4 - \pi$ is a possible answer
9 There are many possible answers. You could just give the same answer as question 5.
10 a $\frac{1}{28}$ b $\frac{1}{2.8} = \frac{10}{28} = \frac{5}{14}$ c $\frac{8}{1} = 8$ d $5\frac{3}{4} = \frac{23}{4}$. The reciprocal is $\frac{4}{23}$

Notice that in part **b** you could use a calculator to get $1 \div 2.8 = 0.3571$ to 4 d.p.

This is only an approximate answer. For an exact answer you must use fractions.

Answers to Chapter 2

2.1 Equivalent fractions

Exercise 2A

1 a $\frac{8}{20}$ b $\frac{3}{12}$ c $\frac{15}{40}$
d $\times 6$, $\frac{12}{18}$ e $\times 3$, $\frac{9}{12}$ f $\times 5$, $\frac{25}{40}$
2 a $\frac{2}{3}$ b $\frac{4}{5}$ c $\frac{5}{7}$ d $\div 6$, $\frac{2}{3}$
e $25 \div 5$, $\frac{3}{5}$ f $30 \div 3$, $\frac{7}{10}$
3 a $\frac{2}{3}$ b $\frac{1}{3}$ c $\frac{2}{3}$ d $\frac{3}{4}$ e $\frac{1}{3}$
f $\frac{1}{2}$ g $\frac{7}{8}$ h $\frac{4}{5}$ i $\frac{1}{2}$ j $\frac{1}{4}$
4 a $\frac{1}{2}, \frac{2}{3}, \frac{5}{6}$ b $\frac{1}{2}, \frac{5}{8}, \frac{3}{4}$ c $\frac{2}{5}, \frac{1}{2}, \frac{7}{10}$
d $\frac{7}{12}, \frac{2}{3}, \frac{3}{4}$ e $\frac{1}{6}, \frac{1}{4}, \frac{1}{3}$ f $\frac{3}{4}, \frac{4}{5}, \frac{9}{10}$
5 a $\frac{1}{3} + \frac{1}{4} = \frac{4}{12} + \frac{3}{12} = \frac{7}{12}$
Explanations may involve ruling out other combinations
b $\frac{1}{2}$ as the smallest denominator is the biggest unit fraction
Diagrams may be used but must be based on equal sized area.
6 a $2\frac{1}{3}$ b $2\frac{2}{3}$ c $2\frac{1}{4}$
d $1\frac{3}{7}$ e $2\frac{2}{5}$ f $1\frac{2}{5}$
7 a $\frac{10}{3}$ b $\frac{35}{6}$ c $\frac{9}{5}$ d $\frac{37}{7}$ e $\frac{41}{10}$ f $\frac{17}{3}$
g $\frac{5}{2}$ h $\frac{13}{4}$ i $\frac{43}{6}$ j $\frac{29}{8}$ k $\frac{19}{3}$ l $\frac{89}{9}$
8 Students check their own answers.
9 $\frac{27}{4} = 6\frac{3}{4}$, $\frac{31}{5} = 6\frac{1}{5}$, $\frac{13}{2} = 6\frac{1}{2}$, so $\frac{27}{4}$ is the biggest since $\frac{1}{5}$ is less than $\frac{1}{2}$ and $\frac{3}{4}$ is greater than $\frac{1}{2}$
10 Any mixed number which is between 7.7272 and 7.9.
For example $7\frac{4}{5}$

2.2 Fractions and decimals

Exercise 2B

1 a $\frac{7}{10}$ b $\frac{2}{5}$ c $\frac{1}{2}$ d $\frac{3}{100}$ e $\frac{3}{50}$
f $\frac{13}{100}$ g $\frac{1}{4}$ h $\frac{19}{50}$ i $\frac{11}{20}$ j $\frac{16}{25}$
2 a 0.5 b 0.75 c 0.6 d 0.9 e 0.333
f 0.625 g 0.667 h 0.35 i 0.636 j 0.444
3 a 0.3, $\frac{1}{2}$, 0.6 b 0.3, $\frac{2}{5}$, 0.8 c 0.15, $\frac{1}{4}$, 0.35
d $\frac{7}{10}$, 0.71, 0.72 e 0.7, $\frac{3}{4}$, 0.8 f $\frac{1}{20}$, 0.08, 0.1
g 0.4, $\frac{1}{2}$, 0.55 h 1.2, 1.23, $1\frac{1}{4}$
4 Store A – $\frac{1}{3}$ (0.33) is greater than $\frac{1}{4}$ (0.25)

5 a $\frac{12}{30} = \frac{2}{5}$ b 0.4

6 $\frac{7}{8}$ (= 0.875)

7 $\frac{2}{3}$ (= 0.67)

2.3 Recurring decimals

Exercise 2C

1 a 0.333… or $0.\dot{3}$ b 0.75 c 0.8333… or $0.8\dot{3}$
d 0.222… or $0.\dot{2}$ e 0.65 f 0.8181… or $0.\dot{8}\dot{1}$
g 0.1875 h 0.916 66… or $0.91\dot{6}$

2 a 0.4666… or $0.4\dot{6}$ b 0.9333… or $0.9\dot{3}$

3 a 0.1111… b 0.1666… c 0.2777… d 0.0555…

4 $\frac{8}{9}$

5 $\frac{8}{33}$

6 $\frac{11}{30}$

7 $\frac{1}{12}$

8 $2\frac{7}{15}$

9 $0.\dot{2}3076\dot{9}$

10 a $0.\dot{0}\dot{9}$ b $0.\dot{1}\dot{8}$ c $0.\dot{2}\dot{7}$, $0.\dot{3}\dot{6}$ and $0.\dot{6}\dot{3}$

11 a $0.\dot{2}8571\dot{4}$ b $0.\dot{4}2857\dot{1}$ c $\frac{4}{7} = 0.\dot{5}7142\dot{8}$, $\frac{5}{7} = 0.\dot{7}1428\dot{5}$ and $\frac{6}{7} = 0.\dot{8}5714\dot{2}$

12 a $\frac{1}{5}$, $\frac{1}{8}$, $\frac{1}{10}$
b $\frac{1}{N}$ is a terminating decimal if the only prime factors of N are 2 or 5. Otherwise it is a recurring decimal.

2.4 Percentages, fractions and decimals

Exercise 2D

1 a $\frac{2}{25}$ b $\frac{1}{2}$ c $\frac{1}{4}$ d $\frac{7}{20}$ e $\frac{9}{10}$ f $\frac{3}{4}$

2 a 0.27 b 0.85 c 0.13 d 0.06 e 0.8 f 0.32

3 a $\frac{3}{25}$ b $\frac{2}{5}$ c $\frac{9}{20}$ d $\frac{17}{25}$ e $\frac{1}{4}$ f $\frac{5}{8}$

4 a 29% b 55% c 3% d 16% e 60% f 125%

5 a 28% b 30% c 95% d 34% e 27.5% f 87.5%

6 a 0.6 b 0.075 c 0.76 d 0.3125 e 0.05 f 0.125

7 a 63%, 83%, 39%, 62%, 77% b English

8 34%, 0.34, $\frac{17}{50}$; 85%, 0.85, $\frac{17}{20}$; 7.5%, 0.075, $\frac{3}{40}$; 45%, 0.45, $\frac{9}{20}$; 30%, 0.3, $\frac{3}{10}$; 67%, 0.67, $\frac{2}{3}$; 84%, 0.84, $\frac{21}{25}$; 45%, 0.45, $\frac{9}{20}$; 37.5%, 0.375, $\frac{3}{8}$

2.5 Calculating a percentage

Exercise 2E

1 a 0.88 b 0.3 c 0.25 d 0.08 e 1.15

2 a 78% b 40% c 75% d 5% e 110%

3 a \$45 b \$6.30 c 128.8 kg d 1.125 kg
e 1.08 h f 37.8 cm g \$0.12 h 2.94 m
i \$7.60 j 33.88 min k 136 kg l \$162

4 \$2410

5 a 86% b 215

6 8520

7 287

8 990

9 Mon: 816, Tue: 833, Wed: 850, Thu: 799, Fri: 748

10 a \$3.25 b 2.21 kg c \$562.80
d \$6.51 e 42.93 m f \$24

11 480 cm^3 nitrogen, 120 cm^3 oxygen

12 13

13 \$270

14 More this year as it was 3% of a higher amount than last year.

2.6 Increasing or decreasing quantities by a percentage

Exercise 2F

1 a 1.1 b 1.03 c 1.2 d 1.07 e 1.12

2 a \$62.40 b 12.96 kg c 472.5 g d 599.5 m
e \$38.08 f \$90 g 391 kg h 824.1 cm
i 253.5 g j \$143.50 k 736 m l \$30.24

3 \$29 425

4 1 690 200

5 a Caretaker: \$17 325, Driver: \$18 165, Supervisor: \$20 475, Manager: \$26 565
b 5% of different amounts is not a fixed amount. The more pay to start with, the more the increase (5%) will be.

6 \$411.95

7 193 800

8 575 g

9 918

10 60

11 TV: \$287.88, microwave: \$84.60, CD player: \$135.13, stereo: \$34.66

12 \$10

13 c Both the same as $1.05 \times 1.03 = 1.03 \times 1.05$

14 a Shop A, as $1.04 \times 1.04 = 1.0816$, so an 8.16% increase.

15 \$540.96

Exercise 2G

1 a 0.92 b 0.85 c 0.75 d 0.91 e 0.88

2 a \$9.40 b 23 kg c 212.4 g d 339.5 m
e \$4.90 f 39.6 m g 731 m h 83.52 g
i 360 cm j 117 min k 81.7 kg l \$37.70

3 \$5525

4 a 52.8 kg b 66 kg c 45.76 kg

5 Mr Patel \$176, Mrs Patel \$297.50, Sandeep \$341, Priyanka \$562.50

6 448

7 705

8 a 66.5 km/h b 73.5 km/h

9 No, as the total is \$101. She will save \$20.20, which is less than the \$25 it would cost to join the club.

10 10% off \$50 is \$45; 10% off \$45 is \$40.50; 20% off \$50 is \$40

11 \$765

12 $1.10 \times 0.9 = 0.99$ (99%)

13 Offer A gives 360 grams for \$1.40, i.e. 0.388 cents per gram. Offer B gives 300 grams for \$1.12, i.e 0.373 cents per gram, so Offer B is the better offer.
Or Offer A is 360 for 1.40 = 2.6 grams per cent, offer B is 300 for 1.12 = 2.7 grams per cent, so offer B is better.

2.7 One quantity as a percentage of another

Exercise 2H

1 **a** 25% **b** 60.6% **c** 46.3% **d** 12.5%
e 41.7% **f** 60% **g** 20.8% **h** 10%
i 1.9% **j** 8.3% **k** 45.5% **l** 10.5%

2 32%

3 6.5%

4 33.7%

5 **a** 49.2% **b** 64.5% **c** 10.6%

6 17.9%

7 4.9%

8 90.5%

9 **a** Brit Com: 20.9%, USA: 26.5%, France: 10.3%, Other 42.3%
b Total 100%, all imports

10 Nadia had the greater percentage increase.
Nadia: $(20 - 14) \times 100 \div 14 = 42.9\%$.
Imran: $(17 - 12) \times 100 \div 12 = 41.7\%$

11 Yes, as 38 out of 46 is over 80% (82.6%)

12 Vase 20% loss, radio 25% profit, doll 175% profit, toy train 64% loss

2.8 Simple interest and compound interest

Exercise 2I

1 7420 dollars

2 3600 dollars

3 4 years

4 **a** \$15 600 **b** \$16 224

5 **a** \$1272 **b** \$1348.32 **c** \$1429.22

6 **a** Amar 3200 \$, Mona 3328 \$ **b** Mona, 128 \$

7 **a** \$9528.13 **b** £1528.13

8 £3840

9 **a** Simple **b** 6.5%

10 **a** \$13 800 **b** \$15 870
c Student's own explanation

11 **a** 2652.25 and 5304.50 **b** £796.37

2.9 A formula for compound interest

Exercise 2J

1 \$2249.73

2 \$5681.15

3 **a** \$5071.50 **b** \$5591.33 **c** \$6164.44

4 **a** \$3589.07 **b** \$4458.69

5 \$4272.64

6 **a** \$3941.57 **b** \$441.57

7 8 years

8 The interest in the second five years will be more than the interest in the first five years. The missing number is $5000 \times 1.06^{10} = 8954.24$.

9 **a** \$15 000 **b** \$16 288.95

10 **a** \$1268.24
b The interest over the year is \$268.24.
This is $\frac{268.24}{1000} \times 100\% = 26.824\%$ of \$1000.

2.10 Reverse percentage

Exercise 2K

1 **a** 800 g **b** 250 m **c** 60 cm
d \$3075 **e** \$200 **f** \$400

2 80

3 T shirt: \$8.40, Tights: \$1.20, Shorts: \$5.20, Sweater: \$10.74, Trainers: \$24.80, Boots: \$32.40

4 \$833.33

5 \$300

6 240

7 537.63 dollars

8 4750 blue bottles

9 \$2585

10 \$1440

11 \$2450

12 95 dollars

13 \$140

14 \$945

15 \$1325

16 \$1300

17 Lee has assumed that 291.2 is 100% instead of 112%.
He rounded his wrong answer to the correct answer of \$260.

Answers to Chapter 3

3.1 Order of operations

Exercise 3A

1 **a** 11 **b** 6 **c** 10 **d** 12 **e** 11 **f** 13
g 11 **h** 12 **i** 12 **j** 4 **k** 13 **l** 3

2 **a** 16 **b** 2 **c** 10 **d** 10 **e** 6 **f** 18
g 6 **h** 15 **i** 9 **j** 12 **k** 3 **l** 8

3 **a** (4 + 1) **b** No brackets needed
c (2 + 1) **d** No brackets needed
e (4 + 4) **f** (16 – 4)

g No brackets needed
h No brackets needed
i (20 – 10)
j No brackets needed
k (5 + 5)
l (4 + 2)
m (15 – 5)
n (7 – 2)
o (3 + 3)
p No brackets needed
q No brackets needed
r (8 – 2)

4 No, correct answer is 5 + 42 = 47

5 a $2 \times 3 + 5 = 11$ b $2 \times (3 + 5) = 16$
c $2 + 3 \times 5 = 17$ d $5 - (3 - 2) = 4$
e $5 \times 3 - 2 = 13$ f $5 \times 3 \times 2 = 30$

6 $4 + 5 \times 3 = 19$
$(4 + 5) \times 3 = 27$. So $4 + 5 \times 3$ is smaller

7 $(5 - 2) \times 6 = 18$

8 $8 \div (5 - 3) = 4$

3.2 Choosing the correct operation

Exercise 3B

1 a 6000
b 5 cans cost \$1.95, so 6 cans cost \$1.95. 32 = (5 × 6) + 2. Cost is \$10.53.

2 a 288 b 16

3 a 38
b Coach price for adults = \$8, coach price for juniors = \$4, money for coaches raised by tickets = \$12 400, cost of coaches = \$12 160, profit = \$240

4 (39 × 20) + (90 × 30) = 1050 = \$10.50

5 (18.81...) Kirsty can buy 18 models.

6 (7.58 ...) Michaela must work for 8 weeks.

7 \$8.40 per year, 70 cents per copy

8 \$450

9 15

10 Gustav pays 2296.25 – 1840 = \$456.25

3.3 Finding a fraction of a quantity

Exercise 3C

1 a 18 b 10 c 18 d 28

2 a \$1800 b 128 g c 160 kg
d \$116 e 65 litres f 90 min

3 a $\frac{5}{8}$ of 40 = 25 b $\frac{3}{4}$ of 280 = 210
c $\frac{4}{5}$ of 70 = 56 d $\frac{5}{6}$ of 72 = 60

4 \$6080

5 \$31 500

6 52 kg

7 a 856 b 187 675

8 a \$50 b \$550

9 a \$120 b \$240

10 Lion Autos

11 Offer B

3.4 Adding and subtracting fractions

Exercise 3D

1 a $\frac{5}{7}$ b $\frac{7}{9}$ c $\frac{4}{5}$ d $\frac{6}{7}$

2 a $\frac{3}{7}$ b $\frac{1}{9}$ c $\frac{4}{11}$ d $\frac{7}{13}$

3 a $\frac{6}{8} = \frac{3}{4}$ b $\frac{4}{10} = \frac{2}{5}$ c $\frac{6}{9} = \frac{2}{3}$ d $\frac{2}{4} = \frac{1}{2}$

4 a $\frac{4}{8} = \frac{1}{2}$ b $\frac{4}{10} = \frac{2}{5}$ c $\frac{4}{6} = \frac{2}{3}$ d $\frac{8}{10} = \frac{4}{5}$

5 a $\frac{12}{10} = \frac{6}{5} = 1\frac{1}{5}$ b $\frac{9}{8} = 1\frac{1}{8}$ c $\frac{9}{8} = 1\frac{1}{8}$
d $\frac{13}{8} = 1\frac{5}{8}$ e $\frac{11}{8} = 1\frac{3}{8}$ f $\frac{7}{6} = 1\frac{1}{6}$
g $\frac{9}{6} = \frac{3}{2} = 1\frac{1}{2}$ h $\frac{5}{4} = 1\frac{1}{4}$

6 a $\frac{10}{8} = \frac{5}{4} = 1\frac{1}{4}$ b $\frac{6}{4} = \frac{3}{2} = 1\frac{1}{2}$
c $\frac{5}{5} = 1$ d $\frac{16}{10} = \frac{8}{5} = 1\frac{3}{5}$

7 a $\frac{5}{8}$ b $\frac{5}{10} = \frac{1}{2}$ c $\frac{1}{4}$ d $\frac{3}{8}$
e $\frac{1}{4}$ f $\frac{3}{8}$ g $\frac{4}{10} = \frac{2}{5}$ h $\frac{5}{16}$

Exercise 3E

1 a $\frac{8}{15}$ b $\frac{7}{12}$ c $\frac{3}{10}$ d $\frac{11}{12}$ e $\frac{7}{8}$ f $\frac{1}{2}$
g $\frac{1}{6}$ h $\frac{1}{20}$ i $\frac{1}{10}$ j $\frac{1}{8}$ k $\frac{1}{12}$ l $\frac{1}{3}$
m $\frac{1}{6}$ n $\frac{7}{9}$ o $\frac{5}{8}$ p $\frac{3}{8}$ q $\frac{1}{15}$ r $1\frac{13}{24}$
s $\frac{59}{80}$ t $\frac{22}{63}$ u $\frac{37}{54}$

2 a $3\frac{5}{14}$ b $10\frac{3}{5}$ c $2\frac{1}{6}$ d $3\frac{31}{45}$
e $4\frac{47}{60}$ f $\frac{41}{72}$ g $\frac{29}{48}$ h $1\frac{43}{48}$
i $1\frac{109}{120}$ j $1\frac{23}{30}$ k $1\frac{31}{84}$

3 $\frac{1}{20}$

4 a $\frac{1}{6}$ b 30, must be divisible by 2 and 3

3.5 Multiplying and dividing fractions

Exercise 3F

1 a $\frac{1}{6}$ b $\frac{1}{10}$ c $\frac{3}{8}$ d $\frac{3}{14}$ e $\frac{8}{15}$
f $\frac{1}{5}$ g $\frac{2}{7}$ h $\frac{3}{10}$ i $\frac{1}{2}$ j $\frac{2}{5}$

2 a $\frac{3}{32}$ b $\frac{3}{8}$ c $\frac{7}{20}$
d $\frac{16}{45}$ e $\frac{3}{5}$ f $\frac{5}{8}$

3 $\frac{1}{12}$

4 $\frac{3}{8}$

5 a $\frac{5}{12}$ b $2\frac{1}{12}$
c $6\frac{1}{4}$ d $2\frac{11}{12}$
e $3\frac{9}{10}$ f $3\frac{1}{3}$
g $12\frac{1}{2}$ h 30

6 $\frac{2}{5}$ of $6\frac{1}{2} = 2\frac{3}{5}$

Exercise 3G

1 a $\frac{3}{4}$ b $1\frac{2}{5}$ c $1\frac{1}{15}$ d $1\frac{1}{14}$ e 4
f 4 g 5 h $1\frac{5}{7}$ i $\frac{4}{9}$ j $1\frac{3}{5}$

2 18

3 40

4 15

5 16

6 a $2\frac{2}{15}$ b 38 c $1\frac{7}{8}$ d $\frac{9}{32}$ e $\frac{1}{16}$ f $\frac{256}{625}$

7 a $1\frac{1}{3}$ b $\frac{3}{4}$

Answers to Chapter 4

4.1 Introduction to directed numbers

Exercise 4A

1 a 0 °C b 5 °C c −2 °C d −5 °C e −1 °C

2 a 11 degrees Celsius b 9 degrees Celsius

3 8 degrees Celsius

4 38 degrees Celcius

5 a 2 degrees Celcius between Helsinki and Moscow
b 34 degrees Celcius between Dubai and Helsinki

4.2 Everyday use of directed numbers

Exercise 4B

1 −$5

2 −200 m

3 above

4 −5 h

5 −2 °C

6 − 70 km

7 +5 minutes

8 −5 km/h

9 −2

10 a −11 °C b 6 degrees Celsius

11 1.54 am

4.3 The number line

Exercise 4C

1 a < b > c < d < e > f <
g < h > i > j < k < l >

2 a < b < c < d > e < f <

3 a −5 −4 −3 −2 −1 0 1 2 3 4 5
c −25 −20 −15 −10 −5 0 5 10 15 20 25
d −10 −8 −6 −4 −2 0 2 4 6 8 10
e −50 −40 −30 −20 −10 0 10 20 30 40 50

4 6 °C −2 °C −4 °C 2 °C

5 a 1 or 0 or −1 or −2 are the possible answers **b** No solution
c Any integer larger than 2. That is 3 or 4 or 5 or …
d Any integer smaller than −3. That is −4 or −5 or −6 or …

4.4 Adding and subtracting directed numbers

Exercise 4D

1 a −2° b −3° c −2° d −3° e −2° f −3°
g 3 h 3 i −1 j −1 k 2 l −3
m −4 n −6 o −6 p −1 q −5 r −4
s 4 t −1 u −5 v −4 w −5 x −5

2 a 7 degrees Celsius b −6 °C

3 a 2 − 8
b 2 + 5 − 8 or 2 + 4 − 7 or 8 − 4 − 5 or 8 − 2 − 7 or 5 − 4 − 2
c 2 − 5 − 7 − 8
d 2 + 5 − 4 − 7 − 8

4 250 metres

Exercise 4E

1 a −8 b −10 c −11 d −3 e 2 f −5
g 1 h 4 i 7 j −8 k −5 l −11
m 11 n 6 o 8 p 8 q −2 r −1
s −9 t −5

2 a 10 degrees Celsius
b 7 degrees Celsius
c 9 degrees Celsius

3 a 2 b −3 c −5 d −7 e −10 f −20

4 a 2 b 4 c −1 d −5 e −11 f 8

5 a 13 b 2 c 5 d 4 e 11 f −2

6 a −10 b −5 c −2 d 4 e 7 f −4

7 a +6 + +5 = 11 b +6 + −9 = −3
c +6 − −9 = 15 d +6 − +5 = 1

8 It may not come on as the thermometer inaccuracy might be between 0° and 2° or 2° and 4°

9 −1 and 6

4.5 Multiplying and dividing directed numbers

Exercise 4F

1 a -15 b -14 c -24 d 6 e 14 f 2
g -2 h -8 i -4 j 3 k -24 l -10
m -18 n 16 o 36 p -4 q -12 r -4
s 7 t 25 u 18

2 a -9 b 16 c -3 d -32 e 18 f 18
g 6 h -4 i 20 j 16 k 8 l -48
m 13 n -13 o -8 p 0 q 16 r -42

3 a -2 b 30 c 15 d -27 e -7

4 a 4 b -9 c -3 d 6 e -4

5 a -9 b 3 c 1

6 a 16 b -2 c -12

7 a 24 b 6 c -4 d -2

8 For example: $1 \times (-12)$, -1×12, $2 \times (-6)$, $6 \times (-2)$, $3 \times (-4)$, $4 \times (-3)$

9 For example: $4 \div (-1)$, $8 \div (-2)$, $12 \div (-3)$, $16 \div (-4)$, $20 \div (-5)$, $24 \div (-6)$

10 -5×4, 3×-6, $-20 \div 2$, $-16 \div -4$

11 a 4 b 25 c 12 d 1

12

×	−2	3	−4
−5	10	−15	20
2	−4	6	−8
−6	12	−18	24

Answers to Chapter 5

5.1 Squares and square roots

Exercise 5A

1 a 49 b 100 c 1.44 d 6.25 e 256 f 400
g 9.61 h 20.25 i 9 j 64 k 0.25 l 0.25

2 a 3 and -3 b 10 and -10 c 11 and -11
d 1.2 and -1.2 e 20 and -20 f 3.5 and -3.5
g 1 and -1 h 100 and -100

3 a 5 b 6 c 10 d 7 e 8
f 1.5 g 5.5 h 1.2 i 20 j 0.5

4 a 81 b 40 c 100 d 14 e 36
f 15 g 49 h 12 i 25 j 21

5 a 24 b 31 c 45 d 40 e 67
f 101 g 3.6 h 6.5 i 13.9 j 22.2

6 $\sqrt{50}$, 3^2, $\sqrt{90}$, 4^2

7 a 6^2 is 36 and 7^2 is 49; 40 is between 36 and 49
b 6.3245553……

8 4 and 5

9 a 8 and 9 b 9 and 10
c 12 and 13 d 15 and 16

10 $\sqrt{324} = 18$

11 15

5.2 Cubes and cube roots

Exercise 5B

1 a 8 b 27 c 512 d 1000
e 1.331 f 15.625 g -27 h -125
i 8000 j 68.921 k -68.921

2 a 2 b 5 c 9 d 1 e 3
f -3 g 10 h 1.5 i 4.5 j 0.5

3 a 5 and 6 b 6 and 7
c 7 and 8 d -8 and -7

4 2^3 because it equals 8, the rest equal 9

5 One possible answer is $8^2 = 4^3$

6 $\sqrt[3]{2000}$, $\sqrt{225}$, 2.5^3, 4^2

7

Number	Square	Cube
10	100	1000
5	25	125
4	16	64
11	121	1331
9	81	729

8 0.8^3, 0.8^2, $\sqrt{0.8}$, $\sqrt[3]{0.8}$.

5.3 More powers and roots

Exercise 5C

1 a 243 b 2401 c 1 000 000 d 256

2 a 0 b 118 c 513

3 a 2592 b 227 c or 0.3789 to 4 d.p.

4 LHS = 31; RHS = 32 − 1 = 31

5 a 5 b 11 c 3 d 20

6 a 20 000 b or 1.125

7 $2^3 \times \sqrt[3]{8} = 16$ $3^2 \times \sqrt[10]{1024} = 18$ $\sqrt[3]{64} \times \sqrt[3]{125} = 20$
$\sqrt[4]{14641} \times \sqrt[6]{64} = 22$

8 a 32 b 12.5

9 a $8\frac{1}{3}$ b $\frac{7}{8}$

10 a 3 b 8 c 5

11 a 3 b 4 c 8

12 a 15 625 b 1 953 125

13 a $x = 4$ and $y = 2$ is one possible pair.
b $x = 8$ and $y = 4$ is a second possible pair; $x = 12$ and $y = 6$ is a third possible pair

14 a 1024 b 1 048 576 c 32 d 4
e 2

5.4 Exponential growth and decay

Exercise 5D

1 **a** i 10 million **ii** 20 million **iii** 40 million
b i 15 million **ii** 45 million **iii** 135 million

2 **a** 6000 **b** 9000 **c** 13 500 **d** 20 250

3 **a** 6000 **b** 1500 **c** 375

4 **a** 4800 **b** 768 **c** 123

5 21.8, 23.8, 25.9

6 18.2, 16.6, 15.1

7 **a** \$6312 **b** 6 years

8 **a** \$100 000 **b** \$195 313

9 The correct value is $150\,000 \times 1.2^5 = 373\,248$

10 **a** \$20 **b** \$20 480

11 **a** 1185 **b** 351

12 272 million

13 **a** \$18 **b** \$32 **c** \$340 **d** \$11568

Answers to Chapter 6

6.1 Inequalities

Exercise 6A

1 **a** > **b** < **c** < **d** =
e = **f** > **g** > **h** <

2 $\frac{1}{3} < \frac{1}{2} < \frac{3}{5}$

3 **a** 4,5,6 **b** 1,2 **c** 6 **d** 1,2,3,4,5
e 2,3,4 **f** 4,5 **g** 1,2,3 **h** 6

4 **a** underweight **b** overweight
c normal **d** normal

5 20, 22, 26, 28

6 **a** 49 **b** 45
c 3,6,9 **d** 16,17,18,19,20

7 **a** true **b** false **c** true
d true **e** false **f** true

8 **a** 6,7,8 **b** 26, 27, 28 **c** −7, −6, −5, −4
d −2, −1, 0, 1 **e** there are none **f** 33

9 **a** $N \geqslant 8$ or $N \leqslant -8$ **b** $N \geqslant 4$ **c** $N < -4$ **d** no solution

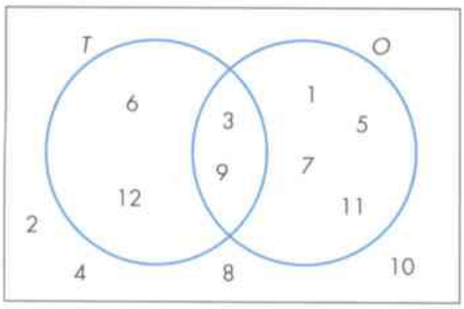
6.2 Sets and Venn diagrams

Exercise 6B

1 **a** The elements can be listed in any order.
i {2, 3, 12, 4, 10, 11} **ii** {4, 5, 11, 10}
iii {4, 10, 11}
b i 6 **ii** 7 **iii** 12
c The first 12 natural numbers.

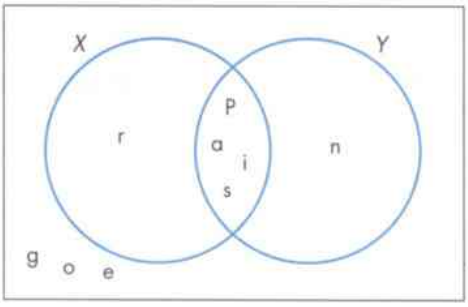
2 **a** i 7 **ii** 5 **iii** 2 **iv** 10
b a, p, r, I or n **c** g or d

3



4 **a** 100 is in A but not in B **b** 6
c multiples of 6.

5



6 **a** i $A = \{4, 22, 2, 20, 12, 10, 28, 30\}$
ii $B = \{2, 20, 12, 6, 26, 16, 18\}$
iii $A \cap B = \{10, 12, 28, 30\}$
iv $A \cap C = \{2, 20, 12\}$ $A \cap B \cap C = \{12\}$
b i 7 **ii** 10 **iii** 12 **iv** 13 **c** The even numbers up to 30.

7 **a** i $X = \{1, 2, 5, 10\}$ **ii** $Y = \{1, 2, 3, 4, 6, 8, 12, 24\}$
b i factors of 10 **ii** factors of 24
c i {1, 2, 3, 6} **ii** {1, 2} **iii** {1, 2, 3}
d i 8 **ii** 10 **iii** 12

8 **a** i {10} **ii** {7, 15} **iii** none
b i 6 **ii** 9 **iii** 7
iv 14 **v** 12

9 **a** i {4, 8, 12} **ii** {2, 4, 6, 8, 10, 12, 14}
iii {10} **iv** {4, 8, 12}
b i 1 **ii** 0

10 **a** 15 **b** 21 **c** 35 **d** 105

11 **a** 8 **b** 10 **c** {1, 2, 4} **d** {2, 7}

12 **a** i {a, b, c, d, e, f} **ii** {d, e, f}
iii {b, c, d, e, f, g} **iv** {b, d, e}
b 3

13 **a** {1, 2, 3, 4, 5, 6, 7, 8, 10, 13} **b** {1, 2, 4, 6, 7, 8, 10, 13}
c {1, 4, 7, 10} **d** {1, 2, 4, 6, 7, 10, 13}

6.3 More about Venn diagrams

Exercise 6C

1 **a** {8, 11, 12, 14, 15} **b** {6, 9, 14, 15} **c** {14, 15}
d {8, 11, 12} **e** {6, 9}

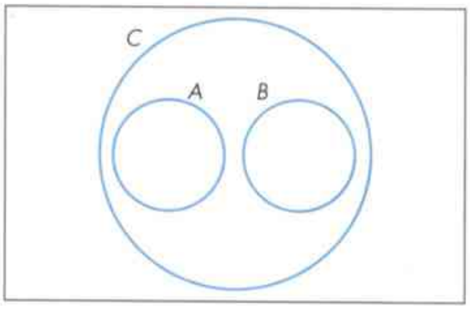
2 **a** $6 \in B'$ **b** $7 \notin A'$ **c** $8 \in (A \cap B)'$
d $9 \in A \cup B'$ **e** $9 \notin A' \cup B$ **f** $9 \notin (A \cup B)'$

3 **a** **i** $X \cap Y'$ is 3 **ii** $(X \cap Y)'$ is 2 **iii** $X \cup Y'$ is 4

iv $(X \cup Y)'$ is 1 **v** $X' \cap Y'$ is 1 **vi** $X' \cup Y'$ is 2

b $(X \cap Y)' = X' \cup Y'$ because they have the same diagram. Also $(X \cup Y)' = X' \cap Y'$

4 **a**



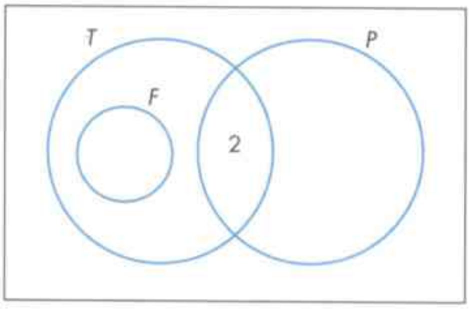
b C

5 $n(A) + n(B)$ counts the elements in the intersection twice but $n(A \cup B)$ only counts them once. This means that $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

If $n(A \cup B) = n(A) + n(B)$ there are no elements in the intersection. Hence $A \cap B = \varnothing$

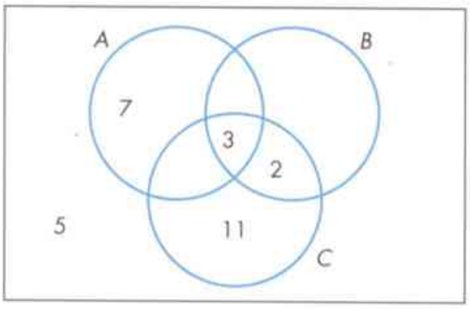
6 $x = 2$ because 2 is the only even prime number.

b



c T = {even numbers} so T' = {odd numbers}

7

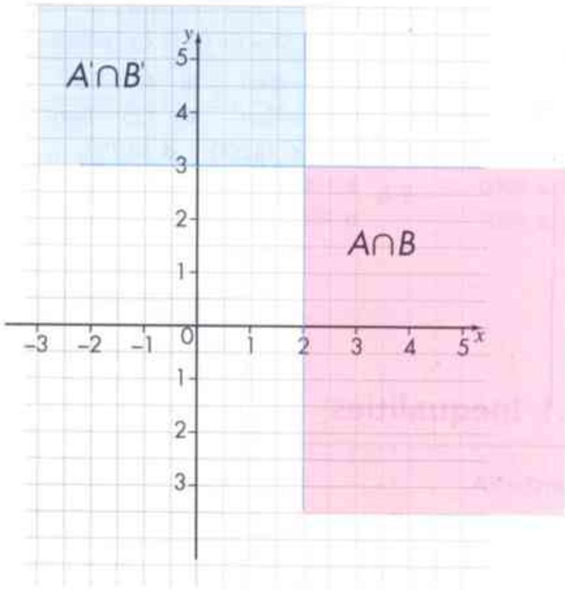


8 **a** $C \subseteq B$ is true **b** $A \cap C$ is true

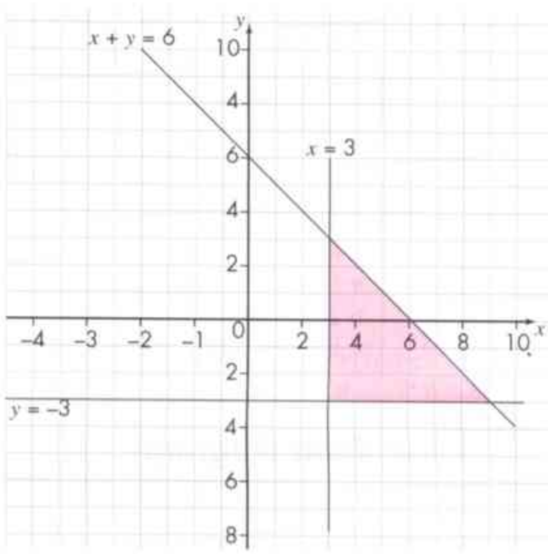
c It is false; $C \cup B = B$ **d** It is false; $B \cap C = C$

e It is false; $A' \cap C = C$ **f** It is false; $C' \cup A' = \xi$

9 a and b

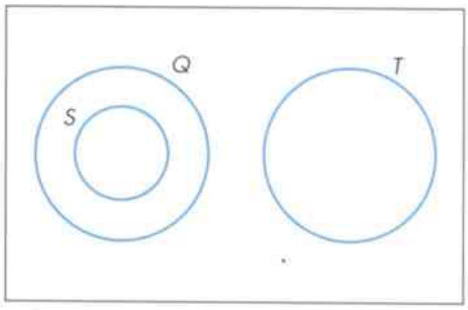


10



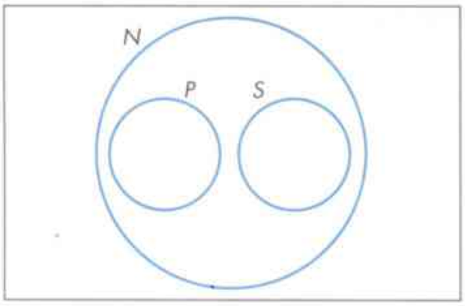
b Any coordinates of the form (0, c) where $-3 \leqslant c \leqslant 6$

11 **a**



b $R \cap T$ = {equilateral triangles}

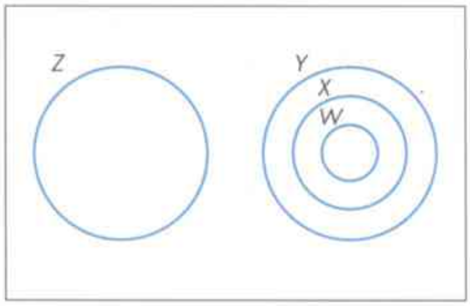
12 a



b No prime number is a square number and so $S \cap P = \varnothing$

c The smallest natural number that is not prime or square is 6.

13 A Venn diagram shows the sets



a $W \cap Y = W$ b $W \cup Y = Y$ c $X' \cap Z = Z$ d $Z' \cup Y = Z'$

Answers to Chapter 7

7.1 Ratio

Exercise 7A

1 a 1 : 3 b 1 : 4 c 2 : 3 d 2 : 1
e 2 : 5 f 2 : 5 g 5 : 8 h 5 : 1

2 a 8 : 1 b 12 : 1 c 5 : 6 d 1 : 24
e 48 : 1 f 5 : 2 g 3 : 8 h 1 : 5

3 $\frac{7}{10}$

4 $\frac{10}{25} = \frac{2}{5}$

5 a $\frac{2}{5}$ b $\frac{3}{5}$

6 a $\frac{7}{10}$ b $\frac{3}{10}$

7 a $\frac{1}{2}$ b $\frac{7}{20}$ c $\frac{3}{20}$

8 3 : 1

Exercise 7B

1 a 160 g, 240 g b 80 kg, 200 kg
c 150, 350 d 950 m, 50 m
e 175 min, 125 min f \$20, \$30, \$50
g \$36, \$60, \$144 h 50 g, 250 g, 300 g
i \$1.40, \$2, \$1.60 j 120 kg, 72 kg, 8 kg

2 a 175 b 30%

3 a 28 b 42

4 21

5 Joshua \$2500, Aicha \$3500, Mariam \$4000

6 a 1 : 400 000 b 1 : 125 000 c 1 : 250 000
d 1 : 25 000 e 1 : 20 000 f 1 : 40 000
g 1 : 62 500 h 1 : 10 000 i 1 : 60 000

7 a 1 : 1 000 000 b 47 km c 8 mm

8 a 1 : 250 000 b 2 km c 4.8 cm

9 a 1 : 20 000 b 0.54 km c 40 cm

10 a 1 : 1.6 b 1 : 3.25 c 1 : 1.125
d 1 : 1.44 e 1 : 5.4 f 1 : 1.5
g 1 : 4.8 h 1 : 42 i 1 : 1.25

11 c 1 : 250 000 At this scale 134 km is 53.6 cm which is a sensible size. The others are too small (5.36 mm or 5.36 cm) or too large (5.36 m).

Exercise 7C

1 a 3 : 2 b 32 c 80

2 1000 g

3 10 125

4 a 14 min b 75 min

5 a 11 pages b 32%

6 Ren \$2040, Shota \$2720

7 a lemonade 20 litres, ginger 0.5 litres
b This one, one-thirteenth is greater than one-fiftieth.

7.2 Increases and decreases using ratios

Exercise 7D

1 a 600 b 300 c 2000 d 350 e 240 f 220

2 a 20 b 60 c 8 d 56 e 16 f 64

3 a 160 cm by 120 cm b 60 cm by 45 cm

4 12.5 cm by 15 cm b 3 : 2

5 a 9 cm by 15 cm b 6.75 cm by 11.25 cm

6 a \$7500 b 50% c \$9375 d 25%
e 87.5% f One way is to cancel 9375 : 5000

7 a One possible answer is to say $\frac{6}{5} = 1.2$ and this is the multiplier for a 20% increase
b 11:10 c 9:10

8 100

9 40 cm^3

7.3 Speed

Exercise 7E

1 18 km/hour

2 440 kilometres

3 52.5 km/hour

4 11.50 am

5 500 s

6 a 75 km/hour b 6.5 hours c 175 km d 240 km
e 64 km/h f 325 km g 4.3 hours (4 h 18 min)

7 a 7.75 h b 85.2 km/hour
8 a 2.25 h b 157.5 km
9 a 1.25 h b 1 h 15 min
10 a 48 km/hour b 6 h 40 min
11 a 120 km b 48 km/h
12 a 30 min b 12 km/h
13 a 10 m/s b 3.3 m/s c 16.7 m/s d 41.7 m/s
e 20.8 m/s
14 a 90 km/h b 43.2 km/h c 14.4 km/h d 108 km/h
e 1.8 km/h
15 a 64.8 km/h b 28 s c 8.07 (37 min journey)
16 a 6.7 m/s b 66 km c 5 minutes d 133.3 metres
17 6.6 minutes

7.4 Rates

Exercise 7F

1 a 3.5 cm b 20 days
2 a 15 litres b 20 seconds
3 a 10.59 g/cm^3 b 50 cm^3
4 1600 days (4.38 years)
5 a 0.5 mm/year b 12.5 mm c 40 years
6 a 4.44 cm^3 to 3 d.p. b 360 g
c No. The density is 8 g/cm^3, not 9 g/cm^3
7 a 62.5 Pa or N/m^2 b 250 Pa or N/m^2
c 187.5 Pa or N/m^2
8 a 31.8 litres b 5.3 litres c 1.06 d 0.106
e 943 km
9 a 14.5 kg b 2900 kg c About 8 kg/day
10 724 N

7.5 Direct proportion

Exercise 7G

1 60 g
2 $5.22
3 45
4 $6.72
5 a $312.50 b 8
6 a 56 litres b 350 km
7 a 300 kg b 9 weeks
8 40 s
9 a i 100 g margarine, 200 g sugar, 250 g flour, 150 g ground rice
ii 150 g margarine, 300 g sugar, 375 g flour, 225 g ground rice
iii 250 g margarine, 500 g sugar, 625 g flour, 375 g ground rice
b 24
10 Pieter's shop as I can buy 24. At Paulo's shop I can only buy 20.

7.6 Inverse proportion

Exercise 7H

1 20 minutes
2 16
3 a 36 b 48
4 $10.80
5 a 16 days b 20
6 The missing times are 5 hours, 1 hour 40 minutes, 1 hour 15 minutes
7 a 6 b 15
8 $8

Answers to Chapter 8

8.1 Rounding whole numbers

Exercise 8A

1 a 20 b 60 c 80 d 50 e 100
f 20 g 90 h 70 i 10 j 30
2 a 200 b 600 c 800 d 500 e 1000
f 100 g 600 h 400 i 1000 j 1100
3 a 2000 b 6000 c 8000 d 5000
e 10 000 f 1000 g 6000 h 3000
i 9000 j 2000
4 a True b False c True d True e True f False
5 a Highest Germany, lowest Italy
b 36 000, 43 000, 25 000, 29 000
c 25 499 and 24 500
6 a 375 b 25 (350 to 374 inclusive)
7 A number between 75 and 84 inclusive added to a number between 45 and 54 inclusive with a total not equal to 130, for example 79 + 49 = 128

8.2 Rounding decimals

Exercise 8B

1 a 4.8 b 3.8 c 2.2 d 8.3 e 3.7
f 46.9 g 23.9 h 9.5 i 11.1 j 33.5
2 a 5.78 b 2.36 c 0.98 d 33.09 e 6.01
f 23.57 g 91.79 h 8.00 i 2.31 j 23.92
3 a 4.6 b 0.08 c 45.716 d 94.85 e 602.1
f 671.76 g 7.1 h 6.904 i 13.78 j 0.1
4 a 8 b 3 c 8 d 6 e 4
f 7 g 2 h 47 i 23 j 96
5 3 + 9 + 6 + 4 = 22 dollars
6 3, 3.46, 3.5
7 4.7275 or 4.7282

8.3 Rounding to significant figures

Exercise 8C

1 a 50000 b 60000 c 30000 d 90000
e 90000 f 0.5 g 0.3 h 0.006
i 0.05 j 0.0009 k 10 l 90
m 90 n 200 o 1000

2 a 56000 b 27000 c 80000 d 31000
e 14000 f 1.7 g 4.1 h 2.7
i 8.0 j 42 k 0.80 l 0.46
m 0.066 n 1.0 o 0.0098

3 a 60000 b 5300 c 89.7 d 110
e 9 f 1.1 g 0.3 h 0.7
i 0.4 j 0.8 k 0.2 l 0.7

4 a 65, 74 b 95, 149 c 950, 1499

5 Satora 750, 849, Nimral 1150, 1249, Korput 164 500, 165 499

6 One, because there could be 450 then 449.

7 Vashti has rounded to 2 significant figures or nearest 10 000.

8.4 Upper and lower bounds

Exercise 8D

1 a 6.5 and 7.5 b 115 and 125
c 3350 and 3450 d 49.5 and 50.5
e 5.50 and 6.50 f 16.75 and 16.85
g 15.5 and 16.5 h 14 450 and 14 550
i 54.5 and 55.5 j 52.5 and 57.5

2 a $5.5 \leqslant$ length in cm < 6.5
b $16.5 \leqslant$ mass in kg < 17.5
c $31.5 \leqslant$ time in minutes < 32.5
d $237.5 \leqslant$ distance in km < 238.5
e $7.25 \leqslant$ distance in m < 7.35
f $25.75 \leqslant$ mass in kg < 25.85
g $3.35 \leqslant$ time in hours < 3.45
h $86.5 \leqslant$ mass in g < 87.5
i $4.225 \leqslant$ distance in mm < 4.235
j $2.185 \leqslant$ mass in kg < 2.195
k $12.665 \leqslant$ time in minutes < 12.675
l $24.5 \leqslant$ distance in metres < 25.5
m $35 \leqslant$ length in cm < 45
n $595 \leqslant$ mass in g < 605
o $25 \leqslant$ time in minutes < 35
p $995 \leqslant$ distance in metres < 1050
q $3.95 \leqslant$ distance in metres < 4.05
r $7.035 \leqslant$ mass in kg < 7.045
s $11.95 \leqslant$ time in seconds < 12.05
t $6.995 \leqslant$ distance in metres < 7.005

3 a 7.5, 8.5 b 25.5, 26.5
c 24.5, 25.5 d 84.5, 85.5
e 2.395, 2.405 f 0.15, 0.25
g 0.055, 0.065 h 250 g, 350 g
i 0.65, 0.75 j 365.5, 366.5
k 165, 175 l 205, 215

4 C: The chain and distance are both any value between 29.5 and 30.5 metres, so there is no way of knowing if the chain is longer or shorter than the distance.

5 2 kg 450 grams

6 a <65.5 g b 64.5 g
c <2620 g d 2580 g

8.5 Upper and lower bounds for calculations

Exercise 8E

1 65 kg and 75 kg

2 a 12.5 kg b 20

3 9 kg 53.5 – 44.5

4 a $26 \text{ cm} \leqslant$ perimeter $< 30 \text{ cm}$
b $25.6 \text{ cm} \leqslant$ perimeter $< 26.0 \text{ cm}$
c $50.5 \text{ cm} \leqslant$ perimeter $< 52.7 \text{ cm}$

5 a $38.25 \text{ cm}^2 \leqslant$ area $< 52.25 \text{ cm}^2$
b $37.1575 \text{ cm}^2 \leqslant$ area $< 38.4475 \text{ cm}^2$
c $135.625 \text{ cm}^2 \leqslant$ area $< 145.225 \text{ cm}^2$

6 $79.75 \text{ m}^2 \leqslant$ area $< 100.75 \text{ m}^2$

7 $216.125 \text{ cm}^3 \leqslant$ volume $< 354.375 \text{ cm}^3$

8 12.5 metres

9 Yes, because they could be walking at 4.5 km/h and 2.5 km/h meaning that they would cover 4.5 km + 2.5 km = 7 km in 1 hour

10 $20.9 \text{ m} \leqslant$ length $< 22.9 \text{ m}$ (3 sf)

11 a $14.65 \text{ s} \leqslant$ time $< 14.75 \text{ s}$
b $99.5 \text{ m} \leqslant$ length $< 100.5 \text{ m}$
c 6.86 m/s (3 sf)

12 $14 \text{ s} \leqslant$ time $< 30 \text{ s}$

13 337.75 and 334.21

14 177.3 and 169.4

Answers to Chapter 9

9.1 Standard form

Exercise 9A

1 a 250 b 34.5 c 0.00467 d 34.6
e 0.020789 f 5678 g 246 h 7600
i 897 000 j 0.008 65 k 60 000 000 l 0.000567

2 a 2.5×10^2 b 3.45×10^{-1} c 4.67×10^4
d 3.4×10^9 e 2.078×10^{10} f 5.678×10^{-4}
g 2.46×10^3 h 7.6×10^{-2} i 7.6×10^{-4}
j 9.99×10^{-1} k 2.3456×10^2 l 9.87654×10^1
m 6×10^{-4} n 5.67×10^{-3} o 5.60045×10^1

3 2.7797×10^4

4 $3.211\,97 \times 10^5$, $4.491\,863 \times 10^6$

5 1.298×10^7, 2.997×10^9, 9.3×10^4

6 100

7 61.8 kilometres.

8 7.78×10^8; 5.8×10^7; 5.92×10^9

9.2 Calculating with standard form

Exercise 9B

1 a 5.67×10^3 b 6×10^2 c 3.46×10^{-1}
d 7×10^{-4} e 5.6×10^2 f 6×10^5
g 7×10^3 h 1.6 i 2.3×10^7

2 a 1.08×10^8 b 4.8×10^6 c 1.2×10^9
d 1.08 e 6.4×10^2 f 1.2×10^1
g 2.88 h 2.5×10^7 i 8×10^{-6}

3 a 2.7×10 b 1.6×10^{-2} c 2×10^{-1}
d 4×10^{-8} e 2×10^5 f 6×10^{-2}

4 2×10^{13}, 1×10^{-10}, mass = 2×10^3 g

5 a (2^{63}), 9.2×10^{18} grains
b $2^{64} - 1 = 1.8 \times 10^{19}$

6 a 1.0×10^8 sq km
b 31%

7 3.80×10^7 sq km

8 5×10^4

9 2.3×10^5

10 4.55×10^8 kg or 455 070 tonnes

11 a 100 000 000 (100 million) b 1.4%

12 a 2.048×10^6 b 4.816×10^6

13 9.41×10^4

14 Any value from 1.00000001×10^8 to 1×10^9 (excluding 1×10^9), i.e. any value of the form $a \times 10^8$ where $1 < a < 10$

15 a India b India and Jamaica
c 2.2×10^7 d 21 or 22 e 480

16 a Togo b Sri Lanka
c Sri Lanka d Russian Federation
e $\frac{1}{261}$

Answers to Chapter 10

10.1 Units of measurement

Exercise 10A

1 a metres b kilometres
c millimetres d kilograms or grams
e litres f tonnes
g millilitres h metres
i kilograms j millimetres

2 Check individual answers.

3 The 5 metre since his height is about 175 cm, the lamp post will be about 525 cm

10.2 Converting between metric units

Exercise 10B

1 a 1.25 m b 8.2 cm c 0.55 m d 4.2 kg
e 5.75 t f 8.5 cl g 0.755 kg h 0.8 l
i 2 l j 1.035 m^3 k 0.53 m^3 l 34 000 m

2 a 3400 mm b 135 mm c 67 cm d 640 m
e 2400 ml f 590 cl g 3750 kg h 0.00094 l
i 2160 cl j 15 200 g k 14 000 l l 0.19 ml

3 He should choose the 2000 mm × 15 mm × 20 mm

4 1 000 000

5 400 hours

6 7.5×10^9

10.3 Time

Exercise 10C

1 a 1 hour 10 minutes; 2 hours 3 minutes; 2 hours 9 minutes; 1 hour 45 minutes
b the 0900

2 a 9:45am, 10:36am, 1:33pm, 4:49pm
b 3 hours and 48 minutes, 6 hours and 13 minutes

3 a 1605 b 0715 c 6 hours 45minutes

4 a 1050 b 1635 c 5 hours 45 minutes

5 a 1210 b 2hours 50 minutes

6 a 12 minutes b 40 minutes c 54 minutes

7 1 hour 13 minutes

8 1415

9 0715 the next day

10 a 0330 b 0600

10.4 Currency conversions

Exercise 10D

1 3197.41

2 164

3 The missing values are 4.30, 7.76, 38.78, 193.88, 387.75, 775.50

4 43.01

5 a 224.91 b 172.74

6 a i 349.83 ii 24692 iii 432.90
b 54000 yen, 500 euros, 650 dollars

7 a 2391.38 b 3489.75
c Taiwan dollar d 1.4593

8 a 74.7755 b 0.14747

10.5 Using a calculator efficiently

Exercise 10E

1 a 144 b 108

2 a 12.54 b 27.45

3 a 196.48 b 1.023 c 0.236 d 4.219

4 a 3.58 b 6

5 a 497.952 b 110.978625

6 a 3.12 b 0.749 c 90.47
d 184.96 e 5.56 f 27.52

Answers to Chapter 11

11.1 The language of algebra

Exercise 11A

1 **a** $x+2$ **b** $x-6$ **c** $x+k$ **d** $x-t$
e $x+3$ **f** $d+m$ **g** $b-y$ **h** $p+t+w$
i $8x$ **j** hj **k** $x \div 4$ or $\frac{x}{4}$ **l** $2 \div x$ or $\frac{2}{x}$
m $y \div t$ or $\frac{y}{t}$ **n** wt **o** a^2 **p** g^2

2 **a** $x+3$ years **b** $x-4$ years

3 $F = 2C + 30$

4 Rule **c**

5 **a** $3n$ **b** $3n+3$ **c** $n+1$ **d** $n-1$

6 Anil: $2n$, Reza: $n+2$, Dale: $n-3$, Chen: $2n+3$

7 **a** \$4 **b** \$$(10-x)$ **c** \$$(y-x)$ **d** \$$2x$

8 **a** \$75 **b** \$$15x$ **c** \$4A **d** \$$Ay$

9 $(A-B)$ dollars

10 \$$A \div 5$ or \$$\frac{A}{5}$.

11 **a** Dad: $(72+x)$ years, me: $(T+x)$ years **b** 31 years

12 **a** $T \div 2$ or $\frac{T}{2}$ **b** $T \div 2 + 4$ or $\frac{T}{2} + 4$ **c** $T - x$

13 **a** $8x$ **b** $12m$ **c** $18t$

14 Andrea: $3n-3$, Barak: $3n-1$, Ahmed: $3n-6$ or $3(n-2)$, Dina: 0, Emma: $3n - n = 2n$. Hana: $3n - 3m$

15 For example, $2 \times 6m$, $1 \times 12m$, $6m + 6m$, etc.

11.2 Substitution into formulae

Exercise 11B

1 **a** 8 **b** 17 **c** 32

2 **a** 3 **b** 11 **c** 43

3 **a** 9 **b** 15 **c** 29

4 **a** 9 **b** 5 **c** −1

5 **a** 13 **b** 33 **c** 78

6 **a** 10 **b** 13 **c** 58

7 **a** \$4 **b** 13 km **c** Yes, the fare is \$5.00

8 **a** $2 \times 8 + 6 \times 11 - 3 \times 2 = 76$
b $5 \times 2 - 2 \times 11 + 3 \times 8 = 12$

9 Any values such that $lw = \frac{1}{2}bh$ or $bh = 2lw$

10 **a** 32 **b** 64 **c** 160

11 **a** 6.5 **b** 0.5 **c** −2.5

12 **a** 2 **b** 8 **c** −10

13 **a** 3 **b** 2.5 **c** −5

14 **a** 6 **b** 3 **c** 2

15 **a** 12 **b** 8 **c** $1\frac{1}{2}$

16 **a** $\frac{1050}{n}$ **b** \$925

17 **a** **i** odd **ii** odd **iii** even **iv** odd
b Any valid expression such as $xy + z$

18 **a** \$20
b **i** −\$40 **ii** Delivery cost will be zero. **c** 40 kilometres

11.3 Rearranging formulae

Exercise 11C

1 $k = \frac{T}{3}$

2 $y = X + 1$

3 $p = 3Q$

4 $r = \frac{A-9}{4}$

5 $n = \frac{W+1}{3}$

6 **a** $m = p - t$
b $t = p - m$

7 $m = gv$

8 $m = \sqrt{t}$

9 $r = \frac{C}{2\pi}$

10 $b = \frac{A}{h}$

11 $l = \frac{P-2w}{2}$

12 $p = \sqrt{m-2}$

13 **a** $-40 - 32 = -72$, $-72 \div 9 = -8$, $5 \times -8 = -40$
b $68 - 32 = 36$, $36 \div 9 = 4$, $4 \times 5 = 20$
c student's own demonstration

14 **a** $a = \frac{v-u}{t}$ **b** $t = \frac{v-u}{a}$

15 $d = \sqrt{\frac{4A}{\pi}}$

16 **a** $n = \frac{W-t}{3}$ **b** $t = W - 3n$

17 **a** $y = \frac{x+w}{5}$ **b** $w = 5y - x$

18 $p = \sqrt{\frac{k}{2}}$

19 **a** $t = u^2 - v$ **b** $u = \sqrt{v+t}$

20 **a** $m = k - n^2$ **b** $n = \sqrt{k-m}$

21 $r = \sqrt{\frac{T}{5}}$

22 **a** $w = K - 5n^2$ **b** $n = \sqrt{\frac{K-w}{5}}$

11.4 More complicated formulae

Exercise 11D

1 **a** 2.5 **b** $a = \sqrt{c^2 - b^2}$

2 **a** 60 **b** $a = \frac{2(s-ut)}{t^2}$

3 **a** $b = ac - 2$ **b** $c = \frac{b+2}{a}$

4 $t = \frac{r}{p} + 3$

5 $e = \left(\frac{12}{d} - 1\right)^2$

6 **a** 5 **b** $u = \sqrt{v^2 - 2as}$ **c** $s = \frac{v^2 - u^2}{2a}$

7 **a** $L = \left(\frac{T}{2\pi}\right)^2 G$ **b** Student's proof

8 **a** $R = \sqrt{\frac{D + \pi r^2}{\pi}}$ **b** $r = \sqrt{\frac{\pi R^2 - D}{\pi}}$ **c** $\pi = \frac{D}{R^2 - r^2}$

9 **a** $x = 5$ or -5 **b** $x = \sqrt{\frac{11 + 4y^2}{3}}$ **c** $y = \sqrt{\frac{3x^2 - 11}{4}}$

10 **a** $a = \left(\frac{T}{2}\right)^2 (c + 3)$ **b** $c = a\left(\frac{2}{T}\right)^2 - 3$

11 $T = \frac{b^2 + c^2 - a^2}{2bc}$

12 **a** 12 **b** $f = \frac{uv}{u + v}$ **c** $u = \frac{fv}{v - f}$ **d** $v = \frac{fu}{u - f}$

Answers to Chapter 12

12.1 Simplifying expressions

Exercise 12A

1 **a** $6t$ **b** $15y$ **c** $8w$ **d** $5b^2$ **e** $2w^2$ **f** $8p^2$ **g** $6t^2$ **h** $15t^2$ **i** $2mt$ **j** $5qt$ **k** $6mn$ **l** $6qt$ **m** $10hk$ **n** $21pr$

2 **a** All except $2m \times 6m$ **b** 2 and 0

3 $4x$ cm

4 **a** y^3 **b** $3m^3$ **c** $4t^3$ **d** $6n^3$ **e** t^4 **f** h^5 **g** $12n^5$ **h** $6a^7$ **i** $4k^7$ **j** t^3 **k** $12d^3$ **l** $15p^6$ **m** $3mp^2$ **n** $6m^2n$ **o** $8m^2p^2$

Exercise 12B

1 **a** \$$t$ **b** \$$(4t + 3)$

2 **a** $10x + 2y$ **b** $7x + y$ **c** $6x + y$

3 **a** $5a$ **b** $6c$ **c** $9e$ **d** $6f$ **e** $4j$ **f** $3q$ **g** 0 **h** $-w$ **i** $6x^2$ **j** $5y^2$ **k** 0

4 **a** $7x$ **b** $3t$ **c** $-5x$ **d** $-5k$ **e** $2m^2$ **f** 0

5 **a** $7x + 5$ **b** $5x + 6$ **c** $5p$ **d** $5x + 6$ **e** $5p + t + 5$ **f** $8w - 5k$ **g** c **h** $8k - 6y + 10$

6 **a** $2c + 3d$ **b** $5d + 2e$ **c** $f + 3g + 4h$ **d** $6u - 3v$ **e** $7m - 7n$ **f** $3k + 2m + 5p$ **g** $2v$ **h** $2w - 3y$ **i** $11x^2 - 5y$ **j** $-y^2 - 2z$ **k** $x^2 - z^2$

7 **a** $8x + 6$ **b** $3x + 16$ **c** $2x + 2y + 8$

8 Any acceptable answers, e.g. $x + 4x + 2y + 2y$ or $6x - x + 6y - 2y$

9 **a** $2x$ and $2y$ **b** a and $7b$

10 **a** $3x - 1 - x$ **b** $10x$ **c** 25 cm

11 Maria is correct, as the two short horizontal lengths are equal to the bottom length and the two short vertical lengths are equal to the side length.

12.2 Expanding brackets

Exercise 12C

1 **a** $6 + 2m$ **b** $10 + 5l$ **c** $12 - 3y$ **d** $20 + 8k$ **e** $6 - 12f$ **f** $10 - 6w$ **g** $10k + 15m$ **h** $12d - 8n$ **i** $t^2 + 3t$ **j** $k^2 - 3k$ **k** $4t^2 - 4t$ **l** $8k - 2k^2$ **m** $8g^2 + 20g$ **n** $15h^2 - 10h$ **o** $y^3 + 5y$ **p** $h^4 + 7h$ **q** $k^3 - 5k$ **r** $3t^3 + 12t$ **s** $15d^3 - 3d^4$ **t** $6w^3 + 3tw$ **u** $15a^3 - 10ab$ **v** $12p^4 - 15mp$ **w** $12h^3 + 8h^2g$ **x** $8m^3 + 2m^4$

2 **a** $5(t - 1)$ and $5t - 5$
b Yes, as $5(t - 1)$ when $t = 4.50$ is $5 \times 3.50 = \$17.50$

3 He has worked out 3×5 as 8 instead of 15 and he has not multiplied the second term by 3. Answer should be $15x - 12$.

4 **a** $3(2y + 3)$ **b** $2(6z + 4)$ or $4(3z + 2)$

Exercise 12D

1 **a** $7t$ **b** $9d$ **c** $3e$ **d** $2t$ **e** $5t^2$ **f** $4y^2$ **g** $5ab$ **h** $3a^2d$

2 **a** $2x$ and $11y$ **b** a and $8b$

3 **a** $2x - 3$ **b** $10x - 16$ or $2(5x - 8)$

4 **a** $22 + 5t$ **b** $21 + 19k$ **c** $22 + 2f$ **d** $14 + 3g$

5 **a** $2 + 2h$ **b** $9g + 5$ **c** $17k + 16$ **d** $6e + 20$

6 **a** $4m + 3p + 2mp$ **b** $3k + 4h + 5hk$ **c** $12r + 24p + 13pr$ **d** $19km + 20k - 6m$

7 **a** $9t^2 + 13t$ **b** $13y^2 + 5y$ **c** $10e^2 - 6e$ **d** $14k^2 - 3kp$

8 **a** $17ab + 12ac + 6bc$ **b** $18wy + 6ty - 8tw$ **c** $14mn - 15mp - 6np$ **d** $8r^3 - 6r^2$

9 For x-coefficients, 3 and 1 or 1 and 4; for y-coefficients, 5 and 1 or 3 and 4 or 1 and 7.

10 $5(3x + 2) - 3(2x - 1) = 9x + 13$

12.3 Factorisation

Exercise 12E

1 **a** $6(m + 2t)$ **b** $3(3t + p)$ **c** $4(2m + 3k)$ **d** $4(r + 2t)$ **e** $m(n + 3)$ **f** $g(5g + 3)$ **g** $2(2w - 3t)$ **h** $y(3y + 2)$ **i** $t(4t - 3)$ **j** $3m(m - p)$ **k** $3p(2p + 3t)$ **l** $2p(4t + 3m)$ **m** $4b(2a - c)$ **n** $5bc(b - 2)$ **o** $2b(4ac + 3de)$ **p** $2(2a^2 + 3a + 4)$ **q** $3b(2a + 3c + d)$ **r** $t(5t + 4 + a)$ **s** $3mt(2t - 1 + 3m)$ **t** $2ab(4b + 1 - 2a)$ **u** $5pt(2t + 3 + p)$

2 **a** Suni has taken out a common factor.
b Because the bracket adds up to \$10.
c \$30

3 **a**, **d**, **f** and **h** do not factorise.
b $m(5 + 2p)$ **c** $t(t - 7)$ **e** $2m(2m - 3p)$ **g** $a(4a - 5b)$ **i** $b(5a - 3bc)$

4 **a** Bernice
b Ahmed has not taken out the largest possible common factor. Craig has taken m out of both terms but there isn't an m in the second term.

5 There are no common factors.

6 numerator $4x^2 - 12x$, denominator $2x - 6$

7 **a** $4(x + 1)$ **b** $2(x + 4)$ **c** $4(x + 1)$ **d** $2(3x + 2)$

12.4 Multiplying two brackets: 1

Exercise 12F

1 $x^2 + 5x + 6$
2 $t^2 + 7t + 12$
3 $w^2 + 4w + 3$
4 $m^2 + 6m + 5$
5 $k^2 + 8k + 15$
6 $a^2 + 5a + 4$
7 $x^2 + 2x - 8$
8 $t^2 + 2t - 15$
9 $w^2 + 2w - 3$
10 $f^2 - f - 6$
11 $g^2 - 3g - 4$
12 $y^2 + y - 12$
13 $x^2 + x - 12$
14 $p^2 - p - 2$
15 $k^2 - 2k - 8$
16 $y^2 + 3y - 10$
17 $a^2 + 2a - 3$
18 $x^2 - 9$
19 $t^2 - 25$
20 $m^2 - 16$
21 $t^2 - 4$
22 $y^2 - 64$
23 $p^2 - 1$
24 $25 - x^2$
25 $49 - g^2$
26 $x^2 - 36$
27 $(x + 2)$ and $(x + 3)$
28 **a** B: $1 \times (x - 2)$ C: 1×2 D: $2 \times (x - 1)$
b $(x - 2) + 2 + 2(x - 1) = 3x - 2$
c Area A $= (x - 1)(x - 2)$ = area of square minus areas (B + C + D)
$= x^2 - (3x - 2)$
$= x^2 - 3x + 2$
29 **a** $x^2 - 9$
b i 9991 **ii** 39991

12.5 Multiplying two brackets: 2

Exercise 12G

1 $6x^2 + 11x + 3$
2 $12y^2 + 17y + 6$
3 $6t^2 + 17t + 5$
4 $8t^2 + 2t - 3$
5 $10m^2 - 11m - 6$
6 $12k^2 - 11k - 15$
7 $6p^2 + 11p - 10$
8 $10w^2 + 19w + 6$
9 $6a^2 - 7a - 3$
10 $8r^2 - 10r + 3$
11 $15g^2 - 16g + 4$
12 $12d^2 + 5d - 2$
13 $8p^2 + 26p + 15$
14 $6t^2 + 7t + 2$
15 $6p^2 + 11p + 4$
16 $-10t^2 - 7t + 6$
17 $-6n^2 + n + 12$
18 $6f^2 - 5f - 6$
19 $-10q^2 + 7q + 12$
20 $-6p^2 - 7p + 3$
21 $-6t^2 + 10t + 4$
22 **a** $x^2 - 1$ **b** $4x^2 - 1$ **c** $4x^2 - 9$
d $9x^2 - 25$
23 **a** $(3x - 2)(2x + 1) = 6x^2 - x - 2$
$(2x - 1)(2x - 1) = 4x^2 - 4x + 1$
$(6x - 3)(x + 1) = 6x^2 + 3x - 3$
$(3x + 2)(2x + 1) = 6x^2 + 7x + 2$
b Multiply the x terms to match the x^2 term and/or multiply the constant terms to get the constant term in the answer.

Exercise 12H

1 $4x^2 - 1$
2 $9t^2 - 4$
3 $25y^2 - 9$
4 $16m^2 - 9$
5 $4k^2 - 9$
6 $16h^2 - 1$
7 $4 - 9x^2$
8 $25 - 4t^2$
9 $36 - 25y^2$
10 $a^2 - b^2$
11 $9t^2 - k^2$
12 $4m^2 - 9p^2$
13 $25k^2 - g^2$
14 $a^2b^2 - c^2d^2$
15 $a^4 - b^4$
16 **a** $a^2 - b^2$
b Dimensions: $a + b$ by $a - b$; Area: $a^2 - b^2$
c Areas are the same, so $a^2 - b^2 = (a + b) \times (a - b)$
17 First shaded area is $(2k)^2 - 1^2 = 4k^2 - 1$
Second shaded area is $(2k + 1)(2k - 1) = 4k^2 - 1$

Exercise 12I

1 $x^2 + 10x + 25$
2 $m^2 + 8m + 16$
3 $t^2 + 12t + 36$
4 $p^2 + 6p + 9$
5 $m^2 - 6m + 9$
6 $t^2 - 10t + 25$
7 $m^2 - 8m + 16$
8 $k^2 - 14k + 49$
9 $9x^2 + 6x + 1$
10 $16t^2 + 24t + 9$
11 $25y^2 + 20y + 4$

12 $4m^2 + 12m + 9$
13 $16t^2 - 24t + 9$
14 $9x^2 - 12x + 4$
15 $25t^2 - 20t + 4$
16 $25r^2 - 60r + 36$
17 $x^2 + 2xy + y^2$
18 $m^2 - 2mn + n^2$
19 $4t^2 + 4ty + y^2$
20 $m^2 - 6mn + 9n^2$
21 $x^2 + 4x$
22 $x^2 - 10x$
23 $x^2 + 12x$
24 $x^2 - 4x$
25 **a** Marcela has just squared the first term and the second term. She hasn't written down the brackets twice.
b Paulo has written down the brackets twice but has worked out $(3x)^2$ as $3x^2$ and not $9x^2$.
c $9x^2 + 6x + 1$
26 Whole square is $(2x)^2 = 4x^2$.
Three areas are $2x - 1$, $2x - 1$ and 1.
$4x^2 - (2x - 1 + 2x - 1 + 1) = 4x^2 - (4x - 1) = 4x^2 - 4x + 1$

12.6 Expanding three brackets

Exercise 12J

1 **a** $x^2 + 2x - 3$ **b** $x^3 + 2x^2 - 3x$ **c** $x^3 + 4x^2 + x - 6$
2 **a** $x^2 - 7x + 10$ **b** $x^3 - 6x^2 + 3x - 10$
c $2x^3 - 13x^2 + 13x + 10$
3 **a** $x^3 - 3x^2 - 13x + 15$ **b** $3x^3 + 31x^2 + 78x + 56$
c $x^3 - 14x^2 + 53x - 40$
4 **a** $x^2 + 4x + 4$ **b** $x^3 + 6x^2 + 12x + 8$
c $8x^3 + 12x^2 + 6x + 1$
5 **a** $x^3 + x^2 - 4x - 4$ **b** $2x^3 - 3x^2 - 11x + 6$
c $x^3 + 4x^2 - 4x - 16$
6 **a** $x^3 + 6x^2 + 11x + 6$ **b** $x^3 - 6x^2 + 11x - 6$
7 **a** $x^3 + 4x^2 - 3x - 18$ **b** $x^3 - 6x^2 - 15x + 100$
c $9x^3 + 78x^2 - 116x + 40$
8 **a** $(x + 1)^3 - (x - 1)^3 = x^3 + 3x^2 + 3x + 1 - (x^3 - 3x^2 + 3x - 1)$
$= x^3 + 3x^2 + 3x + 1 - x^3 + 3x^2 - 3x + 1$
$= 6x^2 + 2 = 2(3x^2 + 1)$
b $4(3x^2 + 4)$
9 The volume of the cube is $(x + 1)^3$
One of the eight pieces is a cube of side x and volume x^3
Three of the eight pieces are cuboids, with sides x, x and 1 and each has volume x^2
Three of the eight pieces are cuboids with sides x, 1 and 1 and each has volume x
One of the eight pieces is a cube of side 1 and volume 1
Add these eight volumes to get $x^3 + 3x^2 + 3x + 1$ which is $(x + 1)^3$
10 **a** $a = 6$ **b** $b = 5$ **c** $c = -8$
11 **a** $x^3 - 1$ **b** $x^3 - 8$
c $x^3 - 27 = (x^2 + 3x + 9)(x - 3)$
12 $6x^3 + 11x^2 + 6x + 1$ cm^3

12.7 Quadratic factorisation

Exercise 12K

1 $(x + 2)(x + 3)$
2 $(t + 1)(t + 4)$
3 $(m + 2)(m + 5)$
4 $(k + 4)(k + 6)$
5 $(p + 2)(p + 12)$
6 $(r + 3)(r + 6)$
7 $(w + 2)(w + 9)$
8 $(x + 3)(x + 4)$
9 $(a + 2)(a + 6)$
10 $(k + 3)(k + 7)$
11 $(f + 1)(f + 21)$
12 $(b + 8)(b + 12)$
13 $(t - 2)(t - 3)$
14 $(d - 4)(d - 1)$
15 $(g - 2)(g - 5)$
16 $(x - 3)(x - 12)$
17 $(c - 2)(c - 16)$
18 $(t - 4)(t - 9)$
19 $(y - 4)(y - 12)$
20 $(j - 6)(j - 8)$
21 $(p - 3)(p - 5)$
22 $(y + 6)(y - 1)$
23 $(t + 4)(t - 2)$
24 $(x + 5)(x - 2)$
25 $(m + 2)(m - 6)$
26 $(r + 1)(r - 7)$
27 $(n + 3)(n - 6)$
28 $(m + 4)(m - 11)$
29 $(w + 4)(w - 6)$
30 $(t + 9)(t - 10)$
31 $(h + 8)(h - 9)$
32 $(t + 7)(t - 9)$
33 $(d + 1)^2$
34 $(y + 10)^2$
35 $(t - 4)^2$
36 $(m - 9)^2$
37 $(x - 12)^2$
38 $(d + 3)(d - 4)$
39 $(t + 4)(t - 5)$
40 $(q + 7)(q - 8)$
41 $(x + 2)(x + 3)$, giving areas of $2x$ and $3x$, or $(x + 1)(x + 6)$, giving areas of x and $6x$.

Exercise 12L

1 $(x + 3)(x - 3)$
2 $(t + 5)(t - 5)$
3 $(m + 4)(m - 4)$
4 $(3 + x)(3 - x)$

5 $(7 + t)(7 - t)$
6 $(k + 10)(k - 10)$
7 $(2 + y)(2 - y)$
8 $(x + 8)(x - 8)$
9 $(t + 9)(t - 9)$
10 **a** x^2
b **i** $(x - 2)$ **ii** $(x + 2)$ **iii** $x(x - 2) = x^2 - 2x$ **iv** 4
c $A + B - C = x^2 - 4$, which is the area of D, which is $(x + 2)(x - 2)$.
11 **a** $x^2 + 4x + 4 - (x^2 + 2x + 1) = 2x + 3$
b $(a + b)(a - b)$
c $(x + 2 + x + 1)(x + 2 - x - 1) = (2x + 3)(1) = 2x + 3$
d The answers are the same.
e $(x + 1 + x - 1)(x + 1 - x + 1) = (2x)(2) = 4x$
12 $(x + y)(x - y)$
13 $(x + 2y)(x - 2y)$
14 $(x + 3y)(x - 3y)$
15 $(3x + 1)(3x - 1)$
16 $(4x + 3)(4x - 3)$
17 $(5x + 8)(5x - 8)$
18 $(2x + 3y)(2x - 3y)$
19 $(3t + 2w)(3t - 2w)$
20 $(4y + 5x)(4y - 5x)$

Exercise 12M

1 $(2x + 1)(x + 2)$
2 $(7x + 1)(x + 1)$
3 $(4x + 7)(x - 1)$
4 $(3t + 2)(8t + 1)$
5 $(3t + 1)(5t - 1)$
6 $(4x - 1)^2$
7 $3(y + 7)(2y - 3)$
8 $4(y + 6)(y - 4)$
9 $(2x + 3)(4x - 1)$
10 $(2t + 1)(3t + 5)$
11 $(x - 6)(3x + 2)$
12 $(x - 5)(7x - 2)$
13 $4x + 1$ and $3x + 2$

14 **a** All the terms in the quadratic have a common factor of 6.
b $6(x + 2)(x + 3)$. This has the highest common factor taken out.
c For example, 'A rectangle could be split in many different ways.'

12.8 Algebraic fractions

Exercise 12N

1 **a** $\frac{5x}{6}$ **b** $\frac{19x}{20}$ **c** $\frac{23x}{20}$ **d** $\frac{3x + 2y}{6}$
e $\frac{x^2y + 8}{4x}$ **f** $\frac{5x + 7}{6}$ **g** $\frac{7x + 3}{4}$ **h** $\frac{13x + 5}{15}$
i $\frac{3x - 7}{4}$ **j** $\frac{5x - 10}{4}$

2 **a** $\frac{x}{6}$ **b** $\frac{11x}{20}$ **c** $\frac{7x}{20}$ **d** $\frac{3x - 2y}{6}$
e $\frac{xy^2 - 8}{4y}$ **f** $\frac{x - 1}{6}$ **g** $\frac{x + 1}{4}$ **h** $\frac{-7x - 5}{15}$
i $\frac{x - 1}{4}$ **j** $\frac{2 - 3x}{4}$

3 **a** $\frac{x^2}{6}$ **b** $\frac{3xy}{14}$ **c** $\frac{8}{3}$ **d** $\frac{2xy}{3}$
e $\frac{x^2 - 2x}{10}$ **f** $\frac{1}{6}$ **g** $\frac{6x^2 + 5x + 1}{8}$ **h** $\frac{2x^2 + x}{15}$
i $\frac{2x - 4}{x - 3}$ **j** $\frac{1}{2x}$

4 **a** x **b** $\frac{x}{2}$ **c** $\frac{3x^2}{16}$ **d** 3
e $\frac{17x + 1}{10}$ **f** $\frac{13x + 9}{10}$ **g** $\frac{3x^2 - 5x - 2}{10}$ **h** $\frac{x + 3}{2}$
i $\frac{2x^2 - 6y^2}{9}$

5 **a** $\frac{7x + 9}{(x + 1)(x + 2)}$ **b** $\frac{11x - 10}{(x - 2)(x + 1)}$ **c** $\frac{2 - 13x}{(4x + 1)(x + 2)}$
d $\frac{8 - 10x}{(2x - 1)(x + 1)}$ **e** $\frac{x + 1}{(2x - 1)(3x - 1)}$

6 First, he did not factorise and just cancelled the x^2s. Then he cancelled 2 and 6 with the wrong signs. Then he said two minuses make a plus when adding, which is not true.

7 $\frac{2x^2 + x - 3}{4x^2 - 9}$

8 **a** $\frac{9x + 13}{(x + 1)(x + 2)}$ **b** $\frac{14x + 19}{(4x - 1)(x + 1)}$ **c** $\frac{2x^2 + x - 13}{2(x + 1)}$ **d** $\frac{x + 1}{(2x - 1)(3x - 1)}$

9 **a** $\frac{x - 1}{2x + 1}$ **b** $\frac{2x + 1}{x + 3}$ **c** $\frac{2x - 1}{3x - 2}$ **d** $\frac{x + 1}{x - 1}$ **e** $\frac{2x + 5}{4x - 1}$

Answers to Chapter 13

13.1 Solving linear equations

Exercise 13A

1 **a** 56 **b** 2 **c** 6 **d** 3 **e** 4
f $2\frac{1}{2}$ **g** $3\frac{1}{2}$ **h** $2\frac{1}{2}$ **i** 4 **j** 21
k 72 **l** 56 **m** 0 **n** −7 **o** −18
p 36 **q** 36 **r** 60 **s** 7 **t** 11
u 2 **v** 7 **w** 2.8 **x** 1 **y** 11.5
z 0.2

2 **a** −4 **b** 15

3 **a** Elif
b Second line: Mustafa subtracts 1 instead of adding 1; fourth line: Mustafa subtracts 2 instead of dividing by 2.

Exercise 13B

1 **a** 3 **b** 7 **c** 5 **d** 3 **e** 4 **f** 6
g 8 **h** 1 **i** $1\frac{1}{2}$ **j** $2\frac{1}{2}$ **k** $\frac{1}{2}$ **l** $1\frac{1}{5}$
m 2 **n** −2 **o** −1 **p** −2 **q** −2 **r** −1

2 Any values that work, e.g. $a = 2$, $b = 3$ and $c = 30$.

3 55

Answers to Chapter 13

Exercise 13C

1 a $x = 2$ b $y = 1$ c $a = 7$ d $t = 4$
e $p = 2$ f $k = -1$ g $m = 3$ h $s = -2$

2 $3x - 2 = 2x + 5$, $x = 7$

3 a $d = 6$ b $x = 11$ c $y = 1$ d $h = 4$
e $b = 9$ f $c = 6$

4 $6x + 3 = 6x + 10$; $6x - 6x = 10 - 3$; $0 = 7$, which is obviously false. Both sides have $6x$, which cancels out.

5 Check student's example.

13.2 Setting up equations

Exercise 13D

1 90 cents or 0.90 dollars

2 a 1.5 b 2

3 a 1.5 cm b 6.75 cm^2

4 17

5 8

6 a $8c - 10 = 56$ b $8.25

7 a B: 450 cars, C: 450 cars, D: 300 cars
b 800 c 750

8 360 dollars

9 3 years

10 9 years

11 3 cm

12 5

13 a $4x + 40 = 180$ b $x = 35°$

14 a $\frac{x+10}{5} = 9.50$ b $37.50

15 No, as $x + x + 2 + x + 4 + x + 6 = 360$ gives $x = 87°$ so the consecutive numbers (87, 89, 91, 93) are not even but odd

16 $4x + 18 = 3x + 1 + 50$, $x = 33$. Large bottle 1.5 litres, small bottle 1 litre

13.3 Solving quadratic equations by factorisation

Exercise 13E

1 –2, –5

2 –3, –1

3 –6, –4

4 –3, 2

5 –1, 3

6 –4, 5

7 1, –2

8 2, –5

9 7, –4

10 3, 2

11 1, 5

12 4, 3

13 –4, –1

14 –9, –2

15 2, 4

16 3, 5

17 –2, 5

18 –3, 5

19 –6, 2

20 –6, 3

21 –1, 2

22 –2

23 –5

24 4

25 –2, –6

26 7

27 a $x(x - 3) = 550$, $x^2 - 3x - 550 = 0$
b $(x - 25)(x + 22) = 0$, $x = 25$ years

28 $x(x + 40) = 48000$, $x^2 + 40x - 48000 = 0$,
$(x + 240)(x - 200) = 0$. Fence is $2 \times 200 + 2 \times 240 = 880$ m.

29 –6, –4

30 2, 16

31 –6, 4

32 –9, 6

33 –10, 3

34 –4, 11

35 –8, 9

36 8, 9

37 1

38 Mario was correct. Sylvan did not make it into a standard quadratic and only factorised the x terms. She also incorrectly solved the equation $x - 3 = 4$.

Exercise 13F

1 a $\frac{1}{3}, -3$ b $1\frac{1}{3}, -\frac{1}{2}$ c $-\frac{1}{5}, 2$
d $-2\frac{1}{2}, 3\frac{1}{2}$ e $-\frac{1}{6}, -\frac{1}{3}$ f $\frac{2}{3}, 4$
g $\frac{1}{2}, -3$ h $\frac{5}{2}, -\frac{7}{6}$ i $-1\frac{2}{3}, 1\frac{2}{5}$
j $1\frac{3}{4}, 1\frac{2}{7}$ k $\frac{2}{3}, \frac{1}{8}$ l $\pm\frac{1}{4}$
m $-2\frac{1}{4}, 0$ n $\pm 1\frac{2}{5}$ o $-\frac{1}{3}, 3$

2 a –6, 7 b $-\frac{5}{2}, \frac{3}{2}$ c –6, 7
d $-1, \frac{11}{13}$ e –2, 3 f $-\frac{2}{5}, \frac{1}{2}$
g $-\frac{1}{2}, -\frac{1}{3}$ h $-2, \frac{1}{5}$ i 4
j $-2, \frac{1}{8}$ k $-\frac{1}{3}, 0$ l –5, 5
m $-\frac{5}{3}$ n $-\frac{7}{2}, \frac{7}{2}$ o $-\frac{5}{2}, 3$

3 a Both have only one solution: $x = 1$.
b B is a linear equation, but A and C are quadratic equations.

4 a $(5x - 1)^2 = (2x + 3)^2 + (x + 1)^2$, when expanded and collected into the general quadratics, gives the required equation.
b $(10x + 3)(2x - 3)$, $x = 1.5$; area = 7.5 cm^2.

13.4 Solving quadratic equations by the quadratic formula

Exercise 13G

1 1.77, –2.27

2 –0.23, –1.43

3 3.70, −2.70
4 0.29, −0.69
5 −0.19, −1.53
6 −1.23, −2.43
7 −0.41, −1.84
8 −1.39, −2.27
9 1.37, −4.37
10 2.18, 0.15
11 −0.39, −5.11
12 0.44, −1.69
13 1.64, 0.61
14 0.36, −0.79
15 1.89, 0.11
16 13
17 $x^2 - 3x - 7 = 0$
18 Hasan gets $x = \frac{4 \pm \sqrt{0}}{8}$ and Miriam gets $(2x - 1)^2 = 0$; each method only gives one solution, $x = \frac{1}{2}$

13.5 Solving quadratic equations by completing the square

Exercise 13H

1 **a** $(x + 2)^2 - 4$ **b** $(x + 7)^2 - 49$
c $(x - 3)^2 - 9$ **d** $(x + 3)^2 - 9$
e $(x - 1.5)^2 - 2.25$ **f** $(x - 4.5)^2 - 20.25$
g $(x + 6.5)^2 - 42.25$ **h** $(x + 5)^2 - 25$
i $(x + 4)^2 - 16$ **j** $(x - 1)^2 - 1$
k $(x + 1)^2 - 1$

2 **a** $(x + 2)^2 - 5$ **b** $(x + 7)^2 - 54$
c $(x - 3)^2 - 6$ **d** $(x + 3)^2 - 2$
e $(x - 1.5)^2 - 3.25$ **f** $(x + 3)^2 - 6$
g $(x - 4.5)^2 - 10.25$ **h** $(x + 6.5)^2 - 7.25$
i $(x + 4)^2 - 22$ **j** $(x + 1)^2 - 2$
k $(x - 1)^2 - 8$ **l** $(x + 1)^2 - 10$

3 **a** $-2 \pm \sqrt{5}$ **b** $-7 \pm 3\sqrt{6}$ **c** $3 \pm \sqrt{6}$
d $-3 \pm \sqrt{2}$ **e** $1.5 \pm \sqrt{3.25}$ **f** $-3 \pm \sqrt{6}$
g $4.5 \pm \sqrt{10.25}$ **h** $-6.5 \pm \sqrt{7.25}$ **i** $-4 \pm \sqrt{22}$
j $-1 \pm \sqrt{2}$ **k** $1 \pm 2\sqrt{2}$ **l** $-1 \pm \sqrt{10}$

4 **a** 1.45, −3.45 **b** 5.32, −1.32 **c** −4.16, 2.16

5 **a** $x = 1.5 \pm \sqrt{3.75}$ **b** $x = 1 \pm \sqrt{0.75}$
c $x = -1.25 \pm \sqrt{6.5625}$ **d** $x = 7.5 \pm \sqrt{40.25}$

6 $p = -14$, $q = -3$

7 **a** 3rd, 1st, 4th and 2nd – in that order

13.6 Simultaneous equations

Exercise 13I

1 **a** $x = 5, y = 10$ **b** $x = 18, y = 6$ **c** $x = 12, y = 48$
2 **a** $x = 6, y = 18$ **b** $x = 12.5, y = 2.5$ **c** $x = 0.5, y = 4.5$
3 **a** $x = 13, y = 7$ **b** $x = 9, y = 14$ **c** $x = 10, y = -4$
4 **a** $x = 0.5, y = 4$ **b** $x = 5.5, y = 14.5$ **c** $x = 2, y = 8$
5 Carmen 32, Anish 8
6 11.5 and 25.5
7 8 and −3
8 **a** $x + y = 75$ **b** $y = 2x$ **c** $x = 25, y = 50$
9 **a** $x + y = 300$ **b** $x = y + 60$ or $y = x - 60$
c $x = 180$ and $y = 120$
10 **a** $x = y - 26$ or $y = x + 26$ or $y - x = 26$ **b** $x + y = 50$
c Ahmed is 12 and his mother is 38.
11 **a** $x = y - 0.4$ or $y = x + 0.4$ **b** $x + y = 8.6$ **c** 4.5 m

Exercise 13J

1 **a** $x = 4, y = 1$ **b** $x = 1, y = 4$
c $x = 3, y = 1$ **d** $x = 5, y = -2$
e $x = 7, y = 1$ **f** $x = 5, y = \frac{1}{2}$
g $x = 4\frac{1}{2}, y = 1\frac{1}{2}$ **h** $x = -2, y = 4$
i $x = 2\frac{1}{2}, y = -1\frac{1}{2}$ **j** $x = 2\frac{1}{4}, y = 6\frac{1}{2}$
k $x = 4, y = 3$ **l** $x = 5, y = 3$

2 **a** 3 is the first term. The next term is $3 \times a + b$, which equals 14.
b $14a + b = 47$
c $a = 3, b = 5$
d 146, 443

Exercise 13K

1 **a** $x = 2, y = -3$ **b** $x = 7, y = 3$
c $x = 4, y = 1$ **d** $x = 2, y = 5$
e $x = 4, y = -3$ **f** $x = 1, y = 7$
g $x = 2\frac{1}{2}, y = 1\frac{1}{2}$ **h** $x = -1, y = 2\frac{1}{2}$
i $x = 6, y = 3$ **j** $x = \frac{1}{2}, y = -\frac{3}{4}$
k $x = -1, y = 5$ **l** $x = 1\frac{1}{2}, y = \frac{3}{4}$

2 **a** They are the same equation. Divide the first by 2 and it is the second, so they have an infinite number of solutions.
b Double the second equation to get $6x + 2y = 14$ and subtract to get $9 = 14$. The left-hand sides are the same if the second is doubled so they cannot have different values.

Exercise 13L

1 **a** $x = 5, y = 1$ **b** $x = 3, y = 8$
c $x = 9, y = 1$ **d** $x = 7, y = 3$
e $x = 4, y = 2$ **f** $x = 6, y = 5$
g $x = 3, y = -2$ **h** $x = 2, y = \frac{1}{2}$
i $x = -2, y = -3$ **j** $x = -1, y = 2\frac{1}{2}$
k $x = 2\frac{1}{2}, y = -½$ **l** $x = -1\frac{1}{2}, y = 4\frac{1}{2}$
m $x = -\frac{1}{2}, y = -6\frac{1}{2}$ **n** $x = 3\frac{1}{2}, y = 1\frac{1}{2}$
o $x = -2\frac{1}{2}, y = -3\frac{1}{2}$

2 (1, −2) is the solution to equations A and C; (−1, 3) is the solution to equations A and D; (2, 1) is the solution to B and C; (3, −3) is the solution to B and D.

3 Intersection points are (0, 6), (1, 3) and (2, 4). Area is 2 cm^2

4 Intersection points are (0, 3), (6, 0) and (4, −1). Area is 6 cm^2

13.7 Linear and non-linear simultaneous equations

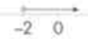
Exercise 13M

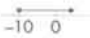
1 a (5, −1) b (4, 1) c (8, −1)
2 a (1, 2) and (−2, −1) b (−4, 1) and (−2, 2)
3 a (3, 4) and (4, 3) b (0, 3) and (−3, 0) c (3, 2) and (−2, 3)
4 a (2, 5) and (−2, −3) b (−1, −2) and (4, 3) c (3, 3) and (1, −1)
5 a (−3, −3), (1, 1) b (3, −2), (−2, 3) c (−2, −1), (1, 2)
d (2, −1), (3, 1) e (−2, 1), (3, 6) f (1, −4), (4, 2)
g (4, 5), (−5, −4)
6 a $x + y = 12$; $x^2 + y^2 = 90$ b Either 391290 or 931290
7 12 years old
8 a $x^2 + y^2 = 85$ and $(x + y)^2 = 121$ b 2 and 9

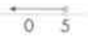
13.8 Solving inequalities

Exercise 13N

1 a $x < 3$ 

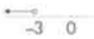
b $t > -2$ 

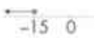
c $p \geq -10$ 

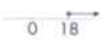
d $x < 5$ 

e $y \leq 3$ 

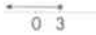
f $t > 3$ 

g $x < 6$ 

h $y \leq -15$ 

i $t \geq 18$ 

j $x < 7$ 

k $x \leq 3$ 

l $t \geq 5.25$ 

2 a 8 b 6 c 16
d 3 e 7
3 a 11 b 16 c 16
4 $2x + 3 < 20$, $x < 8.50$, so the most each could cost is $8.49
5 a Because 3 + 4 = 7, which is less than the third side of length 8
b $x + x + 2 > 10$, $2x + 2 > 10$, $2x > 8$, $x > 4$, so smallest value of x is 5
6 a $x = 6$ and $x < 3$ scores −1 (nothing in common), $x < 3$ and $x > 0$ scores 1 (1 in common for example), $x > 0$ and $x = 2$ scores 1 (2 in common), $x = 2$ and $x \geq 4$ scores −1 (nothing in common), so we get −1 + 1 + 1 − 1 = 0
b $x > 0$ and $x = 6$ scores +1 (6 in common), $x = 6$ and $x \geq 4$ scores +1 (6 in common), $x \geq 4$ and $x = 2$ scores −1 (nothing in common), $x = 2$ and $x \leq 3$ scores +1 (2 in common), +1 + 1 − 1 + 1 = 2
c Any acceptable combination, e.g. $x = 2$, $x \leq 3$, $x > 0$, $x \geq 4$, $x = 6$
7 a $x \geq -6$ b $t \leq \frac{8}{3}$
c $y \leq 4$ d $x \geq -2$
e $w \leq 5.5$ f $x \leq \frac{14}{5}$
8 a $x \leq 2$ b $x > 38$
c $x < 6\frac{1}{2}$ d $x \geq 7$
e $t > 15$ f $y \leq \frac{7}{5}$
9 a 4 b 99 c 11 d 11 e 6
10 a 0, 10 − 10 b $x < 16$
11 a $x < 9$ b $x \geq 11$ c $x \geq 3$
12 a $x \geq 7.5$ b $x \leq -2$ c $x < 6$
d $x > 1.5$ e $x \geq -5$ f $x < 0.5$

Answers to Chapter 14

14.1 Conversion graphs

Exercise 14A

1 a i $8\frac{1}{4}$ kg ii $2\frac{1}{4}$ kg iii 9 lb iv 22 lb
b 2.2 lb
c Read off the value for 12 lb (5.4 kg) and multiply this by 4 (21.6 kg)
2 a i 10 cm ii 23 cm iii 2 in iv $8\frac{3}{4}$ in
b $2\frac{1}{2}$ cm
c Read off the value for 9 in (23 cm) and multiply this by 2 (46 cm)
3 a i $320 ii $100 iii £45 iv £78
b $3.20
c It would become more steep.
4 a i $120 ii $82
b i 32 ii 48
5 a i $100 ii $325
b i 500 ii 250
6 a i $70 ii $29
b i $85 ii $38
7 a i 95 °F ii 68 °F iii 10 °C iv 32 °C
b 32°F
8 a Check student's graph b $50
9 a Student's own graph b about 48 kilometres
c about 16 miles
10 a Student's own graph b about 9 centimetres
c about 4 hours
11 a Student's own graph b about 23 minutes

14.2 Travel graphs

Exercise 14B

1 a i 2 h ii 3 h iii 5 h
b i 40 km/h ii 120 km/h iii 40 km/h
c 5.30 am
2 a i 125 km ii about 25 km/h
b i Between 2 pm and 3 pm ii About 12 km/h

3 a 30 km b 40 km c 100 km/h

4 a i 263 m/min (3 sf)

ii 15.8 km/h (3 sf)

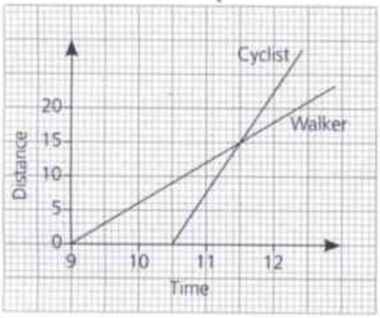
b 500 m/min

c Yuto by 1 minute

5 a Patrick ran quickly at first, then had a slow middle section but he won the race with a final sprint. Araf ran steadily all the way and came second. Sean set off the slowest, speeded up towards the end but still came third.

b i 1.67 m/s ii 6 km/h

6 a



b At 1130

7 a i Because it stopped several times

ii Ravinder

b Ravinder at 1558, Sue at 1620, Michael at 1635

c i 24 km/h

ii 20.6 km/h

iii 5

8 a 50 metres b student's graph c 1 metre/second

9 a student's graph b 80 km/hour

10 a 1300 b 15 km c student's graph

d For the three stages, 5 km/hour, 4 km/hour and 2 km/hour. For the whole trip 3.75 km/hour

14.3 Speed–time graphs

Exercise 14C

1 a 20 m/s b 0.5 m/s^2

c 1 m/s^2 d 600 metres

e 10 m/s

2 a 0.6 m/s^2 b 750 m

3 a 0.2 m/s^2 b 0.1 m/s^2

c 75 metres d 2.5 m/s

4 a 1 m/s^2 b $\frac{2}{3}$ m/s^2

c 6 kilometres (or 6000 metres)

d 30 m/s

5 a student's graph b 0.8 m/s^2 c 80 metres

6 a 26 m/s b student's graph c 144 metres

7 a $1\frac{1}{3}$ m/s^2

b They are together. They have both travelled 450 metres

8 a 2 m/s^2

b i after 20 seconds

ii 100 metres

c 1150 metres

9 a 15 seconds b $1\frac{1}{3}$ m/s^2

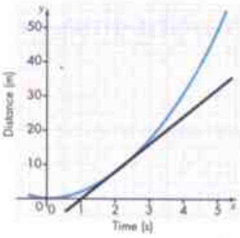
10 a 6 m/s b student's own graph

c 15 metres

14.4 Curved graphs

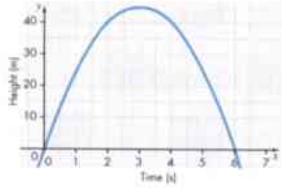
Exercise 14D

1 a and b



c 8 m/s d 16 m/s

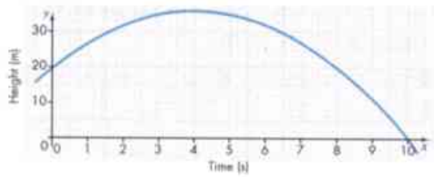
2 a



b 10 m/s c 30 m/s d 0 m/s

e 20 m/s downwards

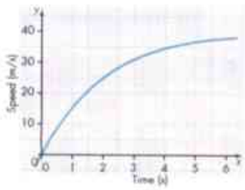
3 a



b 4 m/s c 6 m/s downwards d after 4 s

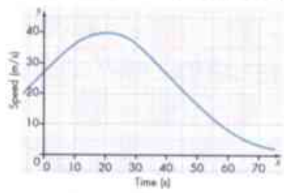
e 12 m/s

4 a



b about 12 m/s^2 c about 7.4 m/s^2 d about 2.7 m/s^2

5 a

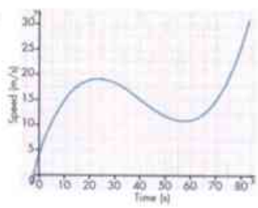


b about 0.73 m/s^2

c after 20 seconds

d about 0.65

6 a



b about 0.72 m/s^2 c about 0.36 m/s^2

d about 0.72 m/s^2 e after about 23 s and 57 s

Answers to Chapter 15

15.1 Drawing straight-line graphs

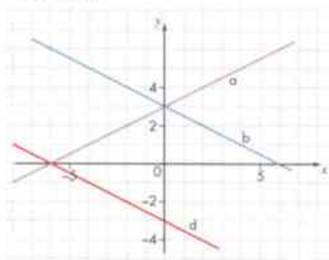
Exercise 15A

1 Extreme points are (0, 4), (5, 19)
2 Extreme points are (0, −5), (5, 5)
3 Extreme points are (0, −3), (10, 2)
4 Extreme points are (−3, −4), (3, 14)
5 Extreme points are (−6, 2), (6, 6)
6 **a** Extreme points are (0, −2), (5, 13) and (0, 1), (5, 11)
 b (3, 7)
7 **a** Extreme points are (0, −5), (5, 15) and (0, 3), (5, 13)
 b (4, 11)
8 **a** Extreme points are (0, −1), (12, 3) and (0, −2), (12, 4)
 b (6, 1)
9 **a** Extreme points are (0, 1), (4, 13) and (0, −2), (4, 10)
 b Do not cross because they are parallel
10 **a** Values of y: 5, 4, 3, 2, 1, 0. Extreme points are (0, 5), (5, 0)
 b Extreme points are (0, 7), (7, 0)
11 **a** yes **b** no **c** yes **d** no **e** yes **f** no
12 **a** 6 **b** 3.5 **c** 2
13 **a** 20 **b** −10

15.2 The equation $y = mx + c$

Exercise 15B

1 **a** 3 **b** 2 **c** $\frac{1}{2}$
 d 3 **e** $\frac{1}{3}$
2 **a** $y = 2x - 2$ **b** $y = x + 1$ **c** $y = 2x - 3$
 d $2y = x + 6$ **e** $y = x$ **f** $y = 2x$
3 **a** $y = 2x + 1$, $y = -2x + 1$ **b** $5y = 2x - 5$, $5y = -2x - 5$
 c $y = x + 1$, $y = -x + 1$
4 **a** $y = -2x + 1$ **b** $2y = -x$ **c** $y = -x + 1$
 d $5y = -2x - 5$ **e** $y = -\frac{3}{2}x - 3$ or $2y = -3x - 6$
5 **a** 3 **b** (0,3)
6 **a** 4 **b** 4
7 The first and last are parallel because they both have a gradient of 4. The middle one has a gradient of 3.
8 $y = 0.6x$
9 **a**, **b** and **d**

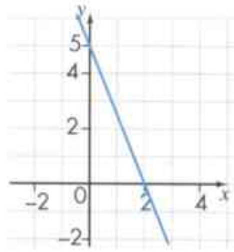


c $y = -0.5x + 3$ **e** $y = -0.5x - 3$

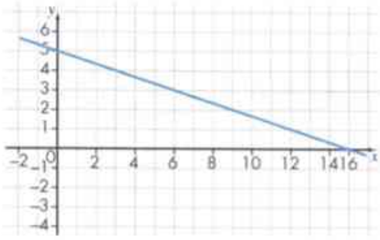
15.3 More about straight-line graphs

Exercise 15C

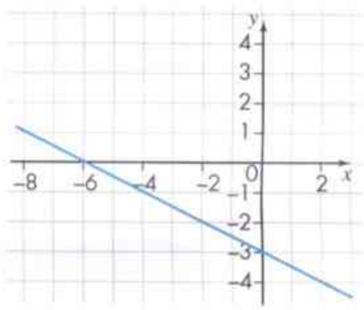
1 **a** $y = -2.5x + 5$ **b** −2.5 and (0, 5)
 c



2 **a** $y = -\frac{1}{3}x + 5$ **b** $-\frac{1}{3}$ and (0, 5)
 c



3 **a** $y = -0.5x - 3$ **b** −0.5 and (0, −3)
 c



4 **a** $y = -x - 20$ **b** $y = 3x + 15$ **c** $y = -0.7x + 3$
 d $y = 0.2x - 8$ **e** $y = 1.5x - 1$ **f** $y = -0.5x + 6$
5 **a** −1 and (0, −20) **b** 3 and (0, 15) **c** −0.7 and (0, 3)
 d 0.2 and (0, −8) **e** 1.5 and (0, −1) **f** −0.5 and (0, 6)
6 **a** −1 **b** $\frac{1}{3}$ **c** 6
 d −0.5 **e** 2 **f** 5.6
7 **a** C **b** A **c** D **d** B
8 line d, $3x - 2y = 12$, all the rest are the same line.
9 **a** (9, 0) and (0, 15)
 b (20, 0) and (0, 10)
 c (0, −10) and (−5, 0)

15.4 Solving equations graphically

Exercise 15D

1 **a** 1.8 **b** −0.4 **c** 2.7
2 **a** −1.6 **b** 3.8 or 3.9 **c** −3.8

3 a 2.3 b 1.6 c 0.9
4 a 23 b −29 c 71
5 a 6.9 b 2.9 c 4.2 or 4.3
6 a −0.8 b 2.6 c 0.3

15.5 Parallel lines

Exercise 15E

1 a $2 \times 3 + 6 = 12$ b 2 c student's graph
d $y = 2x$ e $y = 2x + 3$
2 a (0, −1) and (4, 0) b $y = \frac{1}{4}x + 3$
3 a −2 b (4,0) c student's graph
d $y = -2x$ e $y = -2x + 14$
4 a 1 b $y = 5x - 9$
5 $y = \frac{2}{3}x - 3$
6 a $y = 2x - 4$ b $y = 2x + 8$
7 a $4 + 2 \times 1 = 6$ b $y = -2x + 6$
c −2 d $y = -2x$

15.6 Points and lines

Exercise 15F

1 a 3 b $\frac{1}{2}$ c 4 d −1 e $-\frac{1}{2}$ f $\frac{2}{3}$
2 a $y = 2x - 3$ b $y = \frac{1}{2}x + 4$ c $y = 4x - 2$ d $y = -3x + 8$
3 a (5, 3) b (4, 5) c (3, 2)
d (3, 3) e (1, 3.5) f (−0.5, 0)
4 a student's graph b $y = 0.5x + 6.5$ c (−1,3)
d $y = -x + 8$
5 a 5 b 13 c 10 d 17
6 Show that the distance from each point to (2,1) is 5.
7 AB = $\sqrt{32}$, AC = $\sqrt{80}$, BC = $\sqrt{80}$, so two of the sides are the same length.

15.7 Perpendicular lines

Exercise 15G

1 a −2 b $-\frac{1}{4}$ c $\frac{3}{2}$
2 The gradients are 5 and $-\frac{1}{5}$; $5 \times -\frac{1}{5} = -1$
3 a $-\frac{1}{3}$ b −2 c 2.5 d $-\frac{2}{15}$
4 a 1 b 2 c $\frac{4}{3}$ d −6
5 a $y = -\frac{1}{5}x$ b $y = -\frac{1}{5}x + 10$ c $y = -\frac{1}{5}x + 2$
6 $y = \frac{3}{5}x + 2$
7 $y = -\frac{1}{4}x + \frac{1}{2}$
8 $y = -5x + 11$
9 $6x + 3y = 7$ is the odd one out
10 $y = 2x + 6$

Answers to Chapter 16

16.1 Quadratic graphs

Exercise 16A

1 a x: −3, −2, −1, 0, 1, 2, 3
y: 11, 6, 3, 2, 3, 6, 11
b student's graph
2 a x: −3, −2, −1, 0, 1, 2, 3, 4, 5W
x^2: 9, 4, 1, 0, 1, 4, 9, 16, 25
$-3x$: 9, 6, 3, 0, −3, −6, −9, −12, −15
y: 18, 10, 4, 0, −2, −2, 0, 4, 10
b 1.8 c (1.5, −2.25)
d $x = 1.5$ e $x = 4.2$ or −1.2
3 a y: 7, 0, −5, −8, −9, −8, −5, 0, 7 b $x = 4$ or −2
c The graph should give a value of about −8.75
d The graph should give values of about 4.5 and −2.5
4 a y: 10, 4, 0, −2, −2, 0, 4, 10
b (2.5, −2.25)
c $x = 2.5$
d The graph should give a value of about 6.75
e x = 1 or 4
f The graph should give values of about 0.2 and 4.8
5 a x: −4, −3, −2, −1, 0, 1, 2
y: 7, 2, −1, −2, −1, 2, 7
b 1.6, 0.2
c 0.5
d $x = -2.7$ or 0.7
6 a x: −4, −3, −2, −1, 0, 1, 2, 3, 4
y: −4, 3, 8, 11, 12, 11, 8, 3, −4
b 9.75
c ± 3.5
d 2.2 and −2.2
7 a

x	−5	−4	−3	−2	−1	0	1	2
x^2	25	16	9	4	1	0	1	4
$+4x$	−20	−16	−12	−8	−4	0	4	8
y	5	0	−3	−4	−3	0	5	12

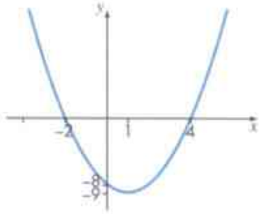
b $x = -4$ and 0
c −3.8
d −4, 0
8 a x: −1, 0, 1, 2, 3, 4, 5, 6, 7
y: 10, 3, −2, −5, −6, −5, −2, 3, 10
b $x = 0.6$ or 5.5
c −5.8
d −0.3, 6.5
9 a y values: −6, 0, 4, 6, 6, 4, 0, −6
b student's graph
c (2.5, 6.25)
d $x = 2.5$
e $x = 4.6$ and 0.4

16.2 Turning points on a quadratic graph

Exercise 16B

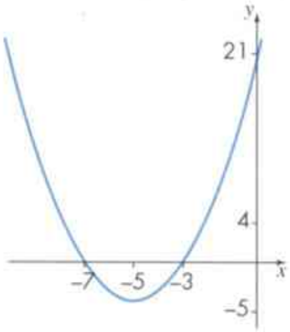
1 **a** $x^2 - 2x - 8 = (x - 1)^2 - 9$ **b** (1, −9) **c** $x = 4$ or −2

d



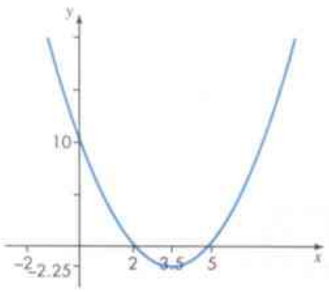
2 **a** $x^2 + 10x + 21 = (x + 5)^2 - 4$ **b** (−5, −4) **c** $x = -3$ or −7

d



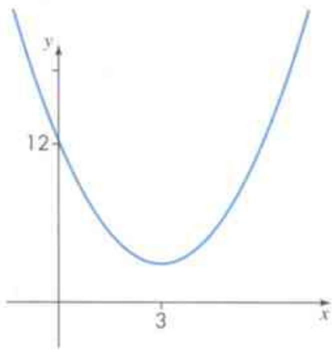
3 **a** $x^2 - 7x + 10 = (x - 3.5)^2 - 2.25$ **b** (3.5, −2.25)

c $x = 2$ or 5 **d**



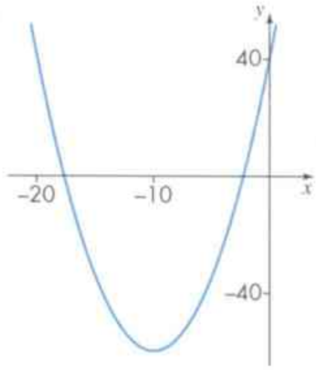
4 **a** (0, 12) **b** (3, 3)

c



d The graph does not cross the x-axis so there is no value of x for which $x^2 - 6x + 12 = 0$

5 **a** (−10, 60) **b**



6 −56.5

7 $b = -10$ and $c = 14$

16.3 Reciprocal graphs

Exercise 16C

1 **a** y values: 10, 5, 4, 2.5, 2, 1.33, 1, 0.67, 0.5
b 0.8 **c** 0.3 **d** −1.6

2 **a** y values: 25, 12.5, 10, 5, 2.5, 1, 0.5, 0.33, 0.25
b student's graph **c** −0.5 and −9.5

3 student's own graph

4 **a** y values 20, 10, 5, 4, 2.5, 2, 1
b student's graph **c** student's graph **d** $x = 6.5$ or −1.5

16.4 More graphs

Exercise 16D

1 student's own graph.

2 **a** y values: −7.81, −4, −1.69, −0.5, −0.06, 0, 0.06, 0.5, 1.69, 4, 7.81
b 2.3

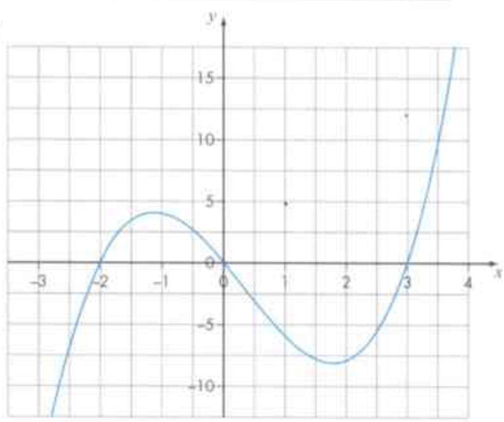
3 **a** y values: −12.63, −5, −0.38, 2, 2.89, 3, 3.13, 4, 6.38, 11, 18.63
b −1.4

4 **a** y values: 1, 4.63, 6, 5.88, 5, 4.13, 4, 5.38, 9
b $x = -1.8$ **c** $x = 1.8$ **d** (0.8, 3.9) and (−0.8, 6.1)

5 **a**

x	−3	−2	−1	0	1	2	3	4
y	−18	0	4	0	−6	−8	0	24

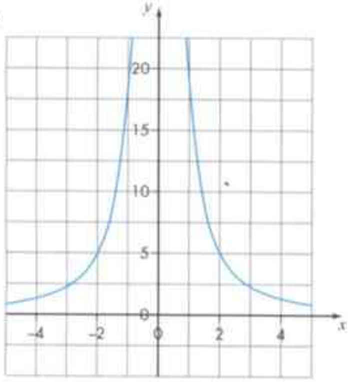
b



c $x = -2.4$, 0.8 or 2.6 **d** (−1.1, 4.1) and (1.8, −8.2)

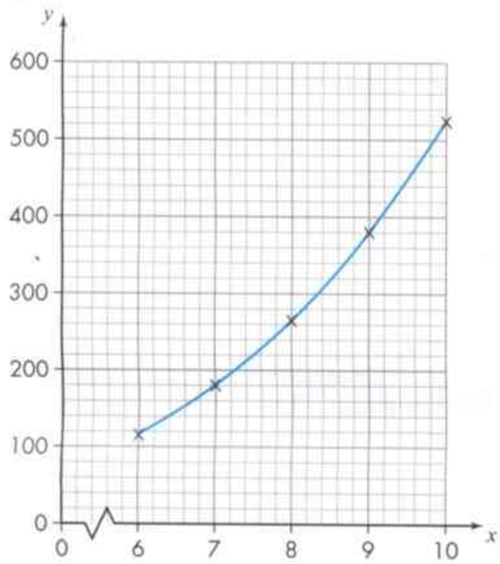
6 **a** y values: 20, 5, 2.22, 1.25, 0.8 **b** student's graph
c The asymptotes are the x-axis and the y-axis (or $y = 0$ and $x = 0$)

7 **a** y values: 4.25, 1.5, −0.22, −1.46, −2.44
b



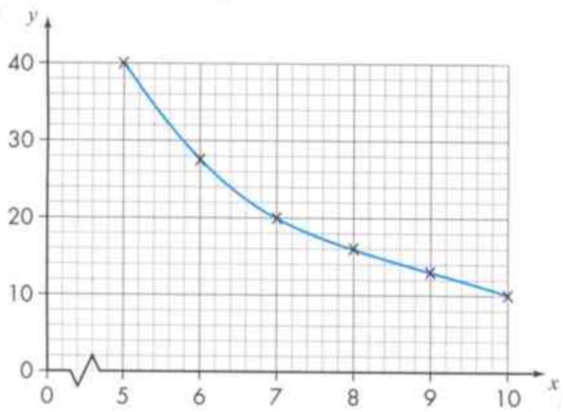
c About 5.85

8 **a**

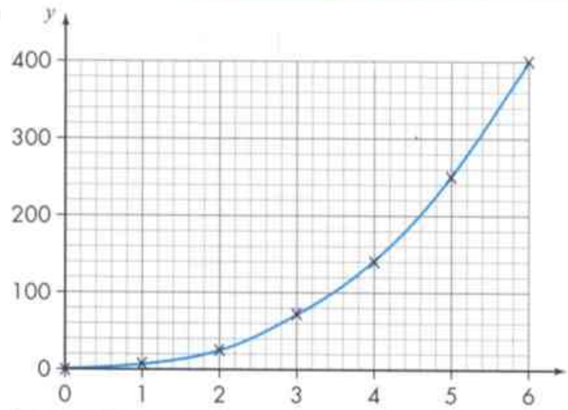


b 8.3 cm

9 **a** $x^2y = 1000 \Rightarrow y = \frac{1000}{x^2}$
b



10 **a**

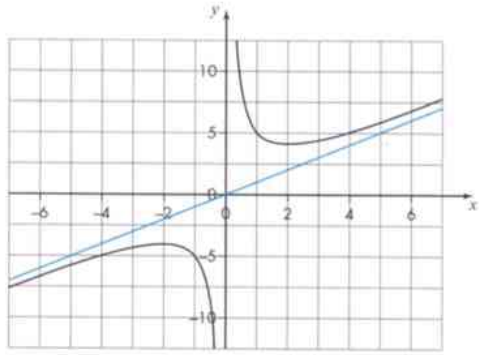


b 4.6 cm, 4.6 cm and 9.6 cm

11 **a**

x	−5	−4	−3	−2	−1	0	1	2	3	4	5
y	−5.8	−5	−4.33	−4	−5	-	5	4	4.33	5	5.8

b and c

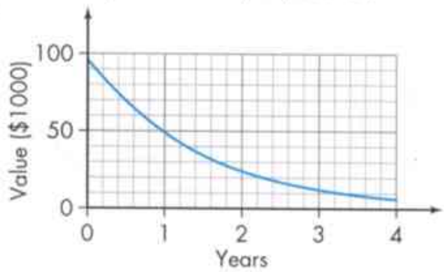


d If x is large, the curve and the straight line are very close together.
e The y-axis (or $x = 0$)

16.5 Exponential graphs

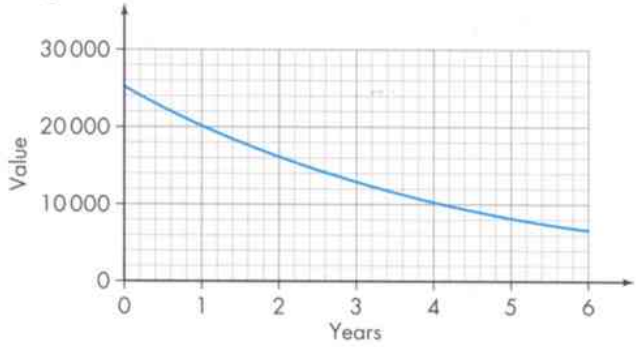
Exercise 16E

1 **a** The missing values are 48, 24, 12 and 6
b



2 **a** 25 000 × 0.8 = 20 000
b The missing values are 16 000, 12 800, 10 240, 8192

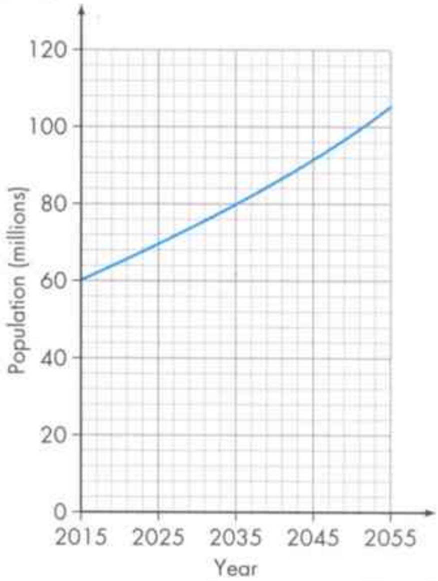
c



d 3.1 years

3 a The missing numbers, rounded to one decimal place, are 79.4, 91.3 and 104.9

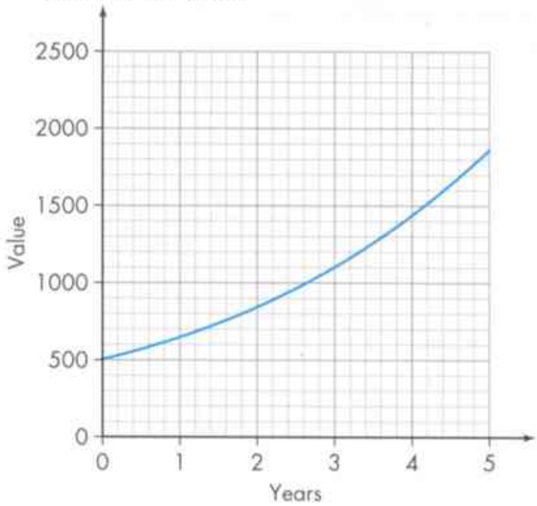
b



c 2051 or 2052

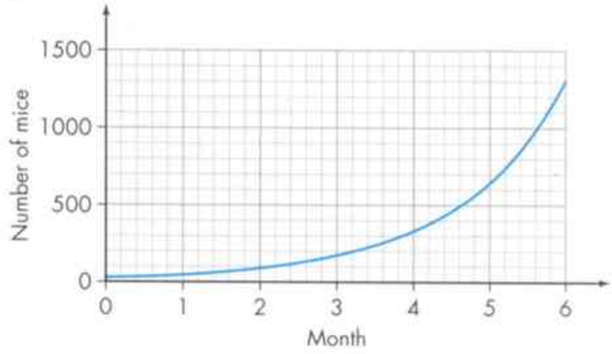
4 a the missing values, to the nearest whole number, are 650, 845, 1099, 1428

b



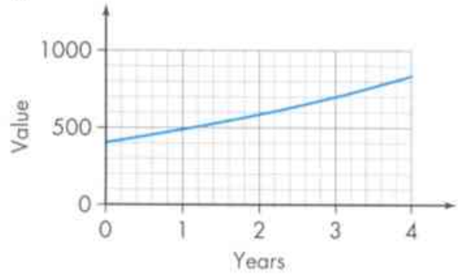
c about 2.6 or 2.7 years

5 a



b about 3.3 months

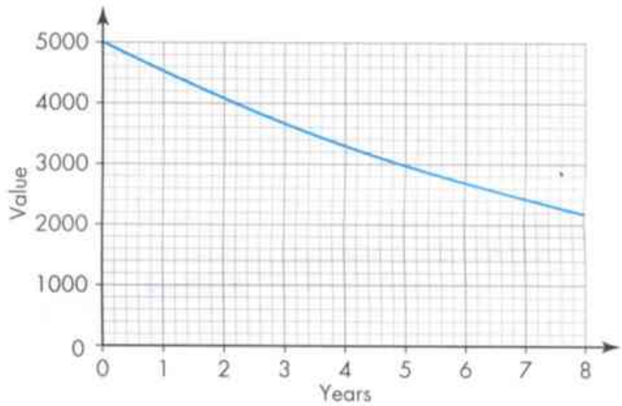
6 a 20% b



c About $630

7 a The missing values are 4500, 3645 and 3280.5

b



c About 6.6 years

8 It is not exponential growth because the gradient is decreasing. The gradient increases as time passes in exponential growth.

9 a 200 b 50% c about 2.7 hours

10 a The initial value is $1000. After two years it is $500. After four years it is $250. After six years it is $125. The value halves every two years.

b 29 or 30%

16.6 Estimating gradients

Exercise 16F

Gradients found in this exercise may vary from the answers given due to variations in drawings of the tangents

1 0.67

2 A: 0.5 B: −2

3 a student's drawing
b student's drawing
c about 1.8
d (1, 1.5)

4 a y values: 0, 0.01, 0.1, 0.34, 0.8, 1.56, 2.7
b student's drawing
c student's drawing
d about 1.2

5 a y values: 0.25, 0.5, 1, 2, 4
b student's drawing
c student's drawing
d about 0.7

6 a y values: 2.5, 1.67, 1.25, 1, 0.83
b student's drawing
c about −0.3

Answers to Chapter 17

17.1 Patterns in number sequences

Exercise 17A

1 a 9, 11, 13: add 2
b 10, 12, 14: add 2
c 80, 160, 320: double
d 81, 243, 729: multiply by 3
e 28, 34, 40: add 6
f 23, 28, 33: add 5
g 20 000, 200 000, 2 000 000: multiply by 10
h 19, 22, 25: add 3
i 114, 105, 96: subtract 9
j 405, 1215, 3645: multiply by 3
k 25, 12.5, 6.25; halve
l 625, 3125, 15 625: multiply by 5

2 a 16, 22 b 26, 37
c 31, 43 d 46, 64
e 121, 169 f 782, 3907
g 22 223, 222 223 h 11, 13
i 33, 65 j 78, 108

3 a 48, 96, 192 b 33, 39, 45
c 4, 2, 1 d 38, 35, 32
e 37, 50, 65 f 26, 33, 41
g 14, 16, 17 h 25, 22, 19
i 28, 36, 45 j 5, 6, 7
k 0.16, 0.032, 0.0064
l 0.0625, 0.031 25, 0.015 625

4 a 21, 34: add previous 2 terms
b 49, 64: next square number
c 47, 76: add previous 2 terms
d 216, 343: cube numbers

5 15, 21, 28, 36

6 61, 91, 127

7 29 and 41

8 No, they both increase by the same number (3).

9 10, 45 and 80

17.2 The nth term of a sequence

Exercise 17B

1 a 3, 5, 7, 9, 11 b 1, 4, 7, 10, 13
c 7, 12, 17, 22, 27 d 1, 4, 9, 16, 25
e 4, 7, 12, 19, 28 f 18, 16, 14, 12, 10

2 a 4, 5, 6, 7, 8 b 2, 5, 8, 11, 14
c 3, 8, 13, 18, 23 d 0, 3, 8, 15, 24
e 9, 13, 17, 21, 25 f 42, 39, 36, 33, 30

3 a 94, 88, 82, 76 b the 17th term, −2

4 a 1, 4, 9, 16 b 3, 6, 11, 18 c 2, 8, 18, 32
d 3, 15, 35, 63 e 199, 196, 191, 184
f 0.25, 1, 2.25, 4

5 a 1, 8, 27, 64 b 2, 9, 28, 65 c −1, 6, 25, 62
d 2, 23, 80, 191 e 0.5, 4, 13.5, 32 f 108, 101, 82, 45

6 a \$305 b \$600 c 3
d 5 (the amount is \$250)

7 $4n - 2 = 3n + 7$ rearranges as $4n - 3n = 7 + 2$, $n = 9$

Exercise 17C

1 a 13, 15, $2n + 1$ b 25, 29, $4n + 1$
c 33, 38, $5n + 3$ d 32, 38, $6n - 4$
e 20, 23, $3n + 2$ f 37, 44, $7n - 5$
g 17, 15; $29 - 2n$ h 22, 18; $46 - 4n$
i 17, 20, $3n - 1$ j 42, 52, $10n - 8$
k 24, 28, $4n + 4$ l 29, 34, $5n - 1$

2 a $3n + 1$, 151 b $2n + 5$, 105
c $5n - 2$, 248 d $4n - 3$, 197
e $8n - 6$, 394 f $n + 4$, 54
g $5n + 1$, 251 h $8n - 5$, 395
i $3n - 2$, 148 j $3n + 18$, 168
k $7n + 5$, 355 l $8n - 7$, 393

3 a i $4n + 1$ ii 401
b i $2n + 1$ ii 201
c i $3n + 1$ ii 301
d i $2n + 6$ ii 206
e i $4n + 5$ ii 405
f i $5n + 1$ ii 501
g i $3n - 3$ ii 297
h i $6n - 4$ ii 596
i i $8n - 1$ ii 799
j i $2n + 23$ ii 223

4 a $8n + 2$ b $8n + 1$ c $8n$ d \$8

5 a n^2 b $n^2 + 2$ c $2n^2$ d $n^2 - 1$

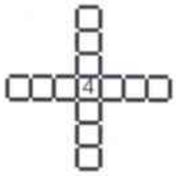
6 a n^3 b $n^3 + 10$ c $0.5n^3$ d $10n^3$

7 a $n + 5$ b $n^2 + 5$ c $n^3 + 5$
d $5n + 1$ e $5n^2$ f $5n^3$

17.3 General rules from patterns

Exercise 17D

1 a



b The missing number is 13
c $4n - 3$
d 97
e 50th diagram

2 a 
b The bottom line is 3, 5, 7, 9, 11
c $2n + 1$
d 121
e 49th set

3 a 18
b the bottom line is 6, 10, 14, 18
c $4n + 2$
d 12

4 a i 24 ii $5n - 1$ iii 224
b 25

5 a i 20 cm ii $(3n + 2)$ cm iii 152 cm
b 332

6 a i 10 ii $2n + 2$ iii 162
b 79.8 km

7 a i 14 ii $3n + 2$ iii 41
b 66

8 a i 5 ii n iii 18
b Formula gives 3 and 6
c 55

9 a

1	2	3
3	9	27
1	4	13
4	13	40

b the numbers in column 4 (top to bottom) are: 4, 81, 40, 121. Student's explanation of method.

10 a Student's drawing – one complete recurring sequence should be added to each one.
b bottom row: 5, 9, 13, 17
c $4n + 1$
d $3n + 1$
e $2n + 1$
f $9n + 3$

17.4 Further sequences

Exercise 17E

1 a 432 b 1053 c 1250 d 41.472
e 640 000 f 15 g 1.6 h 32

2 a 6, 1296 b 16, 128 c 15, 405
d 20, 2.5 e 54, 16

3 a 25×2^n b 1.5×2^n c 2×3^n d 240×0.5^n
e 50×0.9^n f 64×1.25^n

4 a 13 720, 19 208 b 5000×1.4^n

5 a $n^2 + 1$ b $n^2 + 6$ c $n^2 - n$ d $3n^2$
e $3n^2 - 2$ f $3n^2 + n$

6 a 15, 21 b $n^2 + n$ c $0.5(n^2 + n)$ or $0.5n^2 + 0.5n$
d 210 e It is the 50th triangular number because $(50^2 + 50) \div 2 = 1275$

7 a $n^3 - 1$ b $n^3 + 50$ c $n^3 + n$ d $n^3 + 3n$ e $4n^3$ f $4n^3 - n$

8 a 20 b $(4 \times 5 \times 6) \div 6 = 20$ c 7 (using 84 oranges)
d The layers of the tetrahedron are triangular numbers. 20 layers have $(20 \times 21 \times 22) \div 6 = 1540$ oranges.

9 a When $n = 1$ the first term is $a + b$ and this is 6.
b $2a + 4b = 16$ c $a = 4$, $b = 2$; the nth term is $4n + 2n^2$

Answers to Chapter 18

18.1 Using indices

Exercise 18A

1 a 2^4 b 3^5 c 7^2 d 5^3 e 10^7
f 6^4 g 4^1 h 1^7 i 0.5^4 j 100^3

2 a $3 \times 3 \times 3 \times 3$
b $9 \times 9 \times 9$
c 6×6
d $10 \times 10 \times 10 \times 10 \times 10$
e $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$
f 8
g $0.1 \times 0.1 \times 0.1$
h 2.5×2.5
i $0.7 \times 0.7 \times 0.7$
j 1000×1000

3 a 16 b 243
c 49 d 125
e 10 000 000 f 1296
g 4 h 1
i 0.0625 j 1 000 000

4 a 81 b 729
c 36 d 100 000
e 1024 f 8
g 0.001 h 6.25
i 0.343 j 1 000 000

5 125 m^3

6 **b** 10^2 **c** 2^3 **d** 5^2

7 **a** 1 **b** 4 **c** 1 **d** 1 **e** 1

8 Any power of 1 is equal to 1.

9 10^6

10 10^6

11 **a** 1 **b** −1 **c** 1 **d** 1 **e** −1

12 **a** 1 **b** −1 **c** −1 **d** 1 **e** 1

13 2^{24}, 4^{12}, 8^8, 16^6

18.2 Negative indices

Exercise 18B

1 **a** $\frac{1}{5^3}$ **b** $\frac{1}{6}$ **c** $\frac{1}{10^5}$ **d** $\frac{1}{3^2}$
e $\frac{1}{8^2}$ **f** $\frac{1}{9}$ **g** $\frac{1}{w^2}$ **h** $\frac{1}{t}$
i $\frac{1}{x^m}$ **j** $\frac{4}{m^3}$

2 **a** 3^{-2} **b** 5^{-1} **c** 10^{-3} **d** m^{-1} **e** t^{-n}

3 **a i** 2^4 **ii** 2^{-1} **iii** 2^{-4} **iv** -2^3
b i 10^3 **ii** 10^{-1} **iii** 10^{-2} **iv** 10^6
c i 5^3 **ii** 5^{-1} **iii** 5^{-2} **iv** 5^{-4}
d i 3^2 **ii** 3^{-3} **iii** 3^{-4} **iv** -3^5

4 **a** $\frac{5}{x^3}$ **b** $\frac{6}{t}$ **c** $\frac{7}{m^2}$ **d** $\frac{4}{q^4}$
e $\frac{10}{y^5}$ **f** $\frac{1}{2x^3}$ **g** $\frac{1}{2m}$ **h** $\frac{3}{4t^4}$
i $\frac{4}{5y^3}$ **j** $\frac{7}{8x^5}$

5 **a** $7x^{-3}$ **b** $10p^{-1}$ **c** $5t^{-2}$ **d** $8m^{-5}$ **e** $3y^{-1}$

6 **a i** 25 **ii** $\frac{1}{125}$ **iii** $\frac{4}{5}$
b i 64 **ii** $\frac{1}{16}$ **iii** $\frac{5}{256}$
c i 8 **ii** $\frac{1}{32}$ **iii** $\frac{9}{2}$ or $4\frac{1}{2}$
d i 1 000 000 **ii** $\frac{1}{1000}$ **iii** $\frac{1}{4}$

7 24 (32 − 8)

8 $x = 8$ and $y = 4$ (or $x = y = 1$)

9 $\frac{1}{2097152}$

10 **a** x^{-5}, x^0, x^5 **b** x^5, x^0, x^{-5} **c** x^5, x^{-5}, x^0

11 **a** $\frac{M}{3}$ **b** 3M **c** 27M

18.3 Multiplying and dividing with indices

Exercise 18C

1 **a** 5^4 **b** 5^3 **c** 5^2 **d** 5^3 **e** 5^{-5}

2 **a** 6^3 **b** 6^0 **c** 6^6 **d** 6^{-7} **e** 6^2

3 **a** a^3 **b** a^5 **c** a^7 **d** a^4 **e** a^2 **f** a^1

4 **a** Any two values such that $x + y = 10$
b Any two values such that $x - y = 10$

5 **a** 4^6 **b** 4^{15} **c** 4^6
d 4^{-6} **e** 4^6 **f** 4^0

6 **a** $6a^5$ **b** $9a^2$ **c** $8a^6$
d $-6a^4$ **e** $8a^8$ **f** $-10a^{-3}$

7 **a** $3a$ **b** $4a^3$ **c** $3a^4$
d $6a^{-1}$ **e** $4a^7$ **f** $5a^{-4}$

8 **a** $8a^5b^4$ **b** $10a^3b$ **c** $30a^{-2}b^{-2}$
d $2ab^3$ **e** $8a^{-5}b^7$

9 **a** $3a^3b^2$ **b** $3a^2c^4$ **c** $8a^2b^2c^3$

10 **a** Possible answer: $6x^2 \times 2y^5$ and $3xy \times 4xy^4$
b Possible answer: $24x^2y^7 \div 2y^2$ and $12x^6y^8 \div x^4y^3$

11 12 ($a = 2$, $b = 1$, $c = 3$)

12 **a** A^2 **b** A^{-1} **c** $A^{\frac{1}{2}}$ or $\sqrt{A}$ **d** $A^{\frac{1}{3}}$ or $\sqrt[3]{A}$

13 **a** $x^{2n+1} = x^{2n} x = (x^n)^2 \times x = xy^2$ **b** $\frac{y^2}{x}$

18.4 Fractional indices

Exercise 18D

1 **a** 5 **b** 10 **c** 8 **d** 9 **e** 25
f 3 **g** 4 **h** 10 **i** 5 **j** 8
k 12 **l** 20 **m** 5 **n** 3 **o** 10
p 3 **q** 2 **r** 2 **s** 6 **t** 6
u $\frac{1}{4}$ **v** $\frac{1}{2}$ **w** $\frac{1}{3}$ **x** $\frac{1}{5}$ **y** $\frac{1}{10}$

2 **a** $\frac{5}{6}$ **b** $1\frac{2}{3}$ **c** $\frac{8}{9}$ **d** $1\frac{4}{5}$ **e** $\frac{5}{8}$
f $\frac{3}{5}$ **g** $\frac{1}{4}$ **h** $2\frac{1}{2}$ **i** $\frac{4}{5}$ **j** $1\frac{1}{7}$

3 $(x^{\frac{1}{n}})^n = x^{\frac{1}{n} \times n} = x^1 = x$, but $(^n\sqrt{x})^n = {^n\sqrt{x}} \times n\sqrt{x} \ldots n$ times $= x$, so $x^{\frac{1}{n}} = n\sqrt{x}$

4 $64^{-\frac{1}{2}} = \frac{1}{8}$, others are both $\frac{1}{2}$

5 Possible answer: The negative power gives the reciprocal, so $27^{-\frac{1}{3}} = \frac{1}{27^{\frac{1}{3}}}$. The power one-third means cube root, so you need the cube root of 27 which is 3, so $27^{\frac{1}{3}} = 3$ and $\frac{1}{27^{\frac{1}{3}}} = \frac{1}{3}$

6 Possible answer: $x = 1$ and $y = 1$, $x = 8$ and $y = \frac{1}{64}$.

7 **a** 3 **b** $\frac{1}{3}$ **c** 0 **d** $\frac{1}{2}$
e $\frac{1}{2}$ **f** $\frac{1}{4}$ **g** $\frac{1}{4}$ **h** $\frac{1}{3}$
i $\frac{1}{3}$ **j** $\frac{1}{2}$ **k** $\frac{1}{3}$ **l** $\frac{1}{7}$

Exercise 18E

1 **a** 16 **b** 25 **c** 216 **d** 81

2 **a** $t^{\frac{2}{3}}$ **b** $m^{\frac{3}{4}}$ **c** $k^{\frac{2}{5}}$ **d** $x^{\frac{3}{2}}$

3 **a** 4 **b** 9 **c** 64 **d** 3125

4 **a** $\frac{1}{5}$ **b** $\frac{1}{6}$ **c** $\frac{1}{2}$ **d** $\frac{1}{3}$
e $\frac{1}{4}$ **f** $\frac{1}{2}$ **g** $\frac{1}{2}$ **h** $\frac{1}{3}$

5 **a** $\frac{1}{125}$ **b** $\frac{1}{216}$ **c** $\frac{1}{8}$ **d** $\frac{1}{27}$
e $\frac{1}{256}$ **f** $\frac{1}{4}$ **g** $\frac{1}{4}$ **h** $\frac{1}{9}$

6 **a** $\frac{1}{100000}$ **b** $\frac{1}{12}$ **c** $\frac{1}{25}$ **d** $\frac{1}{27}$
e $\frac{1}{32}$ **f** $\frac{1}{32}$ **g** $\frac{1}{81}$ **h** $\frac{1}{13}$

7 $8^{-\frac{2}{3}} = \frac{1}{4}$, others are both $\frac{1}{8}$

8 Possible answer: The negative power gives the reciprocal, so the power one-third means cube root, so we need the cube root of 27 which is 3 and the power 2 means square, so $3^2 = 9$, so $27^{\frac{2}{3}} = 9$ and $\frac{1}{27^{\frac{2}{3}}} = \frac{1}{9}$

9 a $\frac{27}{8}$ b $\frac{9}{25}$ c $\frac{1024}{243}$ d $\frac{8}{343}$
e $\frac{16}{9}$ f $\frac{8}{27}$ g $\frac{625}{256}$ h $\frac{32}{243}$

10 a $\frac{25}{9}$ b $\frac{27}{64}$ c $\frac{125}{729}$ d $\frac{243}{32}$
e $\frac{8}{27}$ f $\frac{243}{32}$ g $\frac{9}{4}$ h $\frac{125}{343}$
i $\frac{16}{25}$ j $\frac{512}{125}$ k $\frac{243}{32}$ l $\frac{32}{243}$

11 a x^4 b x^{-1} c $4y^2$
d $10x^2$ e $20x^{-1}$ f $\frac{1}{3}y$

12 a x b d^{-1} c $t^{\frac{3}{2}}$
d x^2 e $y^{\frac{1}{2}}$ f a^4

13 a $x^{\frac{1}{2}}$ b y^{-1} c $a^{\frac{5}{3}}$
d t^{-2} e d^2 f 1

14 $y^{\frac{9}{4}}$

Answers to Chapter 19

19.1 Direct proportion

Exercise 19A

1 a 15 b 2
2 a 75 b 6
3 a 150 b 6
4 a 22.5 b 12
5 a 175 kilometres b 8 hours
6 a 66.50 dollars b 175 kg
7 a 44 b 84 m^2
8 a 33 spaces
b 66 spaces since new car park has 366 spaces
9 17 minutes 30 seconds

Exercise 19B

1 a 100 b 10
2 a 27 b 5
3 a 56 b 1.69
4 a 192 b 2.25
5 a 25.6 b 5
6 a 80 b 8
7 a $50 b 225
8 a 3.2°C b 10 atm
9 a 388.8 g b 3 mm
10 a 2 J b 40 m/s
11 a 78 dollars b 400 miles
12 4000 cm^3
13 $250
14 a B b A c C
15 a B b A

19.2 Inverse proportion

Exercise 19C

1 $Tm = 12$ a 3 b 2.5
2 $Wx = 60$ a 20 b 6
3 $Q(5 - t) = 16$ a −3.2 b 4
4 $Mt^2 = 36$ a 4 b 5
5 $W\sqrt{T} = 24$ a 4.8 b 100
6 $x^3y = 32$ a 32 b 4
7 $gp = 1800$ a $15 b 36
8 $td = 24$ a 3°C b 12 km
9 $ds^2 = 432$ a 1.92 km b 8 m/s
10 $p\sqrt{h} = 7.2$ a 2.4 atm b 100 m
11 $W\sqrt{F} = 0.5$ a 5 t/h b 0.58 t/h
12 B – This is inverse proportion, as x increases y decreases.
13

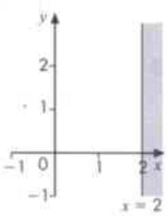
x	8	27	64
y	1	$\frac{2}{3}$	$\frac{1}{2}$

Answers to Chapter 20

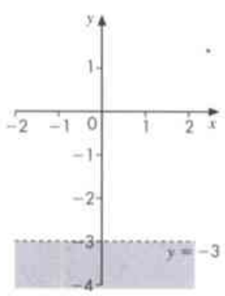
20.1 Graphical inequalities

Exercise 20A

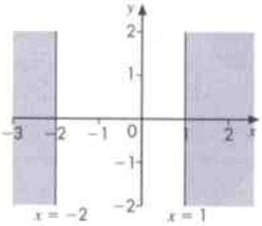
1 a & b



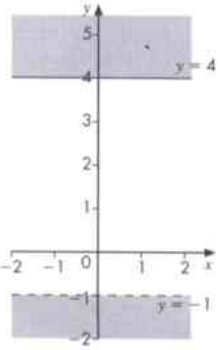
2 a & b



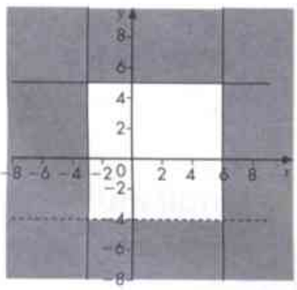
3 a–c



4 a–c

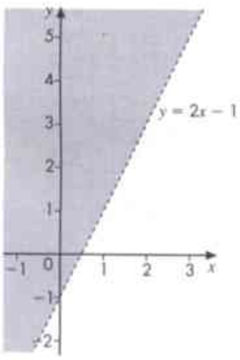


5

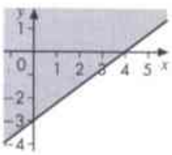


b i Yes ii Yes iii No

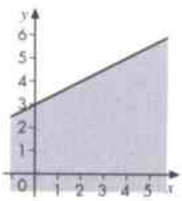
6 a & b



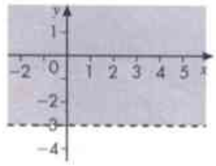
7 a & b



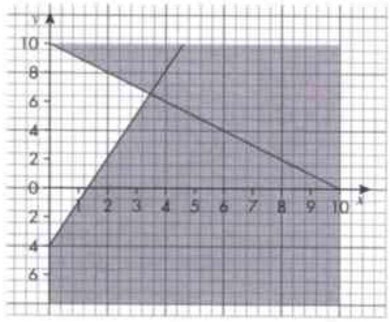
8 a & b



9



10 a–d

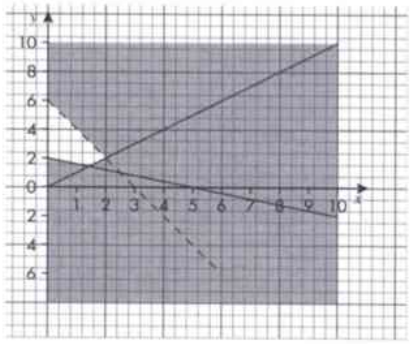


e i No ii Yes iii Yes

20.2 More than one inequality

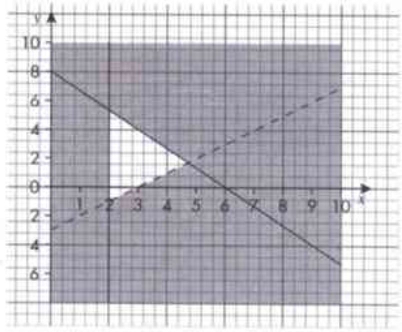
Exercise 20B

1 a–f



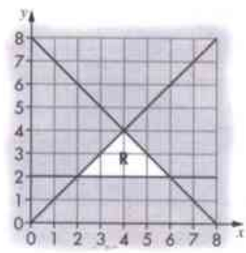
g i No ii No iii Yes

2 a

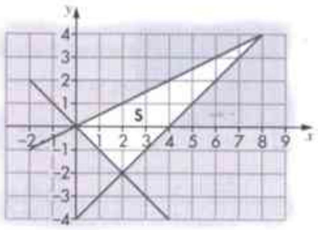


b i No ii Yes iii Yes iv No

3 a & b



4 a & b



c 4 d −2 e 12

5 Test a point such as the origin (0, 0), so $0 < 0 + 2$, which is true. So the side that includes the origin is the required side.

6 a $x + y \geqslant 3$, $y \leqslant \frac{1}{2}x + 3$ and $y \geqslant 5x - 15$

b 9

c 3 at (3,0)

20.3 Linear programming

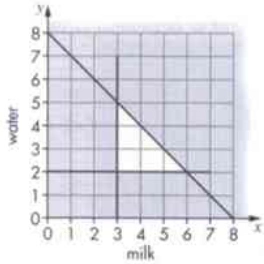
Exercise 20C

1 a He buys at least 3 cartons of milk

b $y \geqslant 2$

c $x + y \leqslant 8$

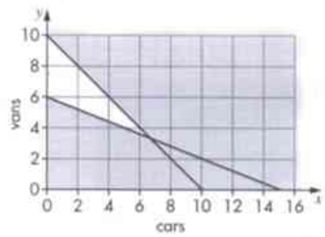
d



2 a $x + y \leqslant 10$

b $2x + 5y \geqslant 30$

c

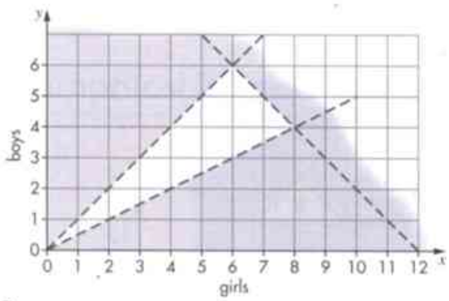


d 6 because cars & vans (x, y) must both be integers

e 4

3 a $x > y$ $x < 2y$ $x + y < 12$

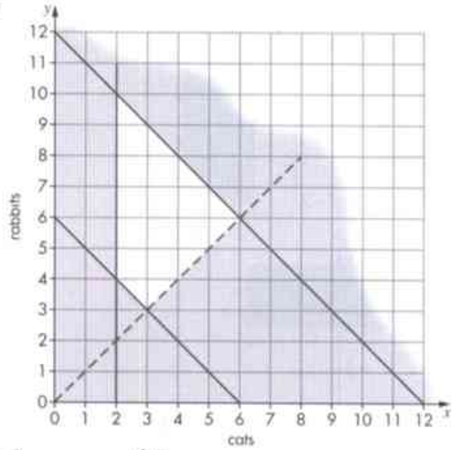
b



c 7

4 a $x + y \geqslant 6$ $x + y \leqslant 12$ $y > x$ $x \geqslant 2$

b

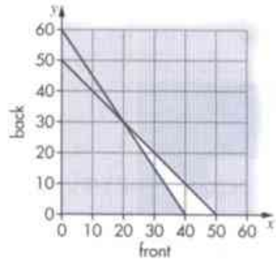


c 4 d 5

5 a Follows from $15x + 10y \geqslant 600$

b $x + y \leqslant 50$

c

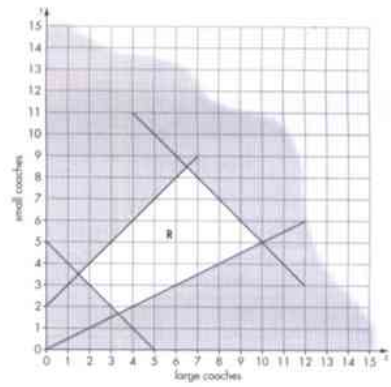


d 30

6 a i $x + y \leqslant 15$ ii $x + y \geqslant 5$ iii $y \geqslant \frac{1}{2}x$

b the number of small coaches is not more than 2 more than the number of large coaches

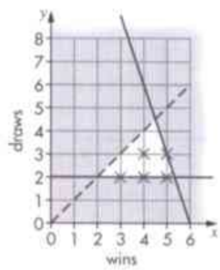
c



d 3, 4, 5, 6, 7 or 8

7 a $3x + y$ b $3x + y \leqslant 18$ $x > y$ $y \geqslant 2$

c



d 3 wins and 2 draws; 4 wins and 2 draws; 5 wins and 2 draws; 4 wins and 3 draws; 5 wins and 3 draws

Answers to Chapter 21

21.1 Function notation

Exercise 21A

1 a 12 b 26 c 7
d −2 e 3

2 a 0.5 b 5 c 50.5
d 2.5 e 0.625 or $\frac{5}{8}$

3 a 5 b −3 c 999801
d 1 e $\frac{1}{8}$

4 a 4 b 32 c 1
d $\frac{1}{2}$ e $\frac{1}{8}$

5 a 3 b 2 c 0
d −1 e 5

6 a 7.5 b −2.5 c −5

7 a 6 b 97 c 3.25

8 a 6 b at (6, 4)

9 a 3 b $-\frac{1}{2}$ c $x = 2$
d $x = 0$

21.2 Inverse functions

Exercise 21B

1 a $x - 7$ b $\frac{x}{8}$ c $5x$ d $x + 3$

2 a 8 b 4 c 5 d −2

3 a $3(x + 2)$ b $\frac{x}{4} + 5$ c $5x - 4$ d $\frac{5x + 6}{3}$
e $2(\frac{x}{3} - 4)$ f $\sqrt[3]{\frac{x}{4}}$

4 a 2 b $\frac{1}{2}$ c −2.5

5 a $10 - x$ b They are identical

6 a $\frac{8}{x}$ b $\frac{20}{x + 1}$ c $\frac{2}{x} - 1$

7 a $\frac{x + 4}{2}$ b student's graph c (4,4)

8 5

9 $\frac{x + 2}{3}$

21.3 Composite functions

Exercise 21C

1 a 6 and 3 b 7 and 3.5
c 10 and 5 d $\frac{x + 4}{2}$
e 1 and 5 f 1.5 and 5.5
g −5 and −1 h $\frac{x}{2} + 4$

2 a 6 and 216 b 10 and 1000 c $(2x)^3$ or $8x^3$
d 64 and 128 e $2x^3$

3 a 1, 9, 25 b 1, 3, 5 c $\sqrt{2x + 1}$

4 a 6 and 18 b 12 and 36 c $9x$

5 a $3(x - 6)$ b $3x - 6$

6 Both are $x - 3$

21.4 More about composite functions

Exercise 21D

1 a 3.5 b 1
c 8 d 5.5

2 a 20 b 9
c 8.75 d 3

3 a 7 b 8
c 256 d 21

4 a $6x$ b $6x - 5$

5 a $9x^2 + 24x + 16$ or $(3x + 4)^2$ b $6x - 5$
c $2x + 3$ d $4 - 2x$

6 a $x - 10$ b $x + 10$ c x

7 a x^4 b $\left(\frac{12}{x}\right)^2$ or $\frac{144}{x^2}$
c $\frac{12}{x^2}$ d x

8 a 80
b $2(2x - 1) - 1$ simplified
c $(2x - 1)^2 + 2(2x - 1)$ simplified

9 a $\frac{1}{3x - 3}$ b $\frac{x - 1}{x - 4}$

10 a $6x - 14$ b $\frac{x + 12}{4}$

11 a 0.5 (1 + 9) = 5 b 7 c 8
d 8.5, 8.75, 8.875, 8.9375, 8.96875, 8.984375
e Getting closer and closer to 9, halving the difference from 9 each time.

12 Student's own description of the convergence towards 9.

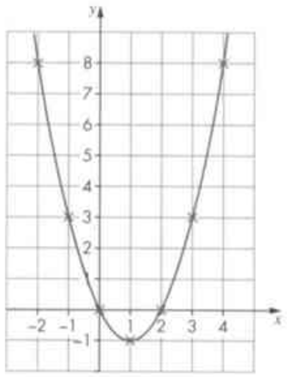
Answers to Chapter 22

22.1 The gradient of a curve

Exercise 22A

1 **a** The missing numbers are 0, –1, 0

b



c $2x - 2$		**d** 4	
e 6		**f** student's choice	
g (1, –1)		**h** student's check	

2 **a** $2x - 6$ **b** –6 **c** 4 **d** (4, 7)

3 **a** $4x$ **b** 8 **c** –4 **d** (3, 8)

4 **a** $4 - 2x$ **b** 4 and –4 **c** (1, 3) **d** (1.5, 3.75)

5 **a** $2x + 1$ **b** $2x - 7$ **c** $8x - 1$ **d** $0.6x - 1.5$
e $-2 + 2x$ **f** $3 - 2x$ **g** 2 **h** 0

6 $2x + 2$

7 **a** $4x + 2$ **b** $2x + 7$ **c** $2x$

8 **a** (0, –5) **b** 2

22.2 More complex curves

Exercise 22B

1 **a** $6x^2$ **b** 6 and 24

2 **a** $3x^2 - 12x + 8$
b If $x = 0$ or 2 or 4, $y = 0$
c 8; –4; 8

3 **a** $1.5x^2 - 6x + 4$
b 4 at (0, 0) and (4, 0); –2 at (2,0)

4 **a** $8x^3$ **b** $6x^2 + 10x$
c $15x^2 - 2$ **d** $-1 - 2x^2$
e $9x^2 + 5$ **f** $-3x^2$
g $4x^3 - 1$ **h** $8x^3 + 18x^2$

5 16 at (2, 0); –16 at (–2, 0); 0 at (0, 0)

6 **a** $dy/dx = 4x^3 - 6x^2$ and if $x = 0$ then $dy/dx = 0$
b –10 **c** 8

7 $x^2 - 5 = 4$ has two solutions, $x = 3$ or -3.
Points are (3, –2) and (–3, 10)

8 $y = 1.5x - 2$

9 **a** 12 **b** 24

22.3 Turning points

Exercise 22C

1 **a** $2x - 4$ **b** $2x - 4 = 0 \Rightarrow x = 2$; (2, –1)
c Minimum

2 **a** (–3, –12) **b** Minimum

3 **a** $5 - 2x$ **b** (2.5, 7.25)
c Maximum

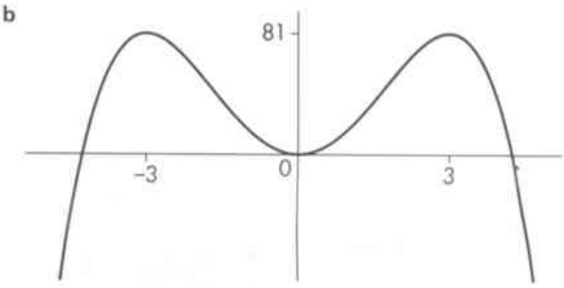
4 2 and –3

5 **a** $3x^2 - 6x$ **b** $x = 0$ or 2
c (0, 0) and (2, –4)

6 **a** If $x = -2$ or 5, $y = 0$ **b** $2x - 3$
c (1.5, –12.25); Minimum **d** $x = 1.5$

7 **a** (–3, 81), (0, 0) and (3, 81)

b



8 **a** $6x^2 - 6$ **b** (1, 0) minimum, (–1, 8) maximum

9 **a** The two sides add up to half the perimeter
b $15 - 2x$
c (7.5, 56.25)
d Maximum
e The largest possible area is 56.25 cm^2, when the rectangle is a square of side 7.5 cm

Answers to Chapter 23

23.1 Angle facts

Exercise 23A

1 **a** 48° **b** 307° **c** 108° **d** 52°
e 59° **f** 81° **g** 139° **h** 58°

2 **a** 82° **b** 105° **c** 75°

3 45° + 125° = 170° and for a straight line it should be 180°.

4 **a** $x = 100°$ **b** $x = 110°$ **c** $x = 30°$

5 **a** $x = 55°$ **b** $x = 45°$ **c** $x = 12.5°$

6 **a** $x = 34°$, $y = 98°$ **b** $x = 70°$, $y = 120°$ **c** $x = 20°$, $y = 80°$

7 $6 \times 60° = 360°$; imagine six of the triangles meeting at a point.

8 $x = 35°$, $y = 75°$; $2x = 70°$ (opposite angles), so $x = 35°$ and $x + y = 110°$ (angles on a line), so $y = 75°$

9 a = 88 b = 132

23.2 Parallel lines

Exercise 23B

1 a 40° b $b = c = 70°$
c $d = 75$, $e = f = 105°$ d $g = 50°$, $h = i = 130°$
e $j = k = l = 70°$ f $n = m = 80°$

2 a $a = 50°$, $b = 130°$ b $c = d = 65°$, $e = f = 115°$
c $g = i = 65°$, $h = 115°$ d $j = k = 72°$, $l = 108°$
e $m = n = o = p = 105°$ f $q = r = s = 125°$

3 a $a = 95°$ b $b = 66°$, $c = 114°$

4 a $x = 30°$, $y = 120°$ b $x = 25°$, $y = 105°$
c $x = 30°$, $y = 100°$

5 a $x = 50°$, $y = 110°$ b $x = 25°$, $y = 55°$
c $x = 20$, $y = 140°$

6 290°; x is double the angle allied to 35°, so is $2 \times 145°$

7 $a = 66$

8 Angle $PQD = 64°$ (alternate angles), so angle $DQY = 116°$ (angles on a line = 180°)

9 Use alternate angles to see b, a and c are all angles on a straight line, and so total 180°.

10 Third angle in triangle equals q (alternate angle), angle sum of triangle is 180°.

11 $A + D = 180°$ because they are allied angles. $C + D = 180°$ because they are allied angles. Hence $A = C$.
In the same way $B + C = 180° = D + C$ because they are pairs of allied angles. Hence $B = D$.

23.3 Angles in a triangle

Exercise 23C

1 a 70° b 50° c 80° d 60°
e 75° f 109° g 38° h 63°

2 a No, total is 190° b Yes, total is 180° c No, total is 170°
d Yes, total is 180° e Yes, total is 180° f No, total is 170°

3 a 60° b Equilateral triangle c Same length

4 a 70° each b Isosceles triangle c Same length

5 a 109° b 130° c 135°

6 Isosceles triangle; angle DFE ∠30° (opposite angles), angle DEF ∠75° (angles on a line), angle FDE ∠75° (angles in a triangle), so there are two equal angles in the triangle and hence it is an isosceles triangle

7 a is ∠80° (opposite angles), b is 65° (angles on a line), c is 35° (angles in a triangle)

8 Missing angle = y, $x + y = 180°$ and $a + b + y = 180°$ so $x = a + b$

9 $b = 240 - a$

23.4 Angles in a quadrilateral

Exercise 23D

1 a $a = 110°$, $b = 55°$ b $c = e = 105°$, $d = 75°$
c $f = 135°$, $g = 25°$ d $h = i = 94°$
e $j = l = 105°$, $k = 75°$ f $m = o = 49°$, $n = 131°$

2 a $x = 25°$, $y = 15°$ b $x = 7°$, $y = 31°$ c $x = 60°$, $y = 30°$

3 a $x = 50°$: 60°, 70°, 120°, 110° – possibly trapezium
b $x = 60°$: 50°, 130°, 50°, 130° – parallelogram or isosceles trapezium
c $x = 30°$: 20°, 60°, 140°, 140° – possibly kite
d $x = 20°$: 90°, 90°, 90°, 90° – square or rectangle

4 52°

5 Both 129°

6 $y = 360° - 4x$

7 36°, 72°, 108° and 144°

23.5 Regular polygons

Exercise 23E

1 a i 45° ii 8 iii 1080°
b i 20° ii 18 iii 2880°
c i 15° ii 24 iii 3960°
d i 36° ii 10 iii 1440°

2 a i 172° ii 45 iii 7740°
b i 174° ii 60 iii 10 440°
c i 156° ii 15 iii 2340°
d i 177° ii 120 iii 21 240°

3 a Exterior angle is 7°, which does not divide exactly into 360°
b Exterior angle is 19°, which does not divide exactly into 360°
c Exterior angle is 11°, which does divide exactly into 360°
d Exterior angle is 70°, which does not divide exactly into 360°

4 a 7° does not divide exactly into 360°
b 26° does not divide exactly into 360°
c 44° does not divide exactly into 360°
d 13° does not divide exactly into 360°

5 $x = 45°$, they are the same, true for all polygons

6 a 36° b 10

7 a The exterior angle is 180 – 170 = 10°; 360 ÷ 10 = 36 so a regular polygon with 36 sides is possible.
b The exterior angle is 180 – 169 = 11°; 360 ÷ 11 is not a whole number so a regular polygon is not possible.

23.6 Irregular polygons

Exercise 23F

1 a 1440° b 2340° c 17 640° d 7740°

2 a 9 b 15 c 102 d 50

3 a 130° b 95° c 130°

4 a 50° b 40° c 59°

5 Hexagon

6 a Octagon b 89°

7 a i 71° ii 109° iii Equal
b If S = sum of the two opposite interior angles, then $S + I = 180°$ (angles in a triangle), and we know $E + I = 180°$ (angles on a straight line), so $S + I = E + I$, therefore $S = E$

8 144°; 360 – (2 × 108)°

9 Three angles are 135° and two angles are 67.5°

23.7 Tangents and diameters

Exercise 23G

1 a 38° b 110° c 15° d 45°

2 a $x = 12°$, $y = 156°$ b $x = 100°$, $y = 50°$
c $x = 62°$, $y = 28°$ d $x = 30°$, $y = 60°$

3 Angle $OCD = 58°$ (triangle OCD is isosceles), angle $OCB = 90°$ (tangent/radius theorem), so angle $DCB = 32°$, hence triangle BCD is isosceles (2 equal angles)

23.8 Angles in a circle

Exercise 23H

1 a 56° b 62° c 105° d 55°
e 45° f 30° g 60° h 145°

2 a 55° b 52° c 50° d 24°
e 39° f 80° g 34° h 30°

3 a 41° b 49° c 41°

4 a 72° b 37° c 72°

5 Angle $AZY = 35°$ (angles in a triangle), $a = 55°$ (angle in a semicircle = 90°)

6 a $x = y = 40°$ b $x = 131°, y = 111°$
c $x = 134°, y = 23°$ d $x = 32°, y = 19°$
e $x = 59°, y = 121°$ f $x = 155°, y = 12.5°$

7 68°

8 Angle $ABC = 180° - x$ (angles on a line), angle $AOC = 360° - 2x$ (angle at centre is twice angle at circumference), reflex angle $AOC = 360° - (360° - 2x) = 2x$ (angles at a point)

9 a x
b $2x$
c From part **b**, angle $AOD = 2x$
Similarly, angle $COD = 2y$
So angle $AOC = AOD + COD = 2x + 2y = 2(x + y)$
$= 2 \times$ angle ABC

23.9 Cyclic quadrilaterals

Exercise 23I

1 a $a = 50°, b = 95°$ b $c = 92°, x = 90°$
c $d = 110°, e = 110°, f = 70°$ d $g = 105°, h = 99°$
e $j = 89°, k = 89°, l = 91°$ f $m = 120°, n = 40°$
g $p = 44°, q = 68°$ h $x = 40°, y = 34°$

2 a $x = 26°, y = 128°$ b $x = 48°, y = 78°$
c $x = 133°, y = 47°$ d $x = 36°, y = 72°$
e $x = 55°, y = 125°$ f $x = 35°$
g $x = 48°, y = 45°$ h $x = 66°, y = 52°$

3 a $x = 49°, y = 49°$ b $x = 70°, y = 20°$
c $x = 80°, y = 100°$ d $x = 100°, y = 75°$

4 a $x = 50°, y = 62°$ b $x = 92°, y = 88°$
c $x = 93°, y = 42°$ d $x = 55°, y = 75°$

5 a $x = 95°, y = 138°$ b $x = 14°, y = 62°$
c $x = 32°, y = 48°$ d 52°

6 a 71° b 125.5° c 54.5°

7 a $x + 2x - 30° = 180°$ (opposite angles in a cyclic quadrilateral), so $3x - 30° = 180°$
b $x = 70°$, so $2x - 30° = 110°$ angle $DOB = 140°$ (angle at centre equals twice angle at circumference), $y = 80°$ (angles in a quadrilateral)

8 a x b $360° - 2x$
c Angle $ADC = \frac{1}{2}$reflex angle $AOC = 180° - x$, so angle $ADC +$ angle $ABC = 180°$

9 Let angle $AED = x$, then angle $ABC = x$ (opposite angles are equal in a parallelogram), angle $ADC = 180° - x$ (opposite angles in a cyclic quadrilateral), so angle $ADE = x$ (angles on a line)

10 Let angle $ABC = x$ and angle $EFG = y$.
Then angle $ADC = 180 - x°$ (opposite angles in a cyclic quadrilateral) and angle $EDG = 180 - y°$.
But angle ADC = angle EDG (opposite angles).
$180 - x° = 180 - y°$ and therefore $x = y$.

23.10 Alternate segment theorem

Exercise 23J

1 a $a = 65°, b = 75°, c = 40°$
b $d = 79°, e = 58°, f = 43°$
c $g = 41°, h = 76°, i = 76°$
d $k = 80°, m = 52°, n = 80°$

2 a $a = 75°, b = 75°, c = 75°, d = 30°$
b $a = 47°, b = 86°, c = 86°, d = 47°$
c $a = 53°, b = 53°$
d $a = 55°$

3 a 36° b 70°

4 a $x = 25°$ b $x = 46°, y = 69°, z = 65°$
c $x = 38°, y = 70°, z = 20°$ d $x = 48°, y = 42°$

5 Angle $ACB = 64°$ (angle in alternate segment), angle $ACX = 116°$ (angles on a line), angle $CAX = 32°$ (angles in a triangle), so triangle ACX is isosceles (two equal angles)

6 Angle $AXY = 69°$ (tangents equal and so triangle AXY is isosceles), angle $XZY = 69°$ (alternate segment), angle $XYZ = 55°$ (angles in a triangle)

7 a $2x$ b $90° - x$ c angle $OPT = 90°$, so angle $APT = x$

Answers to Chapter 24

24.1 Measuring and drawing angles

Exercise 24A

1 a 40° b 125° c 340° d 225°

2 student's drawings of angles

3 AC and BE; AD and CE; AE and CF.

4 Yes, the angle is 75°.

5 Any angle between 90° and 100°.

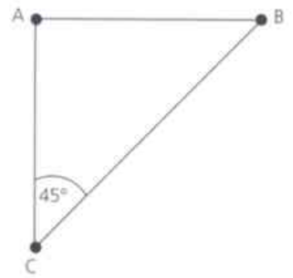
6 a 80° b 50° c 25°

24.2 Bearings

Exercise 24B

1 a 110° b 250° c 091° d 270° e 130° f 180°

2 student's sketches

3 

4 **a** 090°, 180°, 270° **b** 000°, 270°, 180°

5

Leg	Actual distance	Bearing
1	50 km	060°
2	70 km	355°
3	65 km	260°
4	46 km	204°
5	60 km	130°

6 **a** 045° **b** 286°

7 **a** 250° **b** 325° **c** 144°

8 **a** 900 m **b** 280°
c angle *NHS* = 150° and HS = 3 cm

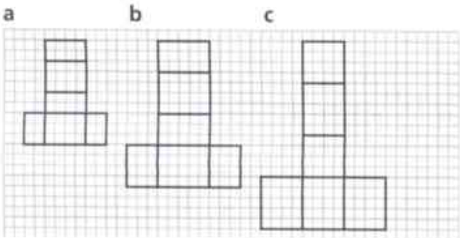
9 108°

10 255°

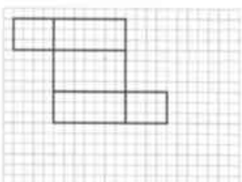
24.3 Nets

Exercise 24C

1 **a** **b** **c**



2 Yes.



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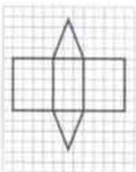
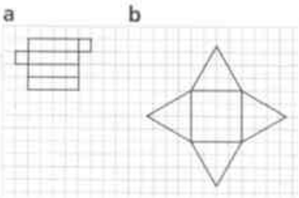
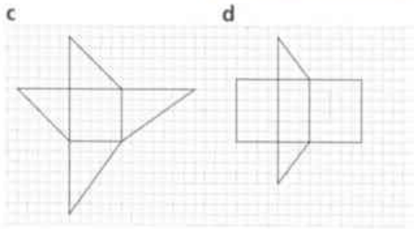


4 **a** **b**



c **d**



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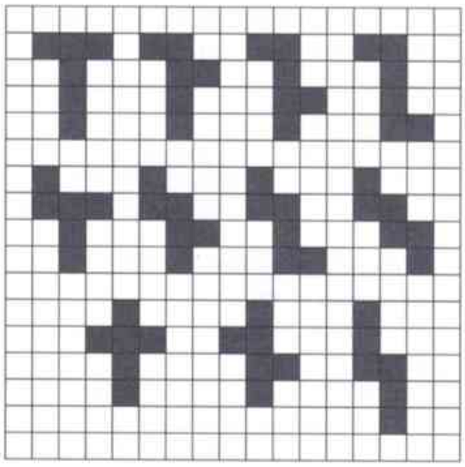


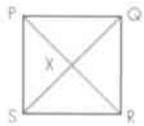
6 a and b

24.4 Congruent shapes

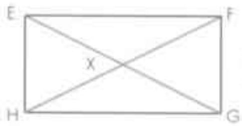
Exercise 24D

1 **a** yes **b** yes **c** no **d** yes **e** no **f** yes

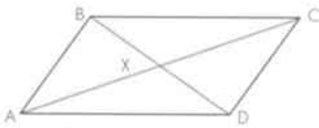
2 **a** triangle ii **b** triangle iii **c** sector i

3 

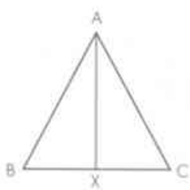
PQR to *QRS* to *RSP* to *SPQ*;
SXP to *PXQ* to *QXR* to *RXS*

4 

EGF to *FHE* to *GEH* to *HFG*;
EFX to *HGX*; *EXH* to *FXG*

5 

ABC to *CDA*; *BDC* to *DBA*;
BXA to *DXC*; *BXC* to *DXA*

6 

AXB to *AXC*

24.5 Congruent triangles

Exercise 24E

1 **a** SAS **b** SSS **c** ASA **d** RHS **e** SSS **f** ASA

2 **a** SSS. A to R, B to P, C to Q
b SAS. A to R, B to Q, C to P

3 a 60° b 80° c 40° d 5 cm
4 a 110° b 55° c 85° d 110° e 4 cm
5 SSS or RHS
6 SSS or SAS or RHS
7 For example, use ΔADE and ΔCDG. $AD = CD$ (sides of large square), $DE = DG$ (sides of small square), angle ADE = angle CDG (angles sum to 90° with angle ADG), so angle ADE = angle CDG (SAS), so $AE = CG$
8 AB and PQ are the corresponding sides to the 42° angle, but they are not equal in length.

24.6 Similar shapes

Exercise 24F

1 a 2 b 3
2 a Yes, 4
 b No, corresponding sides have different ratios.
3 a PQR is an enlargement of ABC
 b 1 : 3 c Angle R d BA
4 a Sides in same ratio
 b Angle P c PR
5 a Same angles b Angle Q c AR
6 a 8 cm b $x = 45$ cm, $y = 9$ cm
 c $x = 19.5$ cm, $y = 10.8$ cm d 4.2 cm
7 a The angles are all 90 degrees. The sides of a square are all equal so the ratio between sides of two different squares will be the same, whatever two sides are chosen.
 b No. They will only be similar if they have the same ratio of length to width.
8 5.2 m

24.7 Areas of similar triangles

Exercise 24G

1 a 2.5 b 125 cm^2
2 a All equilateral triangles are similar b 3.8 cm^2 (to 2 sf)
3 40.32 cm^2
4 75 cm^2
5 a 144 cm^2 b 69.4 cm^2
6 a All angles are the same b 247.7 cm^2
7 a 2 b 10 cm c 7.1 cm
8 354.9 cm^2
9 It will double the area
10 28.3 cm^2

24.8 Areas and volumes of similar shapes

Exercise 24H

1 a 4 : 25 b 8 : 125
2 a 16 : 49 b 64 : 343
3

Linear scale factor	Linear ratio	Linear fraction	Area scale factor	Volume scale factor
2	1 : 2	$\frac{2}{1}$	4	8
3	1 : 3	$\frac{3}{1}$	9	27
$\frac{1}{4}$	4 : 1	$\frac{1}{4}$	$\frac{1}{16}$	$\frac{1}{64}$
5	1 : 5	$\frac{5}{1}$	25	125
$\frac{1}{10}$	10 : 1	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$

4 135 cm^2
5 a 56 cm^2 b 126 cm^2
6 a 48 m^2 b 3 m^2
7 a 2400 cm^3 b 8100 cm^3
8 4 litres
9 1.38 m^3
10 $6
11 4 cm
12 8 × 0.60 = $4.80 which is greater than $4.00 so the large tub is better value
13 a 3 : 4 b 9 : 16 c 27 : 64
14 720 ÷ 8 = 90 cm^3

Exercise 24I

1 6.2 cm, 10.1 cm
2 4.26 cm, 6.74 cm
3 9.56 cm
4 3.38 m
5 8.4 cm
6 26.5 cm
7 47.8 cm
8 a 4.33 cm, 7.81 cm b 143 g, 839 g
9 53.8 kg
10 1.73 kg
11 8.8 cm
12 7.9 cm and 12.6 cm
13 b

Answers to Chapter 25

25.1 Constructing shapes

Exercise 25A

1 a $BC = 2.9$ cm, angle $B = 53°$, angle $C = 92°$
 b $EF = 7.4$ cm, $ED = 6.8$ cm, angle $E = 50°$
 c angle $G = 105°$, angle $H = 29°$, angle $I = 46°$
 d angle $J = 48°$, angle $L = 32°$, $JK = 4.3$ cm
 e angle $N = 55°$, $ON = OM = 7$ cm
 f angle $P = 51°$, angle $R = 39°$, $QP = 5.7$ cm
2 a Students can check one another's triangles.
 b angle $ABC = 44°$, angle $BCA = 79°$, angle $CAB = 57°$
3 5.9 cm
4 student drawing.
5 student drawing.
6 4.3 cm
7 4.3 cm

8 a Right-angled triangle constructed with sides 3, 4, 5 and 4.5, 6, 7.5, and scale marked 1 cm : 1 m
  b Right-angled triangle constructed with 12 equally spaced dots.
9 An equilateral triangle of side 4 cm.
10 Even with all three angles, you need to know at least one length.

25.2 Scale drawings

Exercise 25B

1 a pond: 40 m × 10 m, fruit: 50 m × 10 m, trees: 20 m × 20 m, lawn: 30 m × 20 m, vegetables: 50 m × 20 m
  b pond: 400 m^2, fruit: 500 m^2, trees: 400 m^2, lawn: 600 m^2, vegetables: 1000 m^2
2 a 33 cm b 9 cm
3 a 30 cm × 30 cm b 40 cm × 10 cm c 20 cm × 15 cm
  d 30 cm × 20 cm e 30 cm × 20 cm f 10 cm × 5 cm
4 a student's scale drawing. b 39 plants
5 a 8.4 km b 4.6 km c 6.2 km
  d 6.4 km e 7.6 km f 2.4 km
6 a student's drawing b 12.9 metres
7 a 900 km b 1100 km c 860 km
8 c – 7 cm represents 210 m, so 1 cm represents 30 m

Answers to Chapter 26

26.1 Pythagoras' theorem

Exercise 26A

1 10.3 cm
2 5.9 cm
3 8.5 cm
4 20.6 cm
5 18.6 cm
6 17.5 cm
7 5 cm
8 13 cm
9 10 cm
10 The smaller square in the first diagram and the two smaller squares in the second have the same area.

Exercise 26B

1 a 15 cm b 14.7 cm c 6.3 cm d 18.3 cm
2 a 20.8 m b 15.5 cm c 15.5 m d 12.4 cm
3 a 5 m b 6 m c 3 m d 50 cm
4 There are infinite possibilities, e.g. any multiple of 3, 4, 5 such as 6, 8, 10; 9, 12, 15; 12, 16, 20; multiples of 5, 12, 13 and of 8, 15, 17.
5 42.6 cm

26.2 Trigonometric ratios

Exercise 26C

1 a 0.682 b 0.829 c 0.922 d 1
2 a 0.731 b 0.559 c 0.388 d 0
3 a i 0.574 ii 0.574
  b i 0.208 ii 0.208
  c i 0.391 ii 0.391
  d They are the same.
  e i sin 15° is the same as cos 75°
    ii cos 82° is the same as sin 8°
    iii sin x is the same as cos (90° – x)
4 a 0.933 b 1.48 c 2.38 d Infinite
  e 1 f 0.364 g 0.404 h 0
5 Tan has values > 1
6 a 3.56 b 8.96 c 28.4 d 8.91
7 a 5.61 b 7.08 c 1.46 d 7.77
8 a $\frac{4}{5}, \frac{3}{5}, \frac{4}{3}$ b $\frac{5}{13}, \frac{12}{13}, \frac{5}{12}$ c $\frac{7}{25}, \frac{24}{25}, \frac{7}{24}$

26.3 Calculating angles

Exercise 26D

1 a 30° b 51.7° c 39.8°
  d 61.3° e 87.4° f 45.0°
2 a 60° b 50.2° c 2.6°
  d 45° e 78.5° f 45.6°
3 a 31.0° b 20.8° c 41.8°
  d 46.4° e 69.5° f 77.1°
4 Error message, largest value 1, smallest value –1
5 a i 17.5° ii 72.5° iii 90°
  b Yes

26.4 Using sine, cosine and tangent functions

Exercise 26E

1 a 17.5° b 22.0° c 32.2°
2 a 5.29 cm b 5.75 cm c 13.2 cm
3 a 4.57 cm b 6.86 cm c 100 cm
4 a 5.12 cm b 9.77 cm c 11.7 cm d 15.5 cm

Exercise 26F

1 a 51.3° b 75.5° c 51.3°
2 a 5.35 cm b 14.8 cm c 12.0 cm d 8.62 cm
3 a 5.59 cm b 46.6° c 9.91 cm d 40.1°

Exercise 26G

1 a 33.7° b 36.9° c 52.1°
2 a 9.02 cm b 7.51 cm c 7.14 cm d 8.90 cm
3 a 13.7 cm b 48.4° c 7.03 cm d 41.2°

26.5 Which ratio to use

Exercise 26H

1 a 12.6 b 59.6 c 74.7 d 16.0
  e 67.9 f 20.1
2 a 44.4° b 39.8° c 44.4° d 49.5°
  e 58.7° f 38.7°
3 a 67.4° b 11.3 c 134 d 28.1°
  e 39.7 f 263 g 50.2° h 51.3°
  i 138 j 22.8

4 **a** Sides of right-hand triangle are sine θ and cosine θ
b Pythagoras' theorem
c Students should check the formulae

26.6 Application of trigonometric ratios

Exercise 26I

1 14.0 cm
2 **a** 24.5 cm **b** 20.6 cm **c** 19.4 cm
3 1.46 km
4 3.33
5 10.1 km
6 22°
7 429 m
8 **a** 156 m
b No. the new angle of depression is $\tan^{-1}\left(\frac{200}{312}\right) = 33°$ and half of 52° is 26°
9 **a** 222 m **b** 42°
10 **a** 21.5 m **b** 17.8 m
11 13.4 m
12 19°
13 The angle is 16° so Cara is not quite correct.

26.7 Problems in three dimensions

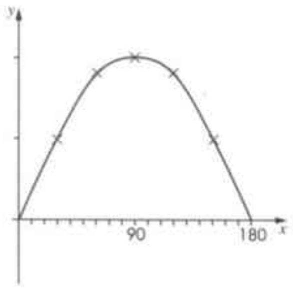
Exercise 26J

1 25.1°
2 **a** 25cm **b** 58.6° **c** 20.5 cm
3 **a** 3.46 m **b** 75.5° **c** 73.2°
4 **a** 24.0° **b** 48.0° **c** 13.5 cm **d** 16.6°
5 It is 44.6°; use triangle XDM where M is the midpoint of BD; triangle DXB is isosceles, as X is over the point where the diagonals of the base cross; the length of DB is ,$\sqrt{656}$. and the cosine of the required angle is $0.5\sqrt{656} \div 18$

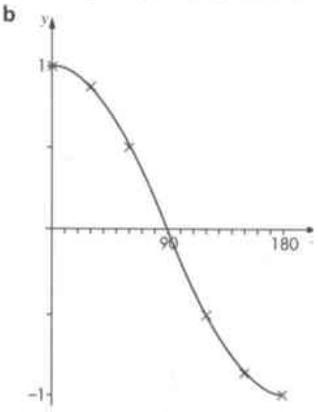
26.8 Sine and cosine of obtuse angles

Exercise 26K

1 **a** The bottom row of the table is 0.174, 0.5, 0.766, 0.996, 1, 0.996, 0.766, 0.5, 0.174.
b



c It has reflection symmetry. The line of symmetry is $x = 90$.
d You could choose 10° and 170°, 30° and 150°, 50° and 130° or 85° and 95°
2 30° and 150°.
3 46° and 134°.
4 122.9°
5 **a** The bottom row of the table is 0.966, 0.819, 0.5, 0.174, 0, −0.174, −0.5, −0.819, −0.966.
b



c It has rotational symmetry of order 2 about the point (90, 0)
6 **a** 31.8° **b** 148.2° **c** 120°
d 90° **e** 82.8° **f** 97.2°
7 **a** 53° **b** 104° **c** 49°, 131° **d** 90°
e 90° **f** 72°, 108° **g** no solution **h** 45°

26.9 The sine rule and the cosine rule

Exercise 26L

1 **a** 3.64 m **b** 8.05 cm **c** 19.4 cm
2 **a** 46.6° **b** 68° **c** 36.2°
3 **a** **i** 30° **ii** 40°
b 19.4 m
4 36.5 m
5 22.2 m
6 3.47 m
7 64.6 km
8 134°

Exercise 26M

1 **a** 7.71 m **b** 29.1 cm **c** 27.4 cm
2 **a** 76.2° **b** 125.1° **c** 90°
d Right-angled triangle
3 5.16 cm
4 65.5 cm
5 **a** 10.7 cm **b** 41.7° **c** 38.3° **d** 6.69 cm
6 58.4 km at 092.5°
7 21.8°
8 42.5 km
9 111°; the largest angle is opposite the longest side

Exercise 26N

1 **a** 8.60 m **b** 90° **c** 27.2 cm
d 26.9° **e** 27.5° **f** 62.4 cm
g 90.0° **h** 866 cm **i** 86.6 cm
2 7 cm
3 11.1 km
4 **a** BAC = 90°; this is Pythagoras' theorem
b BAC is acute
c BAC is obtuse
5 142 m

26.10 Using sine to find the area of a traingle

Exercise 26O

1 a 24.0 cm² b 26.7 cm² c 243 cm² d 21 097 cm² e 1224 cm²
2 4.26 cm
3 a 42.3° b 49.6°
4 2033 cm²
5 21.0 cm²
6 726 cm²
7 149 km²
8 a 66.4 m b 118.9° c 1470 m²
9 43.3 cm²

26.11 Sine, cosine and tangent of any angle

Exercise 26P

1 a 100° b 34° c 325° d 234°
2 a 350° b 235° c 152° d 49°
3 a 27° and 153° b 56° and 124° c 333° and 207° d 304° and 236°
4 a 37° and 323° b 103° and 257° c 157° and 203° d 85° and 275°
5 a 30° and 150° b 60° and 120° c 225° and 315° d 270°
6 a 120° and 240° b 30° and 330° c 45° and 315° d 90° and 270°
7 a 87.1° and 272.9° b 54.3° and 124.7° c 130.5° and 229.5° d 323.1° and 216.9°
8 a 41.8° and 138.2° b 36.9° and 323.1° c 314.4° and 225.6°
9 540°
10 30°, 150°, 210° and 330°
11 45°, 135°, 225° and 315°

Exercise 26Q

1 a 215° b 265° c 298° d 20° e 63° f 157°
2 a 45° and 225° b 135° and 315° c 60° and 240° d 120° and 300°
3 a 11.3° and 191.3° b 78.7° and 258.7° c 160.7° and 340.7° d 103.5° and 283.5°
4 −1.5
5 a 20.6° and 200.6° b 69.4° and 249.4° c 144.2° and 324.2°
6 a 45°, 135°, 225° and 315° b 60°, 120°, 240° and 300° c 30°, 150°, 210° and 330°
7 71.6°, 251.6°, 104.0° and 284.0°
8 a They are the same. **b** and **c** They have the same magnitude but different signs. They add up to 0.

Answers to Chapter 27

27.1 Perimeter and area of a rectangle

Exercise 27A

1 a 35 cm², 24 cm b 33 cm², 28 cm c 45 cm², 36 cm d 70 cm², 34 cm e 56 cm², 30 cm f 10 cm², 14 cm
2 a 53.3 cm², 29.4 cm b 84.96 cm², 38 cm
3 39
4 a 4 b 1 h 52 min
5 40 cm
6 Area B, 44 cm²; perimeter B, 30 cm
7 Never (the area becomes four times greater).
8 a 28 cm, 30 cm² b 28 cm, 40 cm² c 40 cm, 51 cm² d 30 cm, 35 cm² e 32 cm, 43 cm² f 34 cm, 51 cm² g cannot tell what the perimeter is; 48 cm² h 34 cm, 33 cm²
9 72 cm²
10 48 cm

27.2 Area of a triangle

Exercise 27B

1 a 21 cm² b 12 cm² c 14 cm² d 55 cm² e 90 cm² f 140 cm²
2 a 28 cm² b 8 cm c 4 cm d 3 cm e 7 cm f 44 cm²
3 73.9 cm²
4 a 40 cm² b 65 m² c 80 cm²
5 a 65 cm² b 50 m²
6 For example: height 10 cm, base 10 cm; height 5 cm, base 20 cm; height 25 cm, base 4 cm; height 50 cm, base 2 cm
7 Triangle c; a and b each have an area of 15 cm² but c has an area of 16 cm²

27.3 Area of a parallelogram

Exercise 27C

1 a 96 cm² b 70 cm² c 20 m² d 125 cm² e 10 cm² f 112 m²
2 No, it is 24 cm², she used the slanting side instead of the perpendicular height.
3 16 cm
4 a 500 cm² b 15

27.4 Area of a trapezium

Exercise 27D

1 a 30 cm² b 77 cm² c 24 cm² d 42 cm² e 40 cm² f 6 cm g 3 cm
2 a 27.5 cm, 36.25 cm² b 33.4 cm, 61.2 cm² c 38.6 m, 88.2 m²
3 Any pair of lengths that add up to 10 cm. For example: 1 cm, 9 cm; 2 cm, 8 cm; 3 cm, 7 cm; 4 cm, 6 cm; 4.5 cm, 5.5 cm

4 Shape c. Its area is 25.5 cm^2

5 Shape a. Its area is 28 cm^2

6 a

7 2 cm

8 1.4 m^2

27.5 Circumference and area of a circle

Exercise 27E

1 **a** 10π cm and 25π cm^2 **b** 6π cm and 9π cm^2
c 3π cm and 2.25π cm^2 **d** 8π cm and 16π cm^2

2 **a** 25.1 cm and 50.3 cm^2
b 15.7 cm and 19.6 cm^2
c 28.9 cm and 66.5 cm^2
d 14.8 cm and 17.3 cm^2

3 **a i** 56.5 cm **ii** 81π, 254.5 cm^2
b i 69.1 cm **ii** 121π, 380.1 cm^2
c i 40.8 cm **ii** 42.3π, 132.7 cm^2
d i 88.0 cm **ii** 196π, 615.8 cm^2

4 **a** 19.1 cm **b** 9.5 cm
c 286.5 cm^2 (or 283.5 cm^2)

5 962.9 cm^2 (or 962.1 cm^2)

6 **a** 20 cm **b** 400π cm^2

7 **a** 16π m^2 **b** 14π cm^2 **c** 9π cm^2

8 45π cm^2

9 $a^2 = \pi r^2$, so $r^2 = \frac{a^2}{\pi}$ therefore $r = \frac{a}{\sqrt{\pi}}$

10 21.5 cm^2

27.6 Surface area and volume of a cuboid

Exercise 27F

1 **a i** 198 cm^3 **ii** 234 cm^2
b i 90 cm^3 **ii** 146 cm^2
c i 1440 cm^3 **ii** 792 cm^2
d i 525 cm^3 **ii** 470 cm^2

2 24 litres

3 **a** 160 cm^3 **b** 416 cm^3 **c** 150 cm^3

4 **a i** 64 cm^3 **ii** 96 cm^2
b i 343 cm^3 **ii** 294 cm^2
c i 1000 mm^3 **ii** 600 mm^2
d i 125 m^3 **ii** 150 m^2
e i 1728 m^3 **ii** 864 m^2

5 86

6 **a** 180 cm^3 **b** 5 cm **c** 6 cm
d 10 cm **e** 81 cm^3

7 1.6 m

8 48 m^2

9 **a** 3 cm **b** 5 m **c** 2 mm **d** 1.2 m

10 **a** 148 cm^3 **b** 468 cm^3

11 If this was a cube, the side length would be 5 cm, so total surface area would be 5 × 5 × 6 = 150 cm^2; no this particular cuboid is not a cube.

12 **a** 6 cm **b** 216

27.7 Volume and surface area of a prism

Exercise 27G

1 **a i** 21 cm^2 **ii** 63 cm^3
b i 48 cm^2 **ii** 432 cm^3
c i 36 m^2 **ii** 324 m^3

2 **a** 432 m^3 **b** 225 m^3 **c** 1332 m^3

3 **a** A cross-section parallel to the side of the pool always has the same shape.
b About $3\frac{1}{2}$ hours

4 7.65 m^3

5 **a** 21 cm^3, 210 cm^3
b 54 cm^2, 270 cm^2

6 146 cm^3

7 78 m^3 (78.3 m^3)

8 327 litres

9 10.2 tonnes

10 672 cm^3

27.8 Volume and surface area of a cylinder

Exercise 27H

1 **a i** 72π cm^3 **ii** 66π cm^2
b i 4.75π cm^3 **ii** 19.5π cm^2
c i 110π cm^3 **ii** 87.5π cm^2
d i 338π cm^3 **ii** 203π cm^2

2 **a i** 72π cm^3 **ii** 48π cm^2
b i 112π cm^3 **ii** 56π cm^2
c i 180π cm^3 **ii** 60π cm^2
d i 600π m^3 **ii** 120π m^2

3 665 cm^3

4 Label should be less than 10.5 cm wide so that it fits the can and does not overlap the rim and more than 23.3 cm long to allow an overlap.

5 332 litres

6 There is no right answer. Students could start with the dimensions of a real can. Often drinks cans are not exactly cylindrical. One possible answer is height of 6.6 cm and diameter of 8 cm.

7 About 127 cm

8 A diameter of 10 cm and a length of 5 cm give a volume close to 400 cm^3 (0.4 litres).

27.9 Sectors and arcs: 1

Exercise 27I

1 **a** 20π cm **b i** 10π cm **ii** 5π cm **iii** 2.5π cm

2 **a** 100π cm^2 **b i** 50π cm^2 **ii** 25π cm^2 **iii** 12.5π cm^2

3 **a** $\frac{1}{5}$ **b** 10.6 cm **c** 44.3 cm^2

4 **a** 96.5 cm^2 **b** 20.1 cm **c** 39.3 cm

5 **a** 245.4 cm^2 to 1 d.p. **b** 64.3 cm to 1 d.p.

6 **a** The diameter is 80 and the fraction of a circle is $\frac{1}{10}$.
The arc length is $\pi \times 80 \div 10 = 8\pi$ cm.
b 160π cm^2

7 **a** 20π **b** 75π cm^2

27.10 Sectors and arcs: 2

Exercise 27J

1 a i 5.59 cm ii 22.3 cm^2
b i 8.29 cm ii 20.7 cm^2
c i 16.3 cm ii 98.0 cm^2
d i 15.9 cm ii 55.6 cm^2

2 a 9π cm b 54π cm^2

3 a 73.8 cm b 20.3 cm

4 a 107 cm^2 b 173 cm^2

5 43.6 cm

6 $(36\pi - 72)$ cm^2

7 $(32\pi - 64)$

8 a 13.9 cm b 7.07 cm^2

27.11 Volume of a pyramid

Exercise 27K

1 a 56 cm^3 b 168 cm^3 c 1040 cm^3
d 84 cm^3 e 160 cm^3

2 270 cm^3

3 a Put the apexes of the pyramids together. The 6 square bases will then form a cube.
b If the side of the base is a then the height will be $\frac{1}{2}a$.
Total volume of the 6 pyramids is a^3.
Volume of one pyramid is
$\frac{1}{6}a^3 = \frac{1}{3} \times \frac{1}{2} \times a \times a^2 = \frac{1}{3} \times$ height $\times$ base area

4 6.9 m (height of cuboid)

5 a 73.3 m^3 b 45 m^3 c 3250 cm^3

6 1.5 g

7 5.95 cm

8 14.4 cm

9 260 cm^3

27.12 Volume and surface area of a cone

Exercise 27L

1 a i 3560 cm^3 ii 1430 cm^2
b i 314 cm^3 ii 283 cm^2
c i 1020 cm^3 ii 679 cm^2

2 24π cm^2

3 a 816π cm^3 b 720π mm^3

4 a 4 cm b 6 cm
c Various answers, e.g. 60° gives 2 cm, 240° gives 8 cm

5 24π cm^2

6 If radius of base is r, slant height is $2r$.
Area of curved surface $= \pi r \times 2r = 2\pi r^2$, area of base $= \pi r^2$

7 140 g

8 2.81 cm

27.13 Volume and surface of a sphere

Exercise 27M

1 a 36π cm^3 and 36π cm^2
b 288π cm^3 and 144π cm^2
c 1330π cm^3 and 400π cm^2

2 65 400 cm^3, 7850 cm^2

3 a 1960 cm^2
b 8180 cm^3

4 125 cm

5 6231

6 7.8 cm

7 48%.

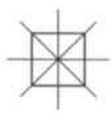
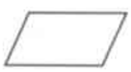
Answers to Chapter 28

28.1 Lines of symmetry

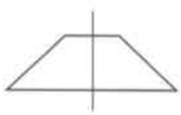
Exercise 28A

1 a

b

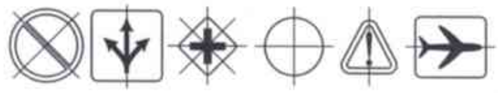
c

d

e

f

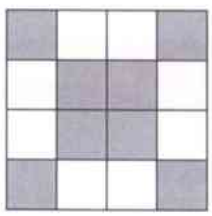
g


2 a i 5 ii 6 iii 8
b 10

3 2, 1, 1, 2, 0

4


5 a 1 b 5 c 1 d 6

6


28.2 Rotational symmetry

Exercise 28B

1 **a** 4 **b** 2 **c** 2 **d** 3 **e** 6

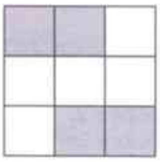
2 **a** 4 **b** 5 **c** 6 **d** 4 **e** 6

3 **a** 2 **b** 2 **c** 2 **d** 2 **e** 2

4 **a** 6

b 9 (the small red circle surrounded by nine 'petals') and 12 (the centre pattern)

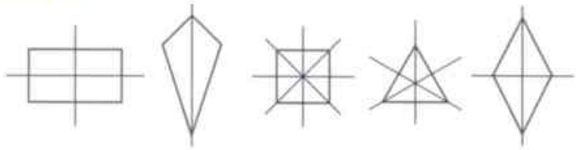
5 For example:



28.3 Symmetry of special two-dimensional shapes

Exercise 28C

1



2 **a** kite

b rectangle 2, square 4, equilateral triangle 3, rhombus 2

3 **a** isosceles **b** no

4 **a** parallelogram **b** square

5 **a** rectangle and rhombus **b** no

6 **a** B and D **b** AB and AD; CB and CD **c** kite

7 **a** diameter **b** infinite **c** infinite

8 **a** A and C; B and D

b AD and BC; AB and DC

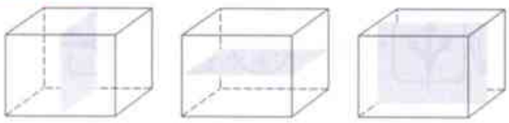
c Parallelogram

9 It will have two pairs of equal angles

28.4 Symmetry of three-dimensional shapes

Exercise 28D

1



2 **a** Diagrams to show axes going through the centres of all three pairs of opposite faces

b 2

3 Two are similar to the one shown, dividing the end triangles in two. The other goes through the centre of each of the long edges, parallel to the end triangles.

4 **a** 3 about AB; 2 about CD

b they are similar to CD, each passing through the centre of a rectangular face.

5 **a** 4 **b** no

6 There are four. All pass through the vertex. Two pass through opposite corners of the square. Two pass through the mid points of opposite sides of the square

7 **a** Any plane dividing each circle in half or the circular plane exactly half way up the cylinder

b any line at right angles to the one shown, passing through the centre of the cylinder

8 **a** one **b** infinite

9 **a** six through the centre of each hexagon; one parallel to the hexagons passing through the centre of the prism

b 4

28.5 Symmetry in circles

Exercise 28E

1 **a–d** student's own drawing

e because the perpendicular bisector of any chord passes through the centre of the circle

f Here is one method: draw two chords; construct the perpendicular bisectors; they meet at the centre

2 **a** Isosceles because OA and OB are radii

b OA = OC; OB = OD; AB = CD so corresponding sides are equal

c 50°

3 **a** EM = FM (given); OE = OF (radii); OM is common to both. Corresponding sides are equal

b EMO and FMO are equal and add up to 180° (because EMF is a straight line) so they must both be 90°

c 18°

4 **a** Angle between a radius and a tangent

b XP = YP (tangents from a point are equal); OX = OY (radii); OP is common. So corresponding sides are equal

c 146°

5 5 cm (use Pythagoras' theorem)

6 40°

Answers to Chapter 29

29.1 Introduction to vectors

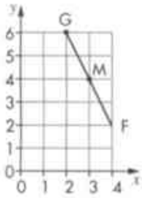
Exercise 29A

1 **a i** $\begin{pmatrix} 4 \\ -1 \end{pmatrix}$ **ii** $\begin{pmatrix} 3 \\ 1 \end{pmatrix}$ **iii** $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$

iv $\begin{pmatrix} -2 \\ 4 \end{pmatrix}$ **v** $\begin{pmatrix} 2 \\ -4 \end{pmatrix}$ **vi** $\begin{pmatrix} -1 \\ 2 \end{pmatrix}$

b Both are $\begin{pmatrix} 1 \\ -2 \end{pmatrix}$. D is the midpoint of AC.

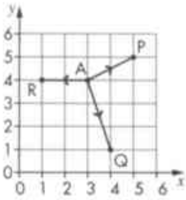
2 **a**



b **i** $\begin{pmatrix}-2\\4\end{pmatrix}$ **ii** $\begin{pmatrix}2\\-4\end{pmatrix}$ **iii** $\begin{pmatrix}3\\4\end{pmatrix}$

iv $\begin{pmatrix}-1\\2\end{pmatrix}$ **v** $\begin{pmatrix}-2\\-6\end{pmatrix}$.

3



4 The diagrams should show the following vectors:

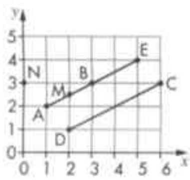
a $\begin{pmatrix}3\\-1\end{pmatrix}$ **b** $\begin{pmatrix}-1\\4\end{pmatrix}$ **c** $\begin{pmatrix}1\\7\end{pmatrix}$

d $\begin{pmatrix}3\\-1\end{pmatrix}$ **e** $\begin{pmatrix}-1\\-7\end{pmatrix}$ **f** $\begin{pmatrix}2\\-8\end{pmatrix}$

5 **a** $\begin{pmatrix}-1\\2\end{pmatrix}$ **b** $\begin{pmatrix}6\\12\end{pmatrix}$ **c** $\begin{pmatrix}-5\\-6\end{pmatrix}$

d $\begin{pmatrix}-5\\6\end{pmatrix}$ **e** $\begin{pmatrix}-12\\-8\end{pmatrix}$ **f** $\begin{pmatrix}1\\6\end{pmatrix}$

6 **a–d**



e k is 4.

29.2 Using vectors

Exercise 29B

1 **a** Any three, of: $\overrightarrow{AC}$, $\overrightarrow{CF}$, $\overrightarrow{BD}$, $\overrightarrow{DG}$, $\overrightarrow{GI}$, $\overrightarrow{EH}$, $\overrightarrow{HJ}$, $\overrightarrow{JK}$

b Any three of: $\overrightarrow{BE}$, $\overrightarrow{AD}$, $\overrightarrow{DH}$, $\overrightarrow{CG}$, $\overrightarrow{GJ}$, $\overrightarrow{FI}$, $\overrightarrow{IK}$

c Any three of: $\overrightarrow{AO}$, $\overrightarrow{CA}$, $\overrightarrow{FC}$, $\overrightarrow{IG}$, $\overrightarrow{GD}$, $\overrightarrow{DB}$, $\overrightarrow{KJ}$, $\overrightarrow{JH}$, $\overrightarrow{HE}$

d Any three of: $\overrightarrow{BO}$, $\overrightarrow{EB}$, $\overrightarrow{HD}$, $\overrightarrow{DA}$, $\overrightarrow{JG}$, $\overrightarrow{GC}$, $\overrightarrow{KI}$, $\overrightarrow{IF}$

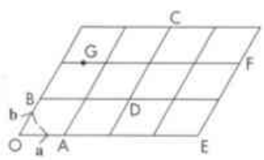
2 **a** 2**a** **b** 2**b** **c** 3**a** + 2**b** **d** **a** + 2**b**

e **a** + **b** **f** 2**a** + 2**b** **g** 3**a** + **b**

3 $\overrightarrow{AI}$, $\overrightarrow{BJ}$, $\overrightarrow{DK}$

4 $\overrightarrow{OF}$, $\overrightarrow{BI}$, $\overrightarrow{EK}$

5



6 **a** **a** – **b** **b** 3**a** – **b** **c** 2**a** – **b** **d** **a** – **b**

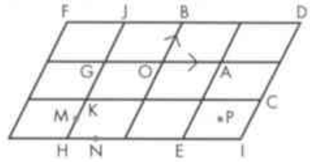
e **a** + **b** **f** –**a** – **b** **g** 2**a** – **b** **h** –**a** – 2**b**

i **a** + 2**b** **j** –**a** + **b** **k** 2**a** – 2**b** **l** **a** – 2**b**

7 **a** $\overrightarrow{BJ}$, $\overrightarrow{CK}$

b $\overrightarrow{EB}$, $\overrightarrow{GO}$, $\overrightarrow{KH}$

8



9 **a** **i** 3**a** + 2**b** **ii** 3**a** + **b**

iii 2**a** –**b** **iv** 2**b** – 2**a**

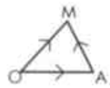
b $\overrightarrow{DG}$ and $\overrightarrow{BC}$

10 **a** 2**a** + **b** **b** 2**b** – **a** **c** **a** + 1.5**b**

d 0.5**a** + 0.5**b** **e** 1.5**a** + 1.5**b** **f** 1.5**a** – 0.5**b**

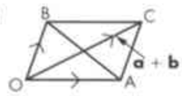
11 **a** **i** –**a** + **b** **ii** $\frac{1}{2}$(–**a** + **b**)

iii



iv $\frac{1}{2}$**a** + $\frac{1}{2}$**b**

b



c M is midpoint of parallelogram of which OA and OB are two sides.

12 **a** **i** –**a** + **b** **ii** $\frac{1}{3}$(–**a** + **b**) **iii** $\frac{2}{3}$**a** + $\frac{1}{3}$**b**

b $\frac{3}{4}$**a** + $\frac{1}{4}$**b**

13 **a** **i** $\frac{2}{3}$**b** **ii** $\frac{1}{2}$**a** + $\frac{1}{2}$**b** **iii** –$\frac{2}{3}$**b**

b $\frac{1}{2}$**a** – $\frac{1}{6}$**b**

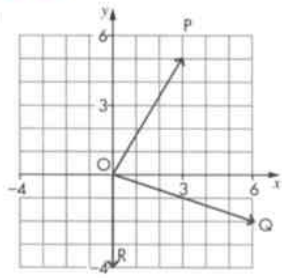
c $\overrightarrow{DE} = \overrightarrow{DO} + \overrightarrow{OE} = \frac{3}{2}\mathbf{a} - \frac{1}{2}\mathbf{b}$

d $\overrightarrow{DE}$ parallel to $\overrightarrow{CD}$ = (multiple of $\overrightarrow{CD}$) and D is a common point

29.3 The magnitude of a vector

Exercise 29C

1 **a**



b $\sqrt{34}$; $\sqrt{40}$; 4

c $\sqrt{58}$

d $\sqrt{40}$

2 **a** 10 and 13 **b** $\begin{pmatrix}11\\-4\end{pmatrix}$ **c** $\sqrt{137}$

d No. 10 + 13 does not equal $\sqrt{137}$

e $\sqrt{401}$ **f** $\sqrt{401}$

g Yes. They are vectors in opposite directions but the same length.

3 **a** 10, 10, 10

b Because they are all the same distance from A. The radius is 10.

4 **a** $\sqrt{17}$ **b** $\sqrt{261}$ **c** 13 **d** 10

Answers to Chapter 30

30.1 Translations

Exercise 30A

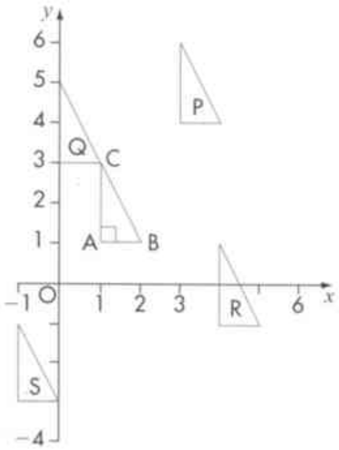
1 **a** i $\begin{pmatrix} 1 \\ 3 \end{pmatrix}$ ii $\begin{pmatrix} 4 \\ 2 \end{pmatrix}$ iii $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$

b i $\begin{pmatrix} 4 \\ -2 \end{pmatrix}$ ii $\begin{pmatrix} -2 \\ 3 \end{pmatrix}$ iii $\begin{pmatrix} 3 \\ 3 \end{pmatrix}$

c i $\begin{pmatrix} -4 \\ -2 \end{pmatrix}$ ii $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ iii $\begin{pmatrix} 0 \\ 4 \end{pmatrix}$

d i $\begin{pmatrix} -2 \\ -7 \end{pmatrix}$ ii $\begin{pmatrix} 5 \\ 0 \end{pmatrix}$ iii $\begin{pmatrix} 1 \\ -5 \end{pmatrix}$

2



3 **a** $\begin{pmatrix} -3 \\ -1 \end{pmatrix}$ **b** $\begin{pmatrix} 4 \\ -4 \end{pmatrix}$ **c** $\begin{pmatrix} -5 \\ -2 \end{pmatrix}$

d $\begin{pmatrix} 4 \\ 7 \end{pmatrix}$ **e** $\begin{pmatrix} -1 \\ 5 \end{pmatrix}$ **f** $\begin{pmatrix} 1 \\ 6 \end{pmatrix}$

g $\begin{pmatrix} -4 \\ 4 \end{pmatrix}$ **h** $\begin{pmatrix} -4 \\ -7 \end{pmatrix}$

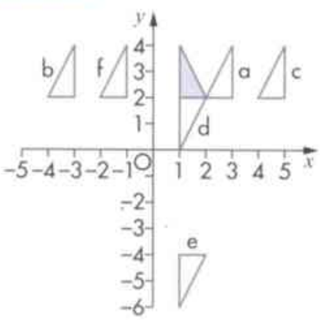
4 $\begin{pmatrix} -x \\ -y \end{pmatrix}$

5 $\begin{pmatrix} -1 \\ 4 \end{pmatrix}$

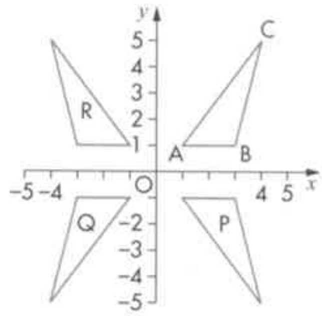
30.2 Reflections: 1

Exercise 30B

1

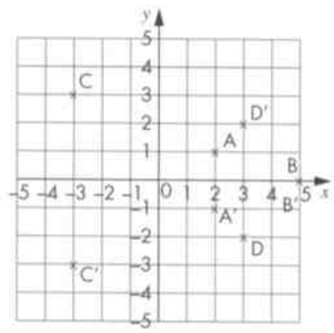


2 **a-e**



f Reflection in the y-axis

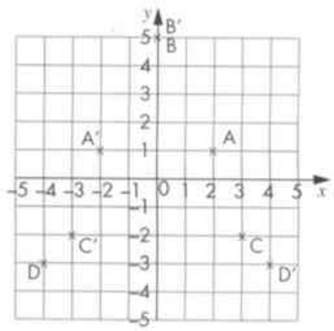
3 **a-b**



c y-value changes sign

d $(a, -b)$

4 **a-b**



c x-value changes sign

d $(-a, b)$

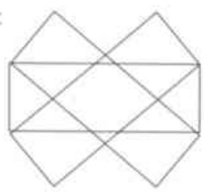
5 Possible answer: Take the centre square as ABCD then reflect this square each time in the line, AB, then BC, then CD and finally AD.

6 $x = -1$

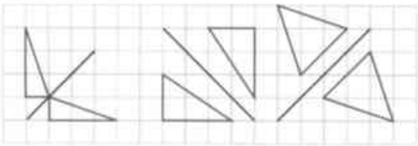
30.3 Reflections: 2

Exercise 30C

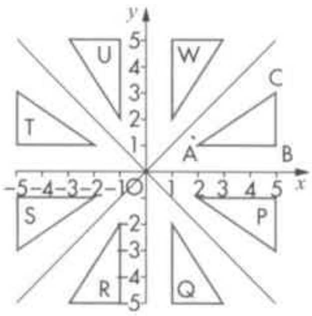
1 Possible answer:



2

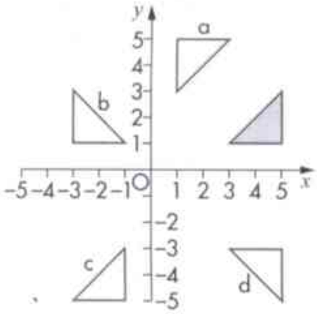


3 a-i

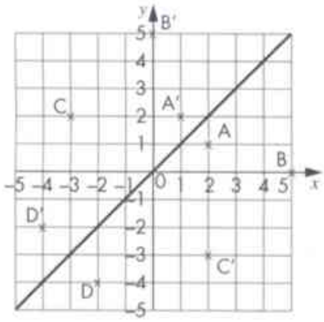


j A reflection in $y = x$

4



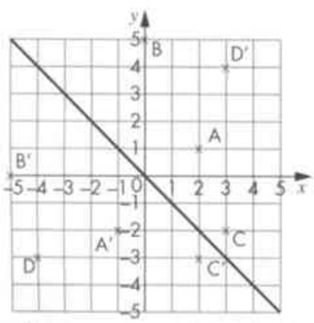
5 a-c



d Coordinates are reversed: x becomes y and y becomes x

e (b, a)

6 a-c



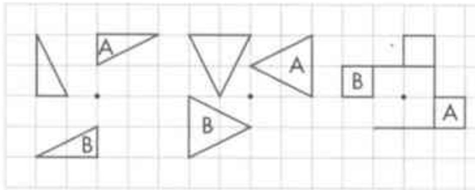
d Coordinates are reversed and the sign changes: x becomes $-y$ and y becomes $-x$

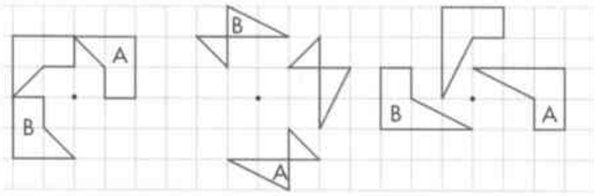
e $(-b, -a)$

30.4 Rotations: 1

Exercise 30D

1 a

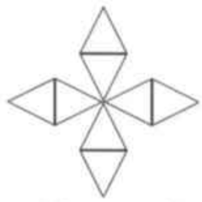




b i Rotation 90° anticlockwise

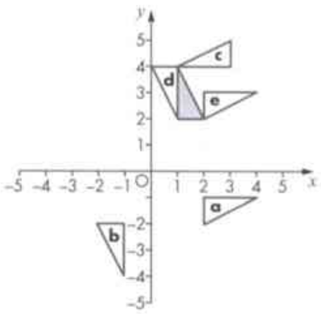
ii Rotation 180°

2



3 Possible answer: If ABCD is the centre square, rotate about A 90° anticlockwise, rotate about new B 180°, now rotate about new C 180°, and finally rotate about new D 180°.

4



5 a (4,5) 180°

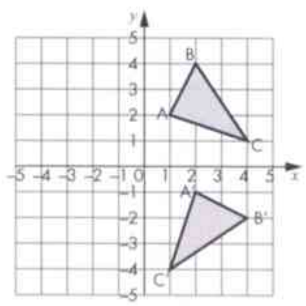
b (5,5) 90° anticlockwise

c (3,3) 180°

b (3,5) 90° clockwise

6 a E b H

7 i

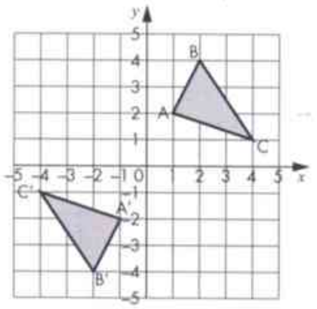


ii A' (2, −1), B' (4, −2), C' (1, −4)

iii Original coordinates (x, y) become $(y, -x)$

iv Yes

8 i



ii A′ (−1, −2), B′ (−2, −4), C′ (−4, −1)

iii Original coordinates (x, y) become $(-x, -y)$

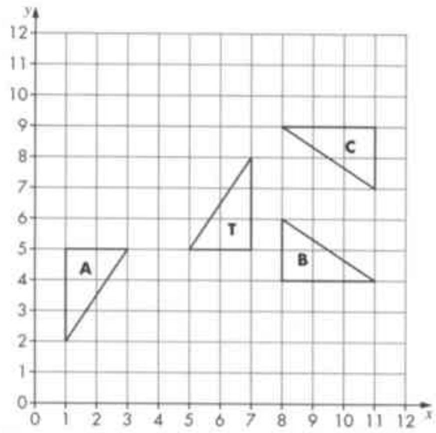
iv Yes

9 Show by drawing a shape or use the fact that (a, b) becomes $(a, -b)$ after reflection in the x-axis, and $(a, -b)$ becomes $(-a, -b)$ after reflection in the y-axis, which is equivalent to a single rotation of 180°.

30.5 Rotations: 2

Exercise 30E

1 a-c



d Rotation of 180° about (9.5, 6.5).

2 **a** (3,0) **b** (0,0) **c** (6,0)

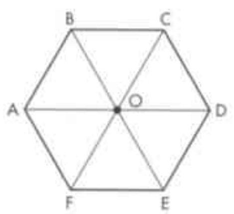
3 **a** (0, −1.5) 180° **b** (−0.5, −1.5) 90° clockwise

c (−3,5,2.5) 90° anti clockwise

d (0.5, 2) 180°

4 Show by drawing a shape or use the fact that (a, b) becomes (b, a) after reflection in the line $y = x$, and (b, a) becomes $(-a, -b)$ after reflection in the line $y = -x$, which is equivalent to a single rotation of 180°.

5 a



b **i** Rotation 60° clockwise about O

ii Rotation 120° clockwise about O

iii Rotation 180° about O

iv Rotation 240° clockwise about O

c **i** Rotation 60° clockwise about O

ii Rotation 180° about O

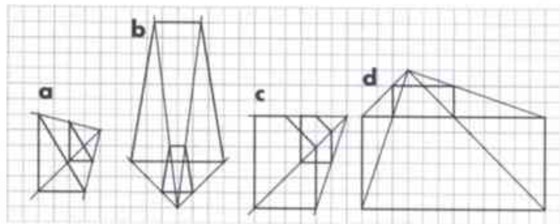
6 Rotation 90° anticlockwise about (3, −2).

7 **a** $y = x$ **b** (1, 1) **c** (6, 6) **d** not possible

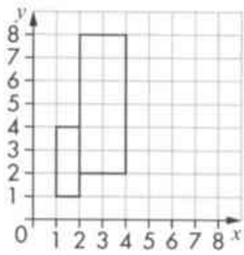
30.6 Enlargements: 1

Exercise 30F

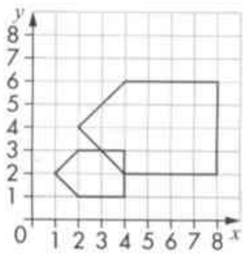
1



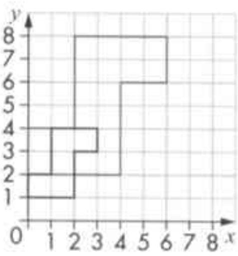
2 a



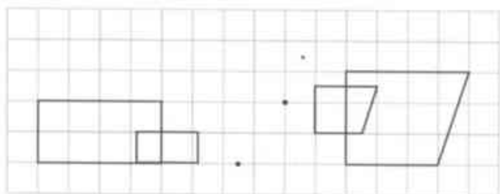
b



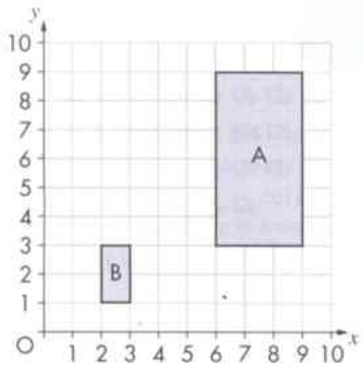
c



3



4 a

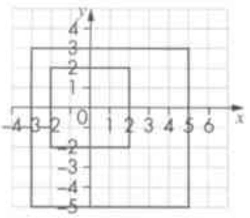


b 3 : 1 **c** 3 : 1 **d** 9 : 1

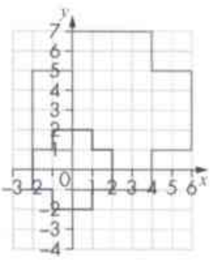
30.7 Enlargements: 2

Exercise 30G

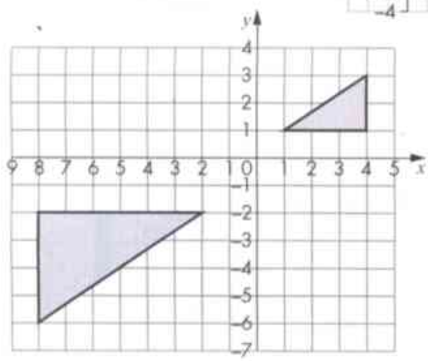
1 a



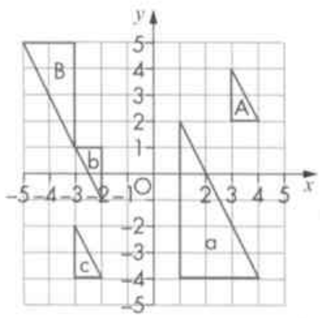
b



2



3 a-c



d Scale factor $-\frac{1}{2}$, centre (1, 3)

e Scale factor –2, centre (1, 3)

f Scale factor –1, centre (–2.5, –1.5)

g Scale factor –1, centre (–2.5, –1.5)

h Same centres, and the scale factors are reciprocals of each other

4 Enlargement, scale factor –2, about (1, 3)

5 **a** 9.6 cm **b** 25 : 1

30.8 Combined transformations

Exercise 30H

1 (–4, –3)

2 **a** (–5, 2) **b** Reflection in y-axis

3 A: translation $\begin{pmatrix} 1 \\ -2 \end{pmatrix}$,

B: reflection in y-axis,

C: rotation 90° clockwise about (0, 0),

D: reflection in $x = 3$,

E: reflection in $y = 4$,

F: enlargement by scale factor 2, centre (0, 1)

4 **a** T_1 to T_2: rotation 90° clockwise about (0, 0)

b T_1 to T_6: rotation 90° anticlockwise about (0, 0)

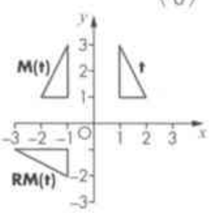
c T_2 to T_3: translation $\begin{pmatrix} 2 \\ 2 \end{pmatrix}$

d T_6 to T_2: rotation 180° about (0, 0)

e T_6 to T_5: reflection in y-axis

f T_5 to T_4: translation $\begin{pmatrix} 4 \\ 0 \end{pmatrix}$

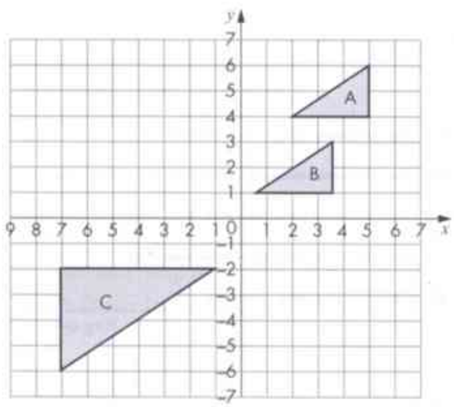
5 **a-c**



d Reflection in the line $y = -x$

6 Reflection in x-axis, translation $\begin{pmatrix} 0 \\ -5 \end{pmatrix}$, rotation 90° clockwise clockwise about (0, 0)

7



b Enlargement, scale factor $-\frac{1}{2}$, centre (1, 2)

Answers to Chapter 31

31.1 Frequency tables

Exercise 31A

1 **a**

Goals	0	1	2	3
Frequency	6	8	4	2

b 1 goal

c 22

2 **a**

Temperature (°C)	14–16	17–19	20–22	23–25	26–28
Frequency	5	10	8	5	2

b 17–19° C

c Getting warmer in the first half and then getting cooler towards the end.

3 **a**

Score	1	2	3	4	5	6
Frequency	5	6	6	6	3	4

b 30

c Yes, frequencies are similar.

4 **a**

Height (cm)	151–155	156–160	161–165	166–170
Frequency	2	5	5	7
Height (cm)	171–175	176–180	181–185	186–190
Frequency	5	4	3	1

b 166–170 cm

c student's survey results

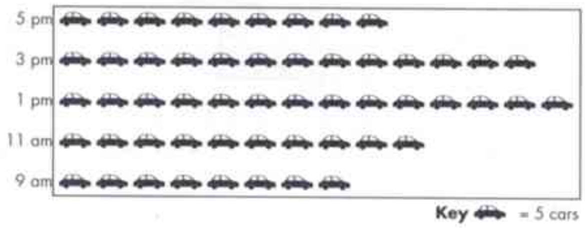
5 Various answers such as 1–10, 11–20, etc. or 1–20, 21–40, 41–60

6 The ages 20 and 25 are in two different groups.

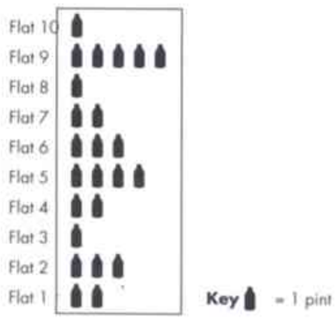
31.2 Pictograms

Exercise 31B

1



2



3 **a** May 10 h, Jun 12 h, Jul 12 h, Aug 12 h, Sep 10 h

b Visual impact, easy to understand.

4 **a** Simon **b** \$165

c Difficult to show fractions of a symbol.

5 **a** **i** 12 **ii** 6 **iii** 13

b Check students' pictograms.

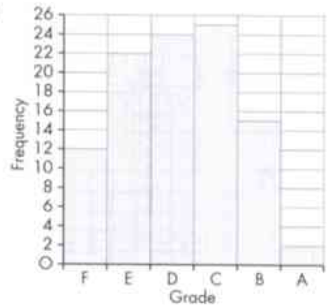
c 63

31.3 Bar charts

Exercise 31C

1 **a** Swimming **b** 74

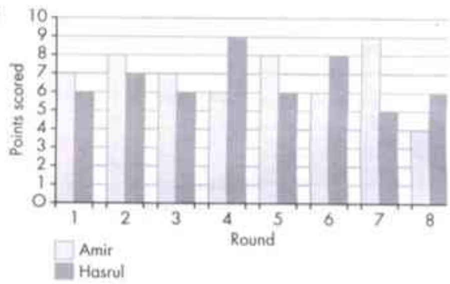
2 **a**



b $\frac{40}{100} = \frac{2}{5}$

c Easier to read the exact frequency.

3 **a**

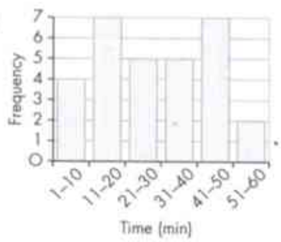


b Amir got more points overall, but Hasrul was more consistent.

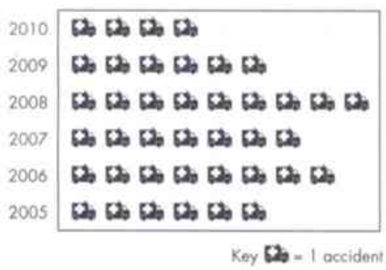
4 **a**

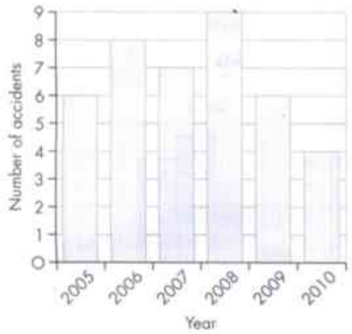
Time (min)	1–10	11–20	21–30	31–40	41–50	51–60
Frequency	4	7	5	5	7	2

b



c For example: Some live close to the school. Some live a good distance away and probably travel to school by bus.

5 a


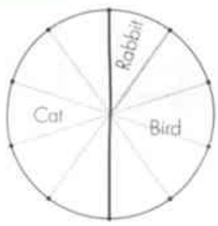
b


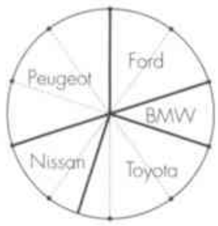
c Use the pictogram because an appropriate symbol makes more impact.

6 Yes. If you double the minimum temperature each time, it is very close to the maximum temperature.

31.4 Pie charts

Exercise 31D

1 a


b


c


2 Pie charts with following angles:
a 36°, 90°, 126°, 81°, 27°
b 168°, 52°, 100°, 40°

3 Pie chart with these angles: 60°, 165°, 45°, 15°, 75°

4 **a** 36
b Pie chart with these angles: 50°, 50°, 80°, 60°, 60°, 40°, 20°
c student's bar chart
d Bar chart, because easier to make comparisons.

5 **a** Pie charts with these angles: 124°, 132°, 76°, 28°
b Split of total data seen at a glance.

6 **a** 55° **b** 22

7 **a** Pie charts with these angles: Strings: 36°, 118°, 126°, 72°, 8°
Brass: 82°, 118°, 98°, 39°, 23°
b Overall, the Strings candidates did better, as a smaller proportion failed. A higher proportion of Brass candidates scored very good or excellent.

8 $\frac{1}{9}$

9 **a** Accept any valid comment that compares the two schools, such as:
School A had a greater percentage of students attaining the top 10 marks than School B
12.5% of School B obtained 30 or less marks: this was half the percentage of School A's results etc.
Reject answers that refer to numbers of students, e.g. more students got marks in the range 61–90 at School B
b Answers could include:
- the actual numbers of students are unknown
- the size of the pie chart can be misleading.

31.5 Scatter diagrams

Exercise 31E

1 **a** No correlation
b Positive correlation

2 **a** No relationship between temperature and speed of cars.
b As people get older, they have more money in the bank.

3 **a** and **b** student's scatter diagram and line of best fit.
c about 20 cm/s
d about 35 cm

4 **a** student's scatter diagram.
b Yes, usually (good correlation).

5 **a** and **b** Student's scatter diagram and line of best fit.
c Sitara
d about 90
e about 55

6 **a** student's scatter diagram.
b no, because there is no correlation.

7 **a** and **b** Student's scatter diagram and line of best fit.
c about 2.4 km
d 8 minutes

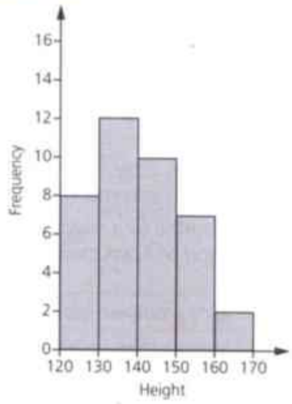
8 23 kilometres/hour

9 Points showing a line of best fit sloping down from top left to bottom right.

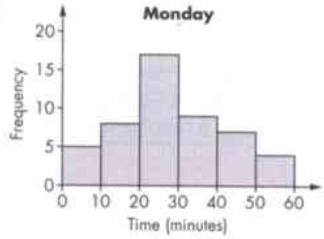
31.6 Histograms

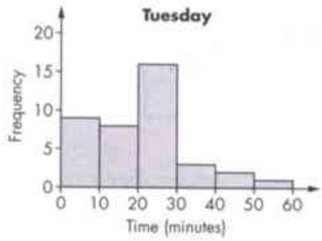
Exercise 31F

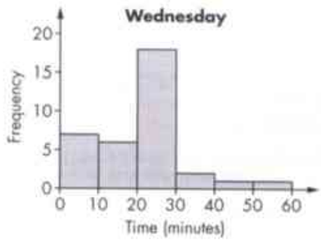
1



2 **a**



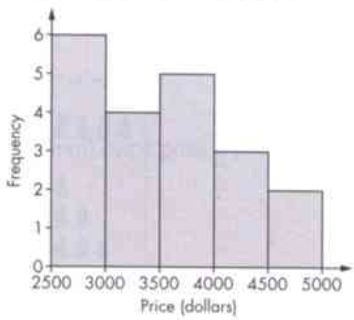




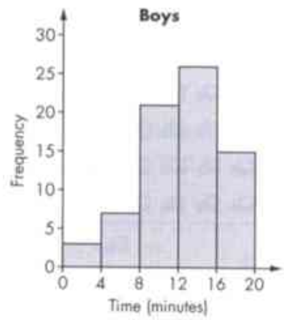
b Monday

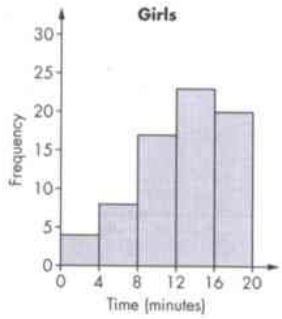
3 **a** The frequencies are 6, 4, 5, 3, 2

b



4 **a**



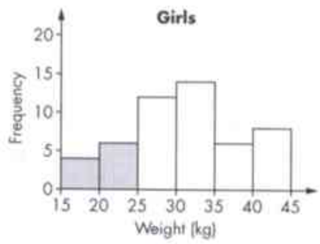


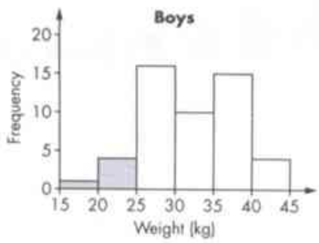
b 72 boys and 72 girls

c Nobody took longer than 20 minutes

d student's own comments

5 **a** and **b**





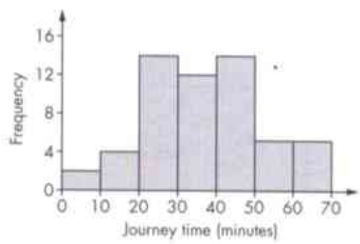
c More girls were underweight.

6 **a** 18

b 36

c The shortest time was at least 10 minutes. The longest time was at most 70 minutes.

d



e Yes. Student's own explanation.

31.7 Histograms with bars of unequal width

Exercise 31G

1 The respective frequency densities on which each histogram should be based are:

a 2.5, 6.5, 6, 2, 1, 1.5

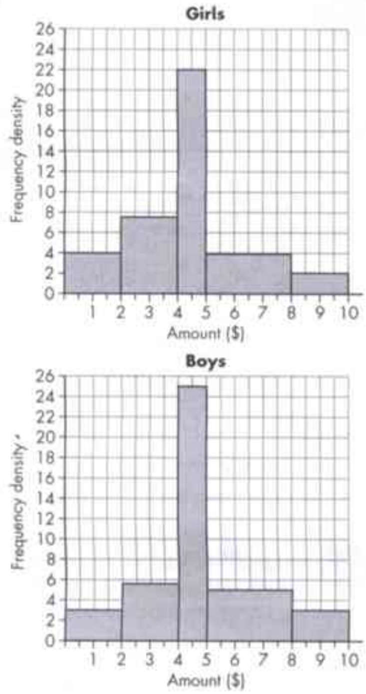
b 4, 27, 15, 3

c 17, 18, 12, 6.67

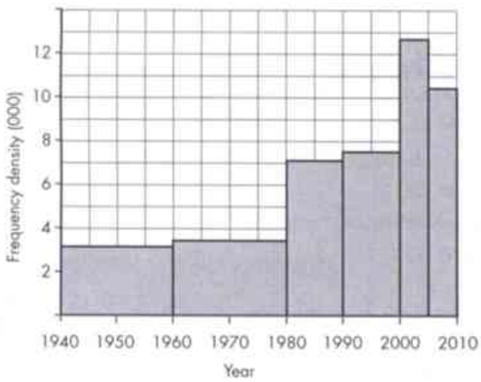
d 0.4, 1.2, 2.8, 1

e 9, 21, 13.5, 9

2 a



3



4 a 775 **b** 400

5 a

Age, y (years)	$9 < y \leq 10$	$10 < y \leq 12$	$12 < y \leq 14$
Frequency	4	12	8
Age, y (years)	$14 < y \leq 17$	$17 < y \leq 19$	$19 < y \leq 20$
Frequency	9	5	1

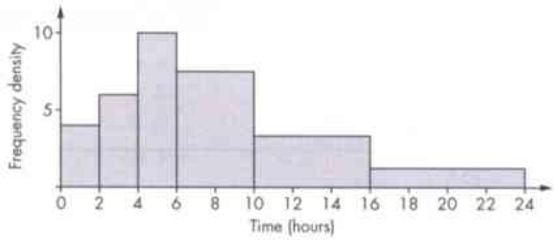
b

Temperature, t (°C)	$10 < t \leq 11$	$11 < t \leq 12$	$12 < t \leq 14$
Frequency	15	15	50
Temperature, t (°C)	$14 < t \leq 16$	$16 < t \leq 19$	$19 < t \leq 21$
Frequency	40	45	15

c

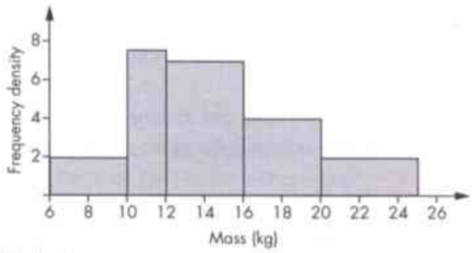
Mass, m (kg)	$50 < m \leq 70$	$70 < m \leq 90$	$90 < m \leq 100$
Frequency	160	200	120
Mass, m (kg)	$100 < m \leq 120$	$120 < m \leq 170$	
Frequency	120	200	

6 a



b 45

7 a



b 33 plants

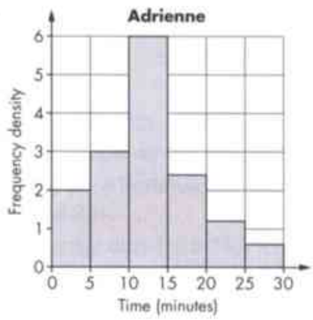
8 a

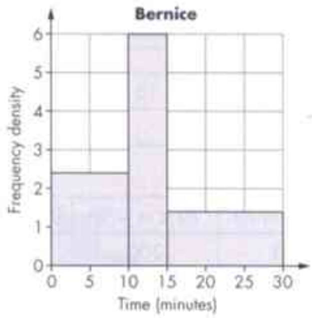
Speed, v (mph)	$0 < v \leq 40$	$40 < v \leq 50$	$50 < v \leq 60$
Frequency	80	10	40
Speed, v (mph)	$60 < v \leq 70$	$70 < v \leq 80$	$80 < v \leq 100$
Frequency	110	60	60

b 360

9 a 80
b 31.25%

10 a





b student's own description

Answers to Chapter 32

32.1 The mode

Exercise 32A

1 a 4 b 48 c −1
d $\frac{1}{4}$ e no mode f 3.21

2 a red b Sun c β d ★

3 a 32 b 6 c no
d no; boys generally take larger shoe sizes

4 a 5
b no; more than half the form got a higher mark

5 The mode will be the most popular item or brand sold in a shop.

6 a 28
b i brown ii blue iii brown
c Both students had blue eyes.

7 a May lose count.
b Put in a table, or arrange in order.
c 4

32.2 The median

Exercise 32B

1 a 5 b 33 c $7\frac{1}{2}$ d 24
e $8\frac{1}{2}$ f 0 g 5.25

2 a $2.20 b $2.25
c median, because it is the central value

3 a 5
b i 15 ii 215 iii 10 iv 10

4 a 13, Ella b 162 cm, Pat c 40 kg, Elisa
d Ella, because she is closest to the 3 medians

5 a 12 b 14

6 Answers will vary

7 12, 14, 14, 16, 20, 22, 24

8 a Possible answer: 11, 15, 21, 21 (one below or equal to 12 and three above or equal)
b Any four numbers higher than or equal to 12, and any two lower or equal
c Eight, all 4 or under

9 A median of $8 does not take into account the huge value of the $3000 so is in no way representative.

32.3 The mean

Exercise 32C

1 a 6 b 24 c 45 d 1.57 e 2

2 a 55.1 b 324.7 c 58.5 d 44.9 e 2.3

3 a 61 b 60 c 59 d Badru e 2

4 42 min

5 a $200 b $260 c $278
d Median, because the extreme value of $480 is not taken into account

6 a 35 b 36

7 a 6
b 16; all the numbers and the mean are 10 more than those in part **a**
c i 56 ii 106 iii 7

8 Possible answers: Speed – Kath, James, John, Joseph; Roberts – Frank, James, Helen, Evie. Other answers are possible.

9 36

10 24

32.4 The range

Exercise 32D

1 a 7 b 26 c 5 d 2.4 e 7

2 a 5°, 3°, 2°, 7°, 3°
b Variable weather over England

3 a $31, $28, $33
b $8, $14, $4
c Not particularly consistent

4 a 82 and 83
b 20 and 12
c Fay, because her scores are more consistent

5 a 5 min and 4 min
b 9 min and 13 min
c Number 50, because times are more consistent

6 a Isaac, Oliver, Evrim, Chloe, Lilla, Badru and Isambard
b 70 cm to 92 cm

7 a Teachers because they have a high mean and students could not have a range of 20.
b Year 11 students as the mean is 15–16 and the range is 1.

32.5 Which average to use

Exercise 32E

1 a i 29 ii 28 iii 27.1
b 14

2 a i Mode 3, median 4, mean 5
ii 6, 7, $7\frac{1}{2}$
iii 4, 6, 8
b i Mean: balanced data
ii Mode: 6 appears five times
iii Median: 28 is an extreme value

3 a Mode 73, median 76, mean 80
b The mean, because it is the highest average

4 a 150 b 20

5 a Mean b Median
c Mode d Median
e Mode f Mean

6 No. Mode is 31, median is 31, and mean is 31½.

7 a Median b Mode c Mean

8 Tom mean, David median, Mohamed mode

9 Possible answers: a 1, 6, 6, 6, 6 b 2, 5, 5, 6, 7

10 Boss chose the mean while worker chose the mode.

11 11.6

12 52.7 kg

32.6. Stem-and-leaf diagrams

Exercise 32F

1 a 40 b 75 marks c 43 marks d 71 marks
e You know that half the students got more marks than the median and half got fewer. The mode does not have such a clear use.

2 a 18 runners b 26.7 s c 4.9 s

3 a 6 people b 35 minutes c 70 minutes

4 a

2	8 9
3	4 5 6 8 8 9
4	1 1 3 3 3 8 8

b 43 cm c 39 cm d 20 cm

5 a

0	2 8 9 9 9
1	2 3 7 7 8
2	0 1 2 3

b 9 messages c 15 messages

6 a

0	7 8 9 9
1	0 2 3 4 5 8 8 9 9
2	0 3 4 4 6 8
3	1

key 2 | 3 = 23

b 18 c 24

7

	Men	Women
Number of people	41	34
Range of ages	42	33
Median age	43 years	32

8 a 8 children

b

	Girls	Boys
Number of children	25	49
Median height	148 cm	146 cm
Range of heights	40 cm	45 cm

c i 2 cm more ii 19.28 cm less

32.7 Using frequency tables

Exercise 32G

1 a i 7 ii 6 iii 6.4
b i 4 ii 4 iii 3.7
c i 8 ii 8.5 iii 8.2
d i 0 ii 0 iii 0.3

2 a 668 b 1.9 c 0 d 328

3 a 2.2, 1.7, 1.3 b Better dental care

4 a 0 b 0.96

5 a 7 b 6.5 c 6.5

6 a 1 b 1 c 0.98

7 a Roger 5, Brian 4
b Roger 3, Brian 8
c Roger 5, Brian 4
d Roger 5.4, Brian 4.5
e Roger, because he has the smaller range
f Brian, because he has the better mean

8 Possible answers: 3, 4, 15, 3 or 3, 4, 3, 15 …

9 Add up the weeks to see she travelled in 52 weeks of the year, the median is in the 26th and 27th week. Looking at the weeks in order, the 23rd entry is the end of 2 days in a week so the median must be in the 3 days in a week.

32.8 Grouped data

Exercise 32H

1 a i $30 < x \leq 40$ ii 29.5
b i $0 < y \leq 100$ ii 158.3
c i $5 < z \leq 10$ ii 9.43
d i 7–9 ii 8.4 weeks

2 a $100 < m \leq 120$ g b 10 860 g c 108.6 g

3 a 207 b 19–22 cm c 20.3 cm

4 a 160 b 52.6 min c modal group
d 65%

5 a $175 < h \leq 200$ b 31% c 193.25
d No: mode, mean and median are all less than 200 hours

6 Average price increases: Soundbuy 17.6p, Springfields 18.7p, Setco 18.2p

7 Yes: average distance is 11.7 miles per day.

8 The first 5 and the 10 are the wrong way round.

9 $740

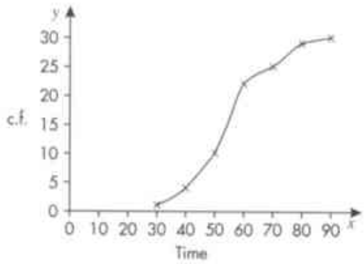
10 As we do not know what numbers are in each group, we cannot say what the median is.

32.9 Cumulative frequency diagrams

Exercise 32I

1 **a** Cumulative frequency 1, 4, 10, 22, 25, 28, 30

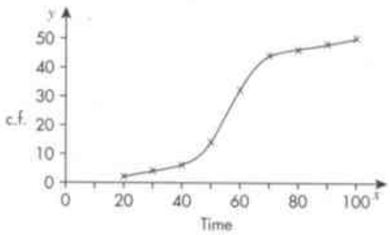
b



c 54 secs, 16 secs

2 **a** Cumulative frequency 1, 3, 5, 14, 31, 44, 47, 49, 50

b

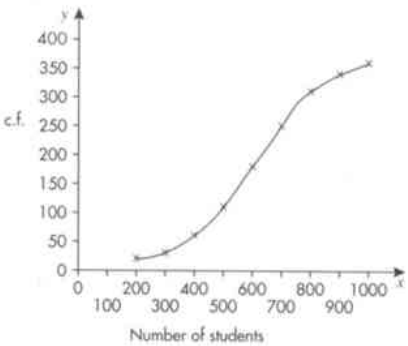


c 56 secs, 17 secs

d Pensioners, median closer to 60 secs

3 **a** Cumulative frequency 12, 30, 63, 113, 176, 250, 314, 349, 360

b



c 605 students, 280 students

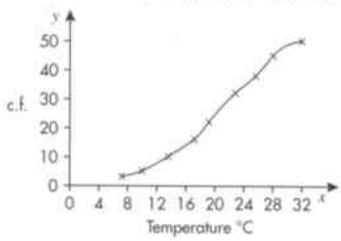
d 46–47 schools

e about 830

f about 550

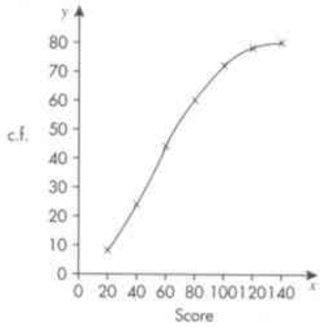
4 **a** Cumulative frequency 2, 5, 10, 16, 22, 31, 39, 45, 50

b



c 20.5°C, 10°C **d** 10.5°C

5 **a**

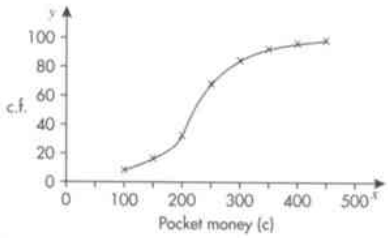


b 56, 43

c about 17.5%

6 **a** Cumulative frequency 6, 16, 36, 64, 82, 93, 98, 100

b



c 225c, 90c

d about 120 cents and about 340 cents

7 **a** Paper A 68, Paper B 57

b Paper A 28, Paper B 18

c Paper B is the harder paper, it has a lower median and a lower upper quartile.

d **i** Paper A 43, Paper B 45 **ii** Paper A 78, Paper B 67

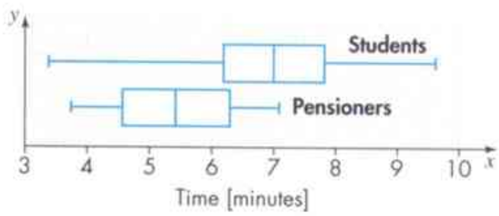
8 **a** about 40% **b** about 6 minutes

9 Find the top 10% on the cumulative frequency scale, read along to the graph and read down to the marks. The mark seen will be the minimum mark needed for this top grade.

32.10 Box-and-whisker plots

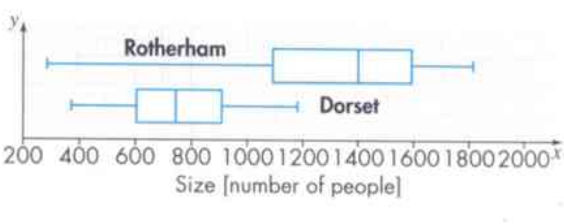
Exercise 32J

1 **a**



b Students are much slower than the pensioners. Both distributions have the same interquartile range, but students' median and upper quartiles are 1 minute, 35 seconds higher. The fastest person to complete the calculations was a student, but so was the slowest.

2 **a**

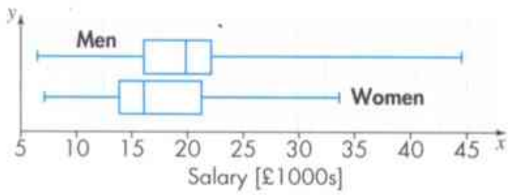


b Schools are much larger in Rotherham than Dorset. The Dorset distribution is symmetrical, but the Rotherham distribution is negatively skewed – so most Rotherham schools are large.

3 **a** The resorts have similar median temperatures, but Resort B has a much wider temperature range, where the greatest extremes of temperature are recorded.

b Resort A is probably a better choice as the weather seems more consistent.

4 **a**

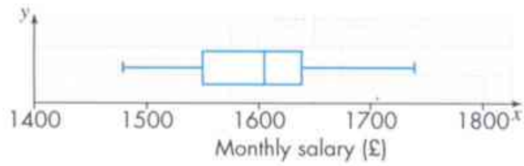


b Both distributions have a similar interquartile range, and there is little difference between the upper quartile values. Men have a wider range of salaries, but the higher men's median and the fact that the men's distribution is negatively skewed and the women's distribution is positively skewed indicates that men are better paid than women.

5 **a**

b £1605, £85

c **i**



ii Negatively

6 **a** **i** 24 min **ii** 12 min **iii** 42 min

b **i** 6 min **ii** 17 min **iii** 9 min

c Either doctor with a plausible reason, e.g. Dr Excel because his waiting times are always shorter than Dr Collins', or Dr Collins because he takes more time with each patient

7 The girls have a mean 2.6 higher than the boys. (Create grouped frequencies using the four quartiles.)

8 Many possible answers but not including numerical values: Bude (Torquay) had a higher median amount of sunshine than Torquay (Bude), Bude had a smaller interquartile range than Torquay, Bude had more sunshine on any one day.

9 **a** Symmetric **b** Negatively skewed
c Negatively skewed **d** Symmetric
e Negatively skewed **f** Positively skewed
g Negatively skewed **h** Positively skewed
i Positively skewed **j** Symmetric

10 A and X, B and Y, C and W, D and Z

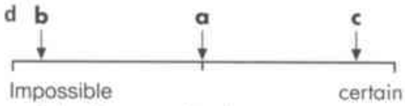
Answers to Chapter 33

33.1 Probability scale

Exercise 33A

1 **a** unlikely **b** unlikely **c** impossible
d very likely **e** even chance

2



e student's own estimate

3 student's own estimate

4 Student to provide own answers.

5 No. What happens today does not depend on what happened yesterday.

33.2 Calculating probabilities

Exercise 33B

1 **a** $\frac{1}{10}$ **b** $\frac{4}{10}$ or $\frac{2}{5}$ **c** $\frac{7}{10}$
d $\frac{1}{2}$ **e** 0

2 **a** $\frac{1}{8}$ **b** $\frac{5}{8}$ **c** $\frac{1}{2}$

3 **a** 0 **b** 1

4 **a** $\frac{1}{10}$ **b** $\frac{1}{2}$ **c** $\frac{2}{5}$ **d** $\frac{1}{5}$ **e** $\frac{2}{5}$

5 **a** $\frac{6}{11}$ **b** $\frac{5}{11}$ **c** $\frac{6}{11}$

6 **a** $\frac{1}{5}$ **b** $\frac{1}{2}$ **c** $\frac{1}{2}$ **d** $\frac{7}{10}$

7 $\frac{1}{25}$

8 **a** AB, AC, AD, AE, BC, BD, BE, CD, CE, DE
b 1 **c** $\frac{1}{10}$ **d** 6 **e** $\frac{3}{5}$ **f** $\frac{3}{10}$

9 **a** **i** $\frac{12}{25}$ **ii** $\frac{7}{25}$ **iii** $\frac{6}{25}$
b They add up to 1.
c All possible outcomes are mentioned.

10 35%

11 0.5

12 Class U

13 There might not be the same number of boys as girls in the class.

Answers to Chapter 33

33.3 Probability that an event will not happen

Exercise 33C

1 **a** $\frac{3}{4}$ **b** 0.55 **c** 0.2

2 **a** $\frac{3}{4}$ **b** $\frac{17}{20}$ **c** $\frac{19}{20}$

3 **a** **i** $\frac{1}{4}$ **ii** $\frac{3}{4}$

b **i** $\frac{3}{11}$ **ii** $\frac{8}{11}$

4 Because it might be possible for the game to end in a draw.

33.4 Probability in practice

Exercise 33D

1 **a** 0.2, 0.08, 0.1, 0.105, 0.148, 0.163, 0.1645
b 6 **c** 1 **d** $\frac{1}{6}$ **e** 1000

2 **a** 0.095, 0.135, 0.16, 0.265, 0.345
b 40 **c** No; all numbers should be close to 40.

3 **a** 0.2, 0.25, 0.38, 0.42, 0.385, 0.3974
b 8

4 **a** 6 **b** and **c** Student to provide own answers.

5 **a** Caryl, threw the greatest number of times.
b 0.39, 0.31, 0.17, 0.14
c Yes; all answers should be close to 0.25.

6 The missing top numbers are 4 and 5, the bottom two numbers are both likely to be close to 20.

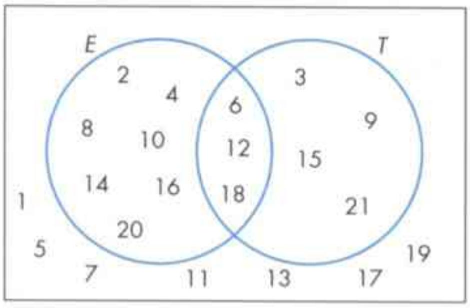
7 Thursday

8 Although he might expect the probability to be close to $\frac{1}{2}$ giving 500 heads, the actual number of heads is unlikely to be exactly 500, but should be close to it.

33.5 Using Venn diagrams

Exercise 33E

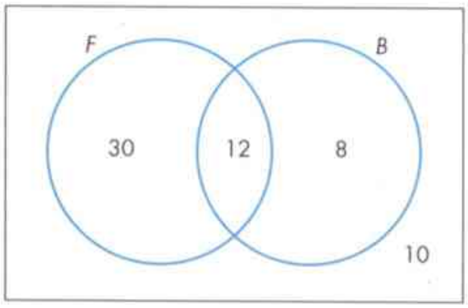
1 **a**



b **i** $\frac{10}{21}$ **ii** $\frac{7}{21} = \frac{1}{3}$ **iii** $\frac{3}{21} = \frac{1}{7}$

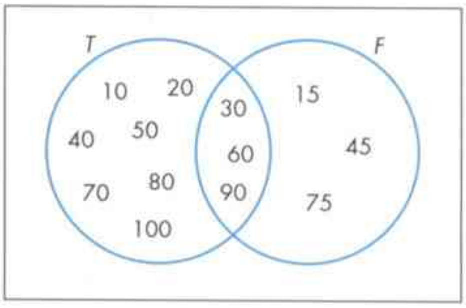
2 **a** $\frac{60}{100} = \frac{3}{5}$ or 0.6 **b** $\frac{35}{100} = \frac{7}{20}$ or 0.35
c $\frac{75}{100} = \frac{3}{4}$ or 0.75 **d** $\frac{25}{100} = \frac{1}{4}$ or 0.25

3 **a**



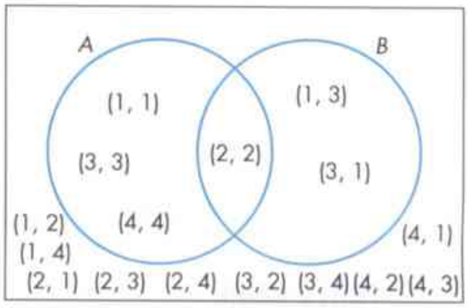
b **i** $\frac{12}{60} = \frac{1}{5}$ or 0.2 **ii** $\frac{42}{60} = \frac{7}{10}$ or 0.7 **c** $\frac{20}{60} = \frac{1}{3}$

4 **a**



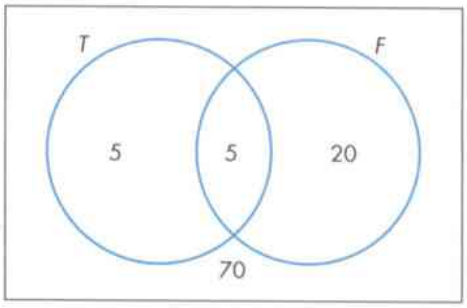
b **i** $\frac{6}{100} = \frac{3}{50}$ **ii** $\frac{3}{100}$ **iii** $\frac{87}{100}$

5 **a**



b **i** $\frac{4}{16} = \frac{1}{4}$ **ii** $\frac{3}{16}$ **iii** $\frac{1}{16}$

6 **a** 10
b



c **i** $\frac{10}{100} = \frac{1}{10}$ **ii** $\frac{25}{100} = \frac{1}{4}$ **iii** $\frac{5}{100} = \frac{1}{20}$ **iv** $\frac{70}{100} = \frac{7}{10}$

33.6 Possibility diagrams

Exercise 33F

1 a 7 b 2 and 12

c $\frac{1}{36}, \frac{1}{18}, \frac{1}{12}, \frac{1}{9}, \frac{5}{36}, \frac{1}{6}, \frac{5}{36}, \frac{1}{9}, \frac{1}{12}, \frac{1}{18}, \frac{1}{36}$

d i $\frac{1}{12}$ ii $\frac{1}{3}$ iii $\frac{1}{2}$ iv $\frac{7}{36}$

v $\frac{5}{12}$ vi $\frac{5}{18}$

2 a $\frac{1}{12}$ b $\frac{11}{36}$ c $\frac{1}{6}$ d $\frac{5}{9}$

3 a $\frac{1}{36}$ b $\frac{11}{36}$ c $\frac{5}{18}$

4

Score on second dice \ Score on first dice	1	2	3	4	5	6
6	5	4	3	2	1	0
5	4	3	2	1	0	1
4	3	2	1	0	1	2
3	2	1	0	1	2	3
2	1	0	1	2	3	4
1	0	1	2	3	4	5

a $\frac{5}{18}$ b $\frac{1}{6}$ c $\frac{1}{9}$

d 0 e $\frac{1}{2}$

5 a $\frac{1}{4}$ b $\frac{1}{2}$

c $\frac{3}{4}$ d $\frac{1}{4}$

6

Score on second spinner \ Score on first spinner	1	2	3	4	5
5	6	7	8	9	10
4	5	6	7	8	9
3	4	5	6	7	8
2	3	4	5	6	7
1	2	3	4	5	6

a 6

b i $\frac{4}{25}$ ii $\frac{13}{25}$ iii $\frac{1}{5}$ iv $\frac{3}{5}$

7 a

Score on spinner 2 \ Score on spinner 1	1	2	3	4	5	6	7	8
8	8	16	24	32	40	48	56	64
7	7	14	21	28	35	42	49	56
6	6	12	18	24	30	36	42	48
5	5	10	15	20	25	30	35	40
4	4	8	12	16	20	24	28	32
3	3	6	9	12	15	18	21	24
2	2	4	6	8	10	12	14	16
1	1	2	3	4	5	6	7	8

b $\frac{8}{64} = \frac{1}{8}$

8 $\frac{7}{36}$; a diagram will help him to see all possible outcomes

33.7 Tree diagrams

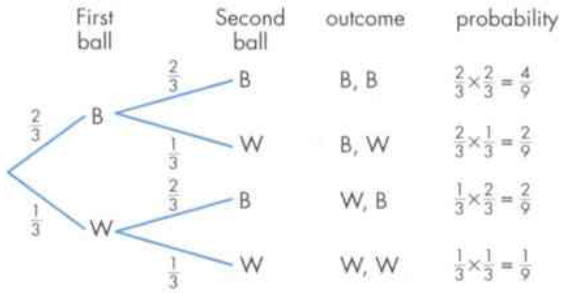
Exercise 33G

1 a $\frac{1}{4}$

b $\frac{1}{2}$

c $\frac{3}{4}$

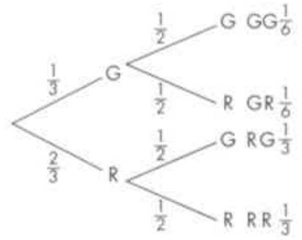
2 a



b i $\frac{1}{9}$ ii $\frac{4}{9}$ iii $\frac{8}{9}$

3 a $\frac{2}{3}$ b $\frac{1}{2}$

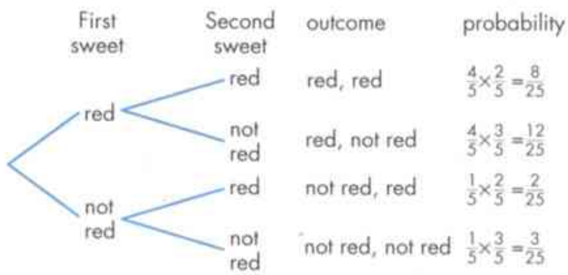
c



d i $\frac{1}{6}$ ii $\frac{1}{2}$ iii $\frac{5}{6}$

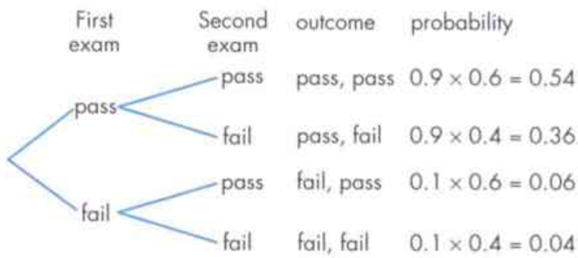
e 15 days

4 a



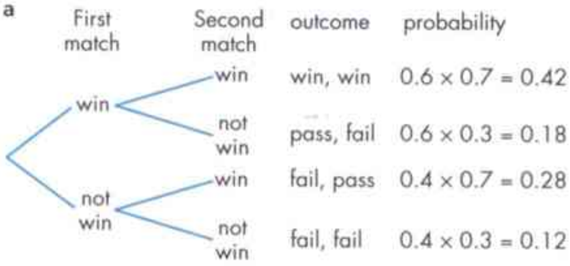
b i $\frac{8}{25}$ ii $\frac{22}{25}$ iii $\frac{3}{25}$

5 a



b i 0.54 ii 0.42

6 a



b 0.88

7 a 0.09 b 0.49 c 0.42

8 a 0.1 b 0.3 c 0.55

9 0.53

10 a i $\frac{5}{13}$ ii $\frac{8}{13}$

b i $\frac{15}{91}$ ii $\frac{4}{13}$

11 a $\frac{1}{120}$ b $\frac{7}{40}$ c $\frac{21}{40}$ d $\frac{7}{24}$

12 a $\frac{1}{9}$ b $\frac{2}{9}$ c $\frac{2}{3}$ d $\frac{7}{9}$

13 a 0.54 b 0.38 c 0.08 d 1

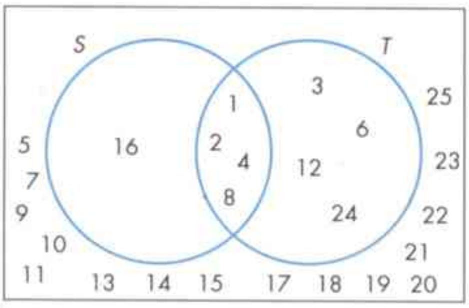
33.8 Conditional probability

Exercise 33H

1 a $\frac{55}{100} = \frac{11}{20}$ or 0.55 b $\frac{22}{55} = \frac{2}{5}$ or 0.4

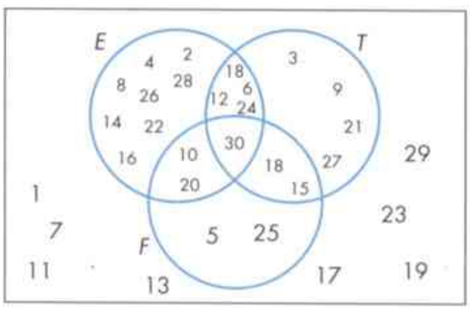
c $\frac{22}{50} = \frac{11}{25}$ or 0.44

2 a



b $\frac{4}{25}$ c $\frac{4}{8} = \frac{1}{2}$ d $\frac{4}{5}$ e $\frac{16}{20} = \frac{4}{5}$

3 a



b $\frac{10}{30} = \frac{1}{3}$ c $\frac{2}{30} = \frac{1}{15}$ d $\frac{2}{6} = \frac{1}{3}$

e $\frac{1}{6}$ f $\frac{2}{10} = \frac{1}{5}$

4 a $\frac{85}{120} = \frac{17}{24}$ b $\frac{30}{120} = \frac{1}{4}$ c $\frac{55}{85} = \frac{11}{17}$

d $\frac{50}{85} = \frac{10}{17}$ e $\frac{20}{35} = \frac{4}{7}$

5 a

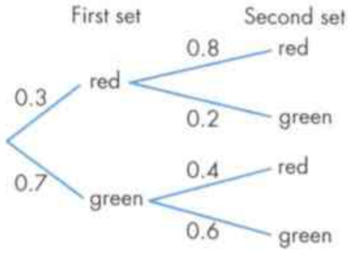
Second dice						
6	7	8	9	10	11	12
5	6	7	8	9	10	11
4	5	6	7	8	9	10
3	4	5	6	7	8	9
2	3	4	5	6	7	8
1	2	3	4	5	6	7
	1	2	3	4	5	6

First dice

b $\frac{5}{36}$ c $\frac{2}{5}$ d $\frac{11}{36}$ e $\frac{2}{11}$

6 a $\frac{6}{16} = \frac{3}{8}$ b $\frac{2}{6} = \frac{1}{3}$ c $\frac{2}{4} = \frac{1}{2}$

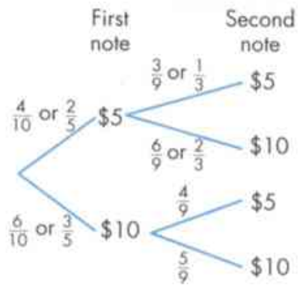
7 a



b 0.24 c 0.42 d 0.34

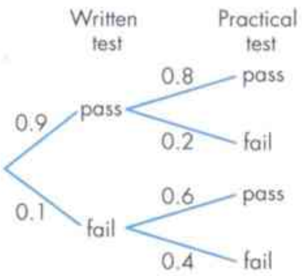
8 a $\frac{4}{10}$ or $\frac{2}{5}$ b $\frac{3}{9}$ or $\frac{1}{3}$

c



d i $\frac{2}{15}$ ii $\frac{8}{15}$ iii $\frac{1}{3}$

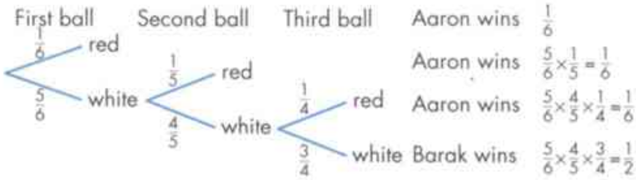
9 **a**



b 0.72 **c** 0.24

10 **a** $\frac{3}{28}$ **b** $\frac{5}{14}$ **c** $\frac{1}{56}+\frac{5}{28}=\frac{11}{56}$

11 The tree diagram looks like this



Aaron wins if the first ball or the second or the third is red.

The probability of this is $\frac{1}{6}+\left(\frac{5}{6}\times\frac{1}{5}\right)+\left(\frac{5}{6}\times\frac{4}{5}\times\frac{1}{4}\right)$

$=\frac{1}{6}+\frac{1}{6}+\frac{1}{6}=\frac{3}{6}=\frac{1}{2}$

Or: Barak wins if there are 3 white balls and the probability of this is $\frac{5}{6}\times\frac{4}{5}\times\frac{3}{4}=\frac{1}{2}$

Hence the probability that Aaron wins is $1-\frac{1}{2}=\frac{1}{2}$

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